Determination of the position error in automatic mass comparators

Measurement 258 (2026) 119357

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Determination of the position error in automatic mass comparators

Andrea Malengo *, Davide Torchio

Istituto Nazionale di Ricerca Metrologica, INRiM, Strada delle Cacce 91, 10135 Torino, Italy

ARTICLE INFO

Keywords:
Mass standards calibration
Balance calibration
Automatic mass comparator
Position error

ABSTRACT

In automatic mass comparators, weighing is carried out automatically without operator intervention, eliminating effects associated with manual handling of mass standards. This allows for significantly more repeatable measurements compared to those obtained manually. Comparisons are performed by automatically exchanging mass standards on the weighing pan, with the standards typically placed on turntables or magazines.

Automatic mass comparators may be affected by a particular systematic error inherent to the weighing system, where the measured mass difference between two mass standards depends on their placement position within the turntable or magazine. This effect is commonly referred to as “position error”. This error can increase over time and lead to significant measurement errors if not properly evaluated, making regular comparator assessment a recommended practice.

The evaluation of such errors is particularly challenging when the number of positions is large, and methods for assessing this effect are poorly documented in the literature or calibration guides. This paper proposes a model to determine these position errors by performing comparisons between mass standards placed at different positions, and applying the weighted least squares method for their estimation. The proposed method provides a practical solution for correcting position errors, leading to improved measurement accuracy and reliability in automatic mass comparators.

1. Introduction

In the last twenty years, significant progress has been made in mass metrology, with the most notable advancement being the redefinition of the kilogram [1]. Additionally, weighing equipment has improved considerably. It is now common for laboratories in national metrological institutes to be equipped with comparators offering very high resolution. Current commercial comparators can achieve resolutions of 0.1 µg across the entire measuring range up to 1 kg, whereas only about ten years ago, most laboratories used balances with 0.1 µg resolution exclusively for weights up to 5 g.

Another important improvement has been the automation of the balances. Automatic mass comparators are essentially of two types: those with a turntable, where weighing is performed through turntable movement, and robot-type systems, where a robotic arm transfers weights from a magazine and places them sequentially on the weighing pan [2–7]. Turntable-based systems are typically compact with a limited number of positions, while robot-type systems are equipped with magazines that can accommodate a large number of weights, up to one hundred or more.

The automation of weighing has undoubtedly contributed to improved measurement accuracy. Automatic mass comparators have increased the efficiency of the weighing process, but most importantly, the repeatability of measurements achieved by automated systems is incomparable to that obtained manually. However, very high resolution and good repeatability in a mass comparator are necessary but not sufficient conditions for obtaining traceable measurements with low uncertainties.

As with traditional balances, automatic comparators are subject to various effects that can significantly contribute to measurement results and, if not properly corrected, can lead to systematic errors. Among the most important are sensitivity, linearity, and effects due to temperature variations. These effects are well known to metrologists, as they are also typical of non-automatic balances [8–10]. However, a less well-known effect is the so-called “position error”, which refers to the phenomenon where the mass difference measured by the balance between two mass standards depends on the position where the standards are placed on the turntable or in the magazine.

This problem can be attributed to the typical effect on balance readings caused by eccentric loading. In conventional balances, the

* Corresponding author.
E-mail addresses: a.malengo@inrim.it (A. Malengo), d.torchio@inrim.it (D. Torchio).

https://doi.org/10.1016/j.measurement.2025.119357
Received 1 August 2025; Received in revised form 15 September 2025; Accepted 14 October 2025
Available online 16 October 2025

reading is influenced by the position where the load is placed on the pan. A similar phenomenon occurs with position error in automatic mass comparators. During the weighing by substitution in automatic comparators, the weights being compared are placed on the comparator pan at positions that depend on their location on the turntable or in the magazine. As with the eccentricity effect in traditional balances, the position error also depends on the load value: the position error increases with increasing load value for the same weight displacement on the pan.

