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Chasing the ohm: a journey from mercury to the quantum realm

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Abstract

The evolution of the ohm as the unit of electrical resistance mirrors the very progress of physics itself. Its realisation has journeyed from mercury columns to mechanical constructs. It now finds permanence in the quantum Hall effect, where semiconductor devices, and their successors in graphene and topological matter, embody Nature's immutable constants.

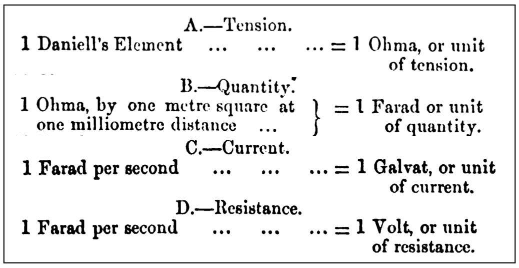
1 Cgs and international units

What's in a name? For over 150 years the ohm stood as the measurement unit of electric resistance for more than 150 years. What we mean by "a resistance of x ohm", however, evolved substantially, both in meaning and in magnitude, over time. The story of such evolution intertwines with breakthroughs in fundamental physics, material science, and electrical engineering.

The first metric system of units that evolved was the *cgs*, centimetre-gram-second. Originally proposed in 1832 by Gauss (in the variant millimetre-milligram-second) it was quickly adopted and recommended in 1873 by the British Association for Advancement of Science (BAAS), the most important scientific body of the time. As the cgs name suggests, in the system there is no space for electromagnetic base units. All quantities of electromagnetism are measured in mechanical units, chosen to be consistent with the mechanical manifestations of electromagnetism. Two options were available. The *electrostatic cgs* was based on the Coulomb's force law between charged particles, with resistance measured in $\mathrm{cm}^{-1}\mathrm{s}$, or statohm); and the *electromagnetic cgs*, which has the Ampère's force between current-carrying conductors as foundation, with the unit $\mathrm{cm\,s}^{-1}$, or abohm.

Quantity	electrostatic cgs		electromagnetic cgs	
Current	statampere	336 pA	abampere	10 A
Voltage	statvolt	300 V	abvolt	10 nV
Resistance	statohm	899 GΩ	abohm	1 nΩ
Capacitance	cm	11 pF	abfarad	1 MF
Inductance	—	—	abhenry	1 nH

Table 1 Some cgs units for electromagnetic quantities. An approximate conversion to the corresponding SI units is also given. The statohm and abohm are related by the huge numerical factor $c^2 \approx 8.987\,552 \times 10^{20}$.



A.—Tension.

1 Daniell's Element ... ... = 1 Ohma, or unit of tension.

B.—Quantity.

1 Ohma, by one metre square at one milliometre distance ... } = 1 Farad or unit of quantity.

C.—Current.

1 Farad per second ... ... = 1 Galvat, or unit of current.

D.—Resistance.

1 Farad per second ... ... = 1 Volt, or unit of resistance.

Fig. 1 Clark and Bright proposal for the names of four practical electrical units [1]. It is worth noting that Ohma was chosen as the unit of voltage, and Volt as the unit of resistance.

Quantity	international unit	definition (practical realisation)
Current	ampere	The unvarying current which, when passed through a solution of silver nitrate in water, deposits silver at the rate of 0.001 118 00 grams per second
Voltage	volt	$\frac{1000}{1434}$ of the electromotive force of a Clark cell at a temperature of 15 °C
Resistance	ohm	The resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14.4521 g in mass, of a constant cross-sectional area and of the length of 106.3 cm

Table 2 The international units, as established in 1893.

Table 1 shows the sizes of some cgs units, and displays how they are unusably large or small for everyday measurements. A need for *practical* electromagnetic units, that could be accurately reproduced using standardized procedures and simple devices, quickly emerged. A first proposal, in 1861 from Clark and Bright [1] (see Fig. 1), introduced the Ohma, Farad, Galvat and Volt—names playfully derived from the founding fathers of electromagnetism. The nomenclature was accepted by the BAAS in 1874

Fig. 2 Mercury ohm according to the 1898 German Imperial Law "Concerning Electrical Units of Measurement", Implementation Regulation for the International Ohm. Museum display of the Physikalisch-Technische Bundesanstalt, courtesy of H. Scherer.

(with some rearrangement: the Ohma was shortened to Ohm and assigned to the unit of resistance). Such names were made permanent by the International Electrical Congress (IEC, now International Electrotechnical Commission) in its meetings of 1881 and 1893 [3], where the international ohm, together with the international ampere, volt, coulomb and farad, were defined as multiples of the corresponding cgs units.