This effect is well known to manufacturers, and various techniques are employed to minimize such errors. The most common approach is the use of a gimbal-mounted hanging weighing pan; however, constructing a perfect, frictionless gimbal system is challenging. Other problems arise from the imperfect parallelism of the spring beams of the measuring cells and the parallelism between the balance pan, the weight carrier, and the plane on which the weight is placed, whether it be the magazine or the turntable [11–14].

These errors are strongly related to the mechanical precision of the balance construction and can worsen over time due to wear of mechanical parts. It is therefore necessary to monitor this effect over time. The effect leads to highly repeatable systematic errors, making determination critical since no degradation of repeatability is detected during routine measurements.

Specific methods for assessing this effect are poorly documented in the literature or calibration guides. To assess this phenomenon, the standard approach involves testing the mass difference between weights by placing them in different positions, verifying that the measurement variations remain within acceptable limits such that the result can be considered independent of the positions used.

Ideally, these variations should remain within the typical repeatability of the mass comparator. However, it is acceptable to obtain results with discrepancies greater than the balance repeatability, provided these discrepancies are not excessive. In such cases, an additional uncertainty contribution must be considered, as mentioned in [15].

When only one position is affected by position error, it is straightforward to evaluate the error. However, when multiple positions are affected, the determination becomes more complex. In such cases, the usual approach of increasing the uncertainty for all positions may not be feasible, as the additional uncertainty could become excessive and may not meet the metrological requirements specified by the laboratory. Under these circumstances, the solution might be to request technical service from the manufacturer to reduce this effect.

An alternative approach would be to perform the measurements between the standards by exchanging the standards among all possible positions (those necessary to accommodate the standards involved) and finally calculate the mean values. However, this method would be extremely time-consuming and impractical for routine measurements. This approach would be equivalent to the transposition weighing method that was used in the past with equal-armed beam balances [9].

Ultimately, the optimal long-term solution is to accurately characterize and quantify these position errors so that the weighing results can be corrected through validated correction factors, preserving both measurement accuracy and operational efficiency.

To address this problem, a measurement model capable of estimating these position errors has been developed. The method identifies the error for each position of the mass comparator. It allows correction of the weighing without significantly affecting the measurement uncertainty while maintaining full metrological traceability.

Table 1
Example of weighing scheme for balance with four positions and two weights.

Row	m_A	m_B	Δp_1	Δp_2	Δp_3	Δp_4
1	1	−1	1	−1	0	0
2	1	−1	1	0	−1	0
3	1	−1	1	0	0	−1
4	1	−1	0	1	−1	0

2. Measurement model

The position error causes variations in the weighing results depending on the positions in which the standards are placed (turntable or magazine). This effect can be modelled by assigning a position-dependent error to the balance reading.

Assuming that a weight is placed at position p_i with position error Δp_i, the reading in this position is given by:

$$L_i = m_w + \Delta p_i \tag{1}$$

where m_w is the so-called apparent mass associated with the weight.

Comparing two mass standards at positions p_i and p_j, from the difference of the readings L_i and L_j we obtain the apparent mass difference between the two weights Δm_w and the error $E_{i,j}$ associated with these two positions p_i and p_j, which is given by:

$$E_{i,j} = \Delta p_i - \Delta p_j \tag{2}$$

and consequently also applies to:

$$E_{j,i} = \Delta p_j - \Delta p_i = -E_{i,j} \tag{3}$$

Following this approach, comparing two mass standards of mass m_A and m_B, placed at position p_i and p_j, respectively, neglecting the correction for the acceleration of gravity, the weighing equation is:

$$m_A - m_B + \Delta p_i - \Delta p_j = \Delta m_w \left(1 - \frac{\rho_a}{\rho_{mAdj}}\right) + \rho_a (V_A - V_B) \tag{4}$$

where:

− ρ_a is the air density;
− ρ_{mAdj} is the density of the balance adjustment weight;
− V_A and V_B are the volumes of the mass standards m_A and m_B, respectively.

In general, in an automatic mass comparator, several weights m_i (of the same nominal mass) can be involved, with $i = A, B, C, \cdots$, which can be placed in different positions, and several mass comparisons can be performed.

For each comparison of weights, an equation such as equation (4) is obtained. Therefore, performing several comparisons leads to an overdetermined system of equations.

In matrix notation, such a system of equations can be conveniently represented using the model:

$$\boldsymbol{X_W m} = \boldsymbol{y_W} + \boldsymbol{\varepsilon} \tag{5}$$

where:

− $\boldsymbol{m} = [m_A, m_B, m_C, \cdots, \Delta p_1, \Delta p_2, \Delta p_3, \cdots]^T$ is the column vector of the measurands (the unknown parameters): mass values of the weights and of the position errors
− $\boldsymbol{X}_W$ is the design matrix, whose entries are $+1$, -1 or 0, according to the role played by each weight and the associated position (LHS of equation (4))
− $\boldsymbol{y}_W = [\Delta m_1, \Delta m_2, \cdots, \Delta m_q]^T$ is the column vector of the mass differences between the standards involved in each equation (RHS of equation (4))
− $\boldsymbol{\varepsilon}$ is the vector of measurement errors

In contrast to the traditional method used to solve systems of weighing equations [16], the parameter vector $\boldsymbol{m}$ includes not only the masses, but also the position errors.

In order to solve the system of equations (5), multiple weighings

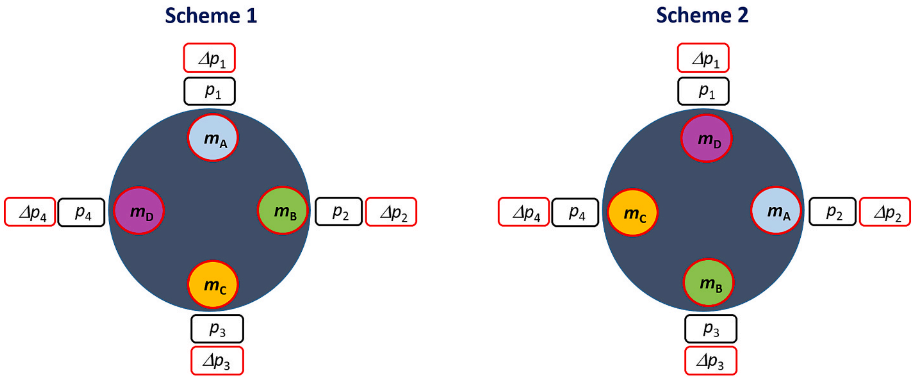

Fig. 1. Diagram of the two weighing schemes for a comparator with four positions and four weights.

Table 2
Enhanced weighing scheme for a balance with four positions and four weights.

Row	m_A	m_B	m_C	m_D	Δp_1	Δp_2	Δp_3	Δp_4
1	**1**	**−1**	**0**	**0**	1	−1	0	0
2	**1**	**0**	**−1**	**0**	1	0	−1	0
3	**1**	**0**	**0**	**−1**	1	0	0	−1
4	**0**	**1**	**−1**	**0**	0	1	−1	0
5	**0**	**1**	**0**	**−1**	0	1	0	−1
6	**0**	**0**	**1**	**−1**	0	0	1	−1
7	**−1**	**0**	**0**	**1**	1	−1	0	0
8	**0**	**−1**	**0**	**1**	1	0	−1	0
9	**0**	**0**	**−1**	**1**	1	0	0	−1
10	**1**	**−1**	**0**	**0**	0	1	−1	0
11	**1**	**0**	**−1**	**0**	0	1	0	−1
12	**0**	**1**	**−1**	**0**	0	0	1	−1

must be carried out involving the differences between the various weights. These must be obtained in different ways by placing the weights in different positions, so as to obtain a number of linearly independent equations at least equal to the number of unknown parameters.

For example, a comparator with four positions could be verified with only two weights, in this case it would be $\boldsymbol{m} = [m_A, m_B, \Delta p_1, \Delta p_2, \Delta p_3, \Delta p_4]^T$, with a very limited number of displacements, for instance, four weighings with the $\boldsymbol{X}_W$ matrix shown in Table 1. However, a limited number of weighings leads to an inefficient system of equations.