The IEC also defined *practical representations* of the units through electrical devices [3], as shown in Table 2. The international ohm was defined as the resistance of a column of mercury of given weight and length, see Figure 2. Because the resistivity of pure metals is highly sensitive to impurity, mercury—the purest available metal, thanks to vapour distillation—became the natural choice.

2 The SI and the absolute realisation of the ohm

International units, although designed to be multiples of cgs units, were in fact independent of them. Over time, with the improvement of experimental techniques, the conversion factors between absolute and international units started to deviate from the intended simple powers of ten. Further, the definition of international units was not *coherent*: for a given realisation of all the three units, Ohm's law is not automatically satisfied.

The very same experimental improvements made direct, *absolute* realisations more feasible over time. As said, electromagnetic units were ultimately mechanical in their definition, so mechanical experiments had to be performed. Dozens of possible absolute realisations of the ohm were considered [4], based on different principles: electrodynamic force, induced voltage in moving coils, calorimetric... Among many ingenious designs, the *calculable inductor* emerged as the most elegant and accurate, either in their mutual or self-inductance (see Figure 3) variants. The measurement of a resistor

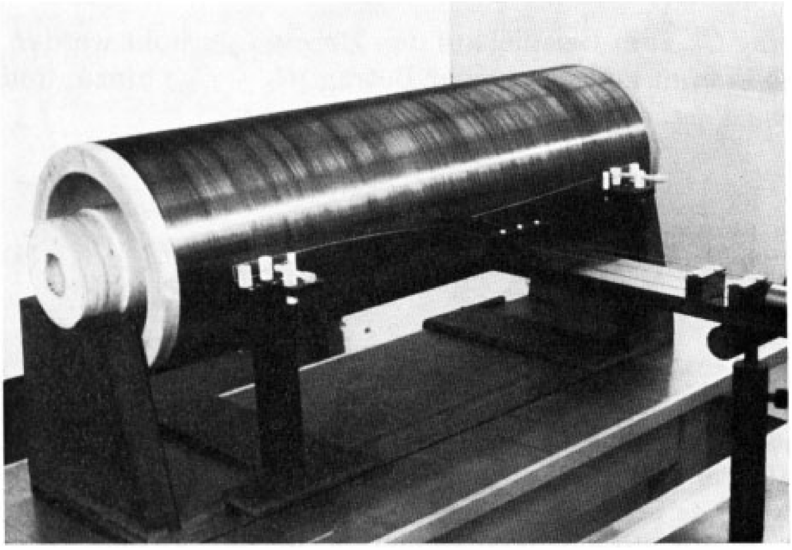


Fig. 3 Calculable inductor. The coil, wound in a single layer on a non-magnetic ceramic core, has 945 turns with a diameter of 310 mm and a length of 709 mm. Adapted from [2].

versus the calculable inductor required one or more dedicated ac impedance bridges, operating at a known frequency. The uncertainty of such absolute realisation was ultimately related to that of the cumbersome mechanical measurements of the 3D helical geometry of the inductor, and was pushed down to 2×10^{-6} in the most recent experiment (1968 [2]).

In 1956 a totally unexpected theoretical result in electrostatics—the first since several decades—opened a new way to realise the farad and the ohm. The *Thompson-Lampard theorem* showed that, for an arrangement of four cylindrical electrodes in a square, the capacitance per unit length between opposite electrodes had the simple expression $C = \epsilon_0 \log 2/\pi = 1.95354904\ldots\,\mathrm{pF\,m^{-1}}$. The value, related to the vacuum permittivity ϵ_0 by an exact numerical constant, is robust with respect to deviations from an ideal symmetric geometry. The measurement of a straight segment length, performed with optical interferometry, is sufficient to calculate a capacitance value. A very small value indeed, which can nevertheless be scaled to higher capacitance, and then to resistance, with the coaxial transformer impedance bridges whose development flourished in the same years. The uncertainty of the absolute realisation of the ohm could now be pushed down to parts in 10^8. In 1960 the adoption of the International System of units, with the choice of the ampere, defined in mechanical terms, as the base unit, consolidated the need of realisation of the electrical units with electromechanical experiments, an specifically the ohm with the calculable capacitor.