The accuracy in the determination of the error of each position is related to the number of times that position is used in the weighings. It should also be noted that the stability of the mass of the standards used in the test has a significant influence on the result. Therefore, a more robust approach is to perform the test with several weights and to exchange the weights at several different positions.

For example, considering four weights m_A, m_B, m_C, m_D on a mass comparator with four positions p_1, p_2, p_3, p_4, it results $\boldsymbol{m} = [m_A, m_B, m_C, m_D, \Delta p_1, \Delta p_2, \Delta p_3, \Delta p_4]^T$. Following the approach shown in Fig. 1, it is possible to determine the mass differences according to the matrix in Table 2. In a first scheme, from row No.1 to row No.6, by placing m_A in position p_1, m_B in position p_2 and so on, performing comparisons between positions $p_1 - p_2$, $p_1 - p_3$, $p_1 - p_4$, etc., to determine all differences between the four weights, as shown in Table 2. Subsequently, for the second scheme, from row No. 7 to row No. 12, the same mass differences can be determined with the weights placed in different positions from the first scheme, for example by moving m_A to position p_2, m_B to position p_3, and so on.

From Table 2, it can be seen that the matrix associated with the position errors (right-hand side of Table 2) remains the same for the two schemes, while the matrix associated with the mass parameters (left-hand side of Table 2) depends on the positions of the weights. Similarly, the equivalent system of equations could be written by keeping the scheme of position errors unchanged and varying that of mass differences, by appropriately changing the order of measurements Δm_i.

To solve the system of equations and determine the values of the mass standards, one constraint has to be assigned to the mass parameters, that is, the value m_R of one reference standard has to be established [16]. Similarly, to determine the values of the position errors, it is also necessary to assign a constraint Δp_R for one of the parameters Δp_i. Since what we are interested in are not the values of the errors Δp_i, but the differences $E_{i,j}$ (equation (2)), we can set a constraint on an arbitrary position and fix an arbitrary value Δp_R. For convenience this could be position p_1, with $\Delta p_1 = \Delta p_R = 0$.

The constraints can be assigned by implementing the problem using Lagrange multipliers or the Gauss Markov method, which are the two most commonly used methods for problems of this type. Since both constraints m_R and Δp_R will be attributed uncertainties that may be significantly different, it is more appropriate to use the Gauss Markov method [17,18]. Following this approach, the vector $\boldsymbol{y_W}$ and the matrix $\boldsymbol{X_W}$ have to be modified.

In order to set one constraint m_R for one of the parameters m_i and the constraint Δp_R for the parameters Δp_i, for the vector $\boldsymbol{y_W}$, the two values m_R and Δp_R are added, for example to the bottom of the vector $\boldsymbol{y_W}$. For the $\boldsymbol{X_W}$ matrix, two rows are added, following this example, to the bottom of the matrix, such that the values for m_R and Δp_R are established to the corresponding parameters.

For example, assigning $m_A = m_R$ and $\Delta p_1 = \Delta p_R$, the new matrices become:

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{X_W} \\ 1\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,1\,0\,0\,0 \end{pmatrix} \tag{6}$$

$$\boldsymbol{y} = \begin{pmatrix} \boldsymbol{y_W} \\ m_R \\ \Delta p_R \end{pmatrix} \tag{7}$$

and the new system of equation is:

$$\boldsymbol{Xm} = \boldsymbol{y} + \boldsymbol{\varepsilon} \tag{8}$$

The solution is provided by:

$$\hat{\boldsymbol{m}} = \left(\boldsymbol{X}^T \boldsymbol{\Psi}_{\boldsymbol{y}}^{-1} \boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \boldsymbol{\Psi}_{\boldsymbol{y}}^{-1} \boldsymbol{y} \tag{9}$$

where $\boldsymbol{\Psi_y}$ is the covariance matrix associated to the vector $\boldsymbol{y}$. A detailed description of how to construct this matrix is provided in [18,19].

The covariance matrix $\boldsymbol{\Psi}_{\hat{m}}$ for $\hat{\boldsymbol{m}}$, from which the uncertainties for the estimated parameters $\hat{m}_i$ and $\widehat{\Delta p}_i$ are obtained, is given by:

$$\boldsymbol{\Psi}_{\hat{m}} = \left(\boldsymbol{X}^T \boldsymbol{\Psi}_{\boldsymbol{y}}^{-1} \boldsymbol{X}\right)^{-1} \tag{10}$$

Table 3
Four weighing schemes with the positions of the weights on the balance.

Scheme	m_A	m_B	m_C	m_D
1	p_1	p_2	p_3	p_4
2	p_2	p_3	p_4	p_1
3	p_3	p_4	p_1	p_2
4	p_4	p_1	p_2	p_3

The constraints m_R and Δp_R have to be assigned their associated uncertainties. For m_R will be the uncertainty $u(m_R)$ of the chosen reference standard. For the uncertainty of Δp_R, since the position error is considered negligible if the value is within the resolution r of the balance, it is reasonable to consider the uncertainty due to resolution. An additional contribution of uncertainty u_{inst} has to be evaluated in order to consider the stability of the mass values as they move from one position to another. The uncertainty $u(\Delta p_R)$ can be defined as:

$$u(\Delta p_R) = \sqrt{\left(\frac{r}{2\sqrt{3}}\right)^2 + u_{inst}^2} \tag{11}$$

To validate the results obtained, the standard deviation of the fit must be checked, which should be similar to the typical standard deviation of the measurements obtained with the balance [18].

Once the position errors have been determined for all positions, the errors $E_{i,j}$ can also be evaluated, which can be used to correct routine measurements during subsequent use of the comparator when these biases are not negligible.

When operating the mass comparator, the user must implement periodic assessments of this effect. The frequency of these evaluation should be based on the variation observed between consecutive checks.

The uncertainty associated with the correction $u(E_{i,j})$, will increase the overall uncertainty of the measured mass difference. This additional uncertainty contribution is evaluated by considering the uncertainties of the position error difference, the covariance term $u\left(\Delta p_i, \Delta p_j\right)$ and the possible temporal instability of these values u_{drift}, as determined through regular periodic assessments:

$$u(E_{i,j}) = \sqrt{u^2(\Delta p_i) + u^2\left(\Delta p_j\right) - 2u\left(\Delta p_i, \Delta p_j\right) + u_{drift}^2} \tag{12}$$

In the next section, a practical example of how to apply the method described is shown. The processing was performed using the RealMass Calibration software from INRiM [20], which makes it possible to implement this problem efficiently.

3. Practical Example: Four-position mass comparator

3.1. Experimental setup

The example concerns the evaluation of the position errors of an automatic comparator where this effect is not negligible. The comparator has a capacity of 10 kg with a resolution of 0.010 mg, and repeatability within 0.020 mg, with four positions. The test was performed with weights of 10 kg. Four weighing schemes were implemented by placing the mass standards in different positions, as shown in Table 3.

For each placement, the six possible differences between the four weights were measured. Scheme 1 corresponds to the first six rows of Table 2, while scheme 2 corresponds to the last six rows. The matrices for the schemes 3 and 4 are not shown but can be easily derived by the reader.

Table 5
Weighing Plan according to the RealMass Calibration software.

Set	ID	Nom.Value/g	Type
Weights	m_A	10 000	*R*
Weights	m_B	10 000	*T*
Weights	m_C	10 000	*T*
Weights	m_D	10 000	*T*
Positions	Δp_1	0	*R*
Positions	Δp_2	0	*T*
Positions	Δp_3	0	*T*
Positions	Δp_4	0	*T*

Table 6
Mass differences for the four weighing schemes.