3 A mechanical and a quantum ohm, together

In the 20th century the ultimately quantum nature of all electromagnetic phenomena was progressively better understood. Nevertheless, the discovery of the quantum Hall effect, by Klaus von Klitzing in 1980 came as a surprise [1]. Consider a solid-state device where charge carriers are constrained in a two-dimensional layer, at a sufficiently low temperature and with a static magnetic field applied perpendicularly to the surface. The Hall resistance, the ratio between the Hall voltage and the current flowing in the device, becomes quantized. The quantum Hall resistance (QHR) value $R_\mathrm{H} = R_\mathrm{K}/i$ is a unit fraction $1/i$ of the resistance quantum $R_\mathrm{K} = h/e^2 \approx 25\,812\,\Omega$, a fundamental constant of nature being a combination of the Planck constant h and the elementary charge e. The robustness of the effect (in proper samples currents in the tens of microampere range can be applied) allows one to perform measurements to the highest accuracy. Tests of the *universality* of the quantum Hall resistance, its independence of the specific device employed, is established to an accuracy of about ten parts per trillion (10^{-12}). QHR experiments were much easier to perform than the delicate calculable capacitor experiment, and were installed in most major national metrology institutes as a representation of the ohm to establish the national resistance standards.

4 The 1990 conventional ohm

The adoption of QHR as a resistance standard raised a problem. It allowed a representations of the ohm with a reproducibility two to three orders of magnitude better than the uncertainty of R_K in SI ohm—the latter becoming the major limitation to the development of resistance metrology. A solution was to introduce, in 1990, a *conventional value* $R_{\mathrm{K}-90} = 25\,812.807\,\Omega$ for the von Klitzing constant, a value chosen to match the best measurements performed in terms of the calculable capacitor. This conventional value (and the corresponding one for the other quantum metrology experiment, the Josephson voltage standard) defined a system of *conventional electrical units*: Ω_{90}, A_{90}, V_{90}, etc. All electrical measurements and calibrations since then were performed in conventional units.

A century later, history repeated itself: the 1990 conventional ohm revived the same problems of the 1893 international ohm. After its introduction further, more accurate determinations of R_K were made: the conventional ohm was shown to be no longer exactly equal to the to the SI ohm. The conventional system of units became, like the 1893 one, noncoherent.

5 The quantum SI

The existence of a parallel system of conventional electrical units, drifting from the corresponding SI one, was recognized as a major problem of the SI.[2] The need of a revision was recognized at the turn of this century, and several solutions proposed and discussed over two decades.

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[1] A detailed history of the discovery of the quantum Hall effect is narrated in excellent recent reviews, e. g. [5], and will not be told again here.

[2] The main problem was the suspected instability of the *International Prototype Kilogram*, which defined the unit of mass.

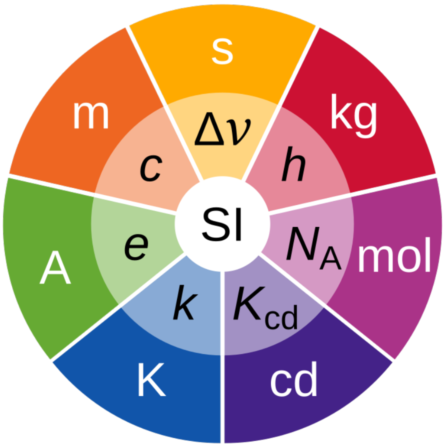

Fig. 4 A pictorial representation of the SI, highlighting the base units and the fundamental constants that enter their definition.

The revision of the SI was approved by the 26th General Conference of Weights and Measures in November 2018 [6] and implemented on 20 May 2019, the *implementation day*. The present SI, see Figure 4, is based on a set of seven constants with exactly specified numerical values. The elementary charge e has the exact value $1.602\,176\,634\times 10^{-19}$ C, and the Planck constant $6.626\,070\,15\times 10^{-34}$ J s. Consequently, The resistance quantum also has the exact value $R_\mathrm{K} = h/e^2 = 25812\ldots\,\Omega$. The QHR is a realisation of the ohm, and all electromagnetic units can be similarly realised with quantum experiments.

6 New physics for resistance metrology

The von Klitzing 1980 experiment was performed on a silicon MOSFET device with a current of 1 µA. The applied magnetic field was 18 T at a temperature of 1.5 K. The two-dimensional electron transport was forced by the electrostatic field generated by a voltage of 23 V applied to the MOSFET gate. The relevance for metrology of the original QHR experiment was immediately recognized, and efforts made to find better QHE devices that could simplify the implementation of QHR experiments for metrology purposes. A major improvement was the development of GaAs heterostructure devices in the 1980s. These devices allow to increase the applied current to tens of µA, moderately reduce the magnetic field needed (to around 10 T, in the range of a standard superconducting magnet) and do not need a gate voltage. The QHR of GaAs devices is presently *the* basis of resistance metrology worldwide.