Scheme	1	2	3	4
Rows	Δm_i/mg	Δm_i/mg	Δm_i/mg	Δm_i/mg
1	0.066	−11.809	26.509	−14.938
2	−14.782	−11.655	14.786	11.670
3	11.758	−26.583	14.861	−0.134
4	−14.854	0.146	−11.733	26.595
5	11.688	−14.774	−11.638	14.805
6	26.529	−14.922	0.090	−11.796

3.2. Software implementation

The system of equations can be easily implemented with the RealMass Calibration software, which has the capability to use the matrix formulation for the weighing design and solve the problem using the weighted least squares method by the Gauss Markov approach.

To use such software, the position parameters Δp_i must be implemented as if they were additional fictitious weights. However, since these are errors attributed to positions, and not weights, no correction of the aerostatic buoyancy must be applied. This is achieved by associating a null value with the volume of the position parameters Δp_i. It should be noted that the value of the position parameters can also be negative, and the software can handle this possibility.

In this example, the reference mass standard is assumed to be weight m_A to which an uncertainty of 0.265 mg is associated (which is dominated by the uncertainty of 0.200 mg for the consensus value [1], without considering such uncertainty, would be approximately 0.170 mg). As described in Section 2, it is also necessary to associate a value and an uncertainty with one position parameter. In this example, the null value is set for the position p_1, that is $\Delta p_1 = 0$, and the uncertainty is evaluated by equation (11), considering $u_{inst} = 0.005$ mg, so that $u(\Delta p_1) = 0.006$ mg.

Table 4
Database information for the RealMass Calibration software.

Set of Weights	ID	Nom [g]	Value [g]	u ($k = 1$) [g]	Volume [cm^3]	$u(k = 1)$ [cm^3]
Weights	m_A	10 000	10 000.002 389	0.000 265	1 249.862	0.007
Weights	m_B	10 000	10 000	1	1 249.826	0.007
Weights	m_C	10 000	10 000	1	1 259.610	0.007
Weights	m_D	10 000	10 000	1	1 243.680	0.019
Positions	Δp_1	0	0	0.000 006	0	0
Positions	Δp_2	0	0	1	0	0
Positions	Δp_3	0	0	1	0	0
Positions	Δp_4	0	0	1	0	0

Table 7
Results obtained without correction for the positions.

Scheme	1	2	3	4	$u(m)$/mg
m_A	10 000.002 389	10 000.002 389	10 000.002 389	10 000.002 389	0.265
m_B	10 000.002 321	10 000.002 241	10 000.002 301	10 000.002 257	0.266
m_C	10 000.017 170	10 000.017 163	10 000.017 169	10 000.017 192	0.266
m_D	9 999.990 635	9 999.990 582	9 999.990 660	9 999.990 592	0.267

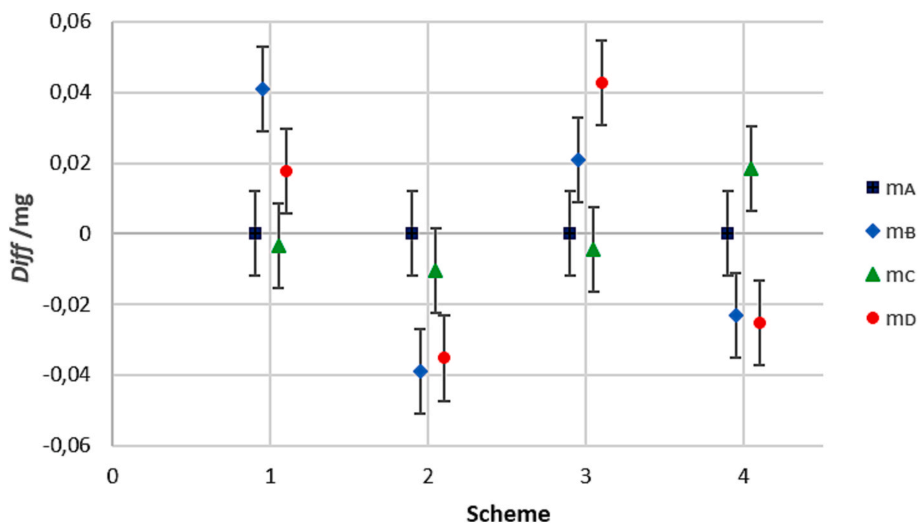

Fig. 2. Variation of the results with respect to the mean value of the four schemes; the error bars represent the standard uncertainty associated with the weighing.