Graphene, the first two-dimensional material, was discovered in 2005. QHR in graphene is more robust than in GaAs, and manifests at higher temperature and lower magnetic fields. After several years of development it is now possible to access QHR in stable devices, at temperatures of several K, by applying a magnetic field of 5 T or lower, and measurement currents in the 100 µA range [7]. These experimental

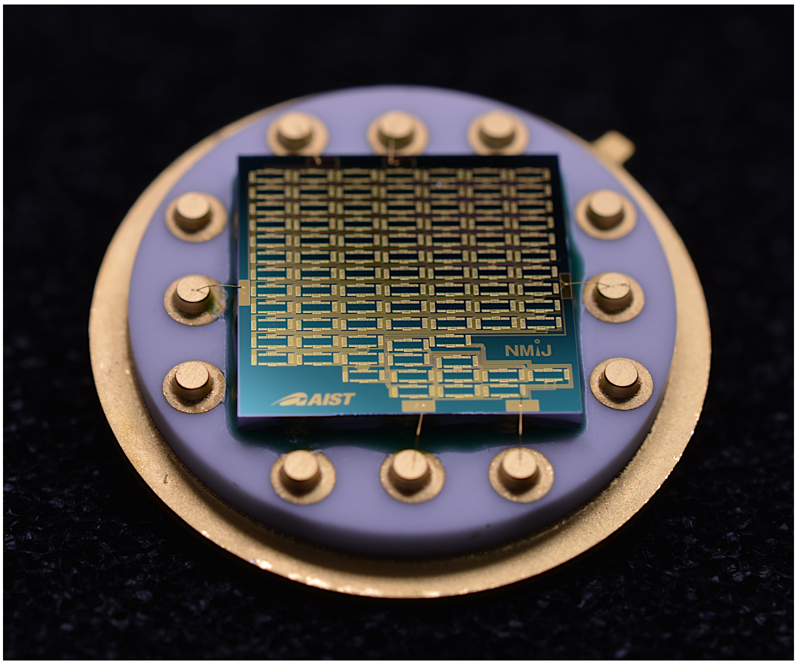

Fig. 5 A GaAs quantum Hall array resistance standard (QHARS). The electrical network of 88 QHE elements gives a 1 MΩ output resistance. Courtesy of N.-H. Kaneko, National Metrology Institute of Japan.

conditions can be achieved in a tabletop dry cryostat refrigerator, thus providing permanent access to the SI ohm with a compact experiment.

The quantum of resistance is an invariant given by Nature, luckily in a resistance range that is easy to access experimentally and of interest for metrology applications. Calibration centers, however, perform calibrations of commercial resistance standards and meters at simple decadal values—such as 1 Ω or 10 kΩ. These values can be obtained by scaling QHR with commercial resistance ratio bridges, but some accuracy is lost in the process.

QHR devices are fabricated with semiconductor foundry techniques, and integration is possible. *Quantum Hall array resistance standards* (QHARS) are integrated circuits of several QHR elements that display perfect fractions (either smaller or greater than 1) of R_{K}. Figure 5 shows an example of a 1 MΩ GaAs QHARS. A recent development is a single graphene device which outputs 100 Ω, 1 kΩ and 10 kΩ resistance values, suitable for performing calibration of artifact standards with commercial room-temperature resistance bridges [8].

Electrical metrologists envision a future where all quantum standards converge to became a single *quantum multimeter*. An integration of QHR with the other quantum effects (Josephson voltage in particular) in a single environment, possibly based on a single chip, to be the basis of an instrument that can measure voltage and current in the dc and ac regime, resistance and impedance, directly traceable to the quantum

SI units. Several steps are being made, in particular on the exploitation of quantum devices in the ac regime. However, the high magnetic field required by QHR—fatal to the Josephson effect, which dies already at very small magnetic fields—remains a formidable obstacle.

A quantized resistance is not exclusive of the QHR. It is related to electron transport without scattering and manifests in other systems, such as quantum point contacts. Stable quantized resistance values have been observed, at room temperature and without applied magnetic field, in *memristors* [9], which also offer resistance programming capabilities. The quantization accuracy is presently insufficient to consider these devices a viable alternative to QHR.

A recent breakthrough is given by the *quantum anomalous Hall effect* (QAHE) in magnetically-doped topological insulator materials. The devices, after an initial magnetisation cycle, display QHR even at zero magnetic field. A very recent experiment [10] shows the universality of QHE in these devices with an accuracy in the 10^{-9} range. A quantum current generator proof of principle, based on a QAHE device and on a Josephson voltage array hosted in the same cryostat, has been published [11]. Presently, the drawbacks of QAHE devices are the need of a very low (100 mK or below) temperature, and very low measurement currents (in the 100 nA range). Ongoing research into these materials, which have many applications, fuels the metrologist's optimism that today's limitations in the QAHE exploitation will be overcome in the near future.

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