Table 8
Results obtained with the proposed model, with the values for the weights and for the position errors.

ID	Value/g	$u(m)$/mg
m_A	10 000.002 389	0.265
m_B	10 000.002 280	0.265
m_C	10 000.017 173	0.265
m_D	9 999.990 617	0.266
Δp_1	0	0.006
Δp_2	0.000 041	0.008
Δp_3	−0.00 0001	0.008
Δp_4	0.00 0023	0.008

The information to be inserted in the database of the RealMass software is shown in Table 4, and the Weighing Plan (WP sheet) is shown in Table 5, where the *R* identifies the references and *T* the unknown parameters.

The Weighing Design (WD sheet) for the first two schemes is shown in Table 2; for simplicity, the complete design matrix is not shown but can be easily derived by the reader. The measurements were carried out at a temperature between 20.04 °C and 20.15 °C and air density between 1.166 kg/m^3 and 1.178 kg/m^3.

The Δm_i values, representing the mass differences corrected for aerostatic buoyancy and obtained as means for the four schemes are shown in Table 6. The uncertainties associated with the difference in mass measured by the balance is 0.012 mg (similar to the resolution), which includes the contribution of the balance resolution and the standard deviation of the mean value.

Table 9
Errors between different weighing positions.

Positions	$E_{i,j}$/mg	$u(E_{i,j})$/mg
p_1-p_2	−0.041	0.005
p_1-p_3	0.001	0.005
p_1-p_4	−0.023	0.005
p_2-p_3	0.042	0.005
p_2-p_4	0.018	0.005
p_3-p_4	−0.024	0.005

3.3. Results without the position error correction

To highlight the effect of the position error, the measurements of the four schemes were processed independently using the design matrix of each individual scheme, that is, using only the first four columns of the design matrix in Table 2. The results are shown in Table 7 and Fig. 2, which shows the variations of each scheme result with respect to the mean value, calculated from the four schemes.

For all four schemes, the standard deviation of the fit was within 0.006 mg, that is in agreement with the standard uncertainty associated with mass difference measured by the balance. It can be seen that the positioning effect is not negligible, the differences in results of the unknown weights m_B, m_C and m_D reach up to 0.080 mg.

4. Results with the position error correction

By processing all measurements from the four schemes and adding the $\Delta p_1, \Delta p_2, \Delta p_3$ and Δp_4 parameters, the results are shown in Table 8. The standard deviation of the fit was 0.006 mg, similar to the value obtained by processing the weighing schemes individually. This goodness of fit demonstrates that the measurement model is correct.

As mentioned in Section 1, the calculated mass values of the standards correspond to those that would be obtained by averaging the four values measured when exchanging the four weights across all positions (see Table 7). However, averaging the values a priori through weight exchange, without employing a measurement model, could yield incorrect results, as weight differences may depend on additional factors beyond positional error alone. Therefore, it is essential to follow the proposed measurement model; any deviation from this model would be evidenced by an increased standard deviation of the fit.

Table 10
Results obtained with the proposed model, with only two weighing schemes.

ID	Value/g	$u(m)$/mg
m_A	10 000.002 389	0.265
m_B	10 000.002 279	0.265
m_C	10 000.017 169	0.265
m_D	9 999.990 614	0.266
Δp_1	0	0.006
Δp_2	0.000 046	0.010
Δp_3	0.000 000	0.011
Δp_4	0.00 0022	0.010

4.1. Analysis of the position errors

The results in Table 8 show that by assigning a null error to position 1, a similar value is also obtained for position 3, but this is not the case for positions 2 and 4. In particular, for position 2 the error is about four times the resolution of the balance.

In this example, according to equation (2), the maximum weighing error due to position error occurs when using positions 2 and 3, where the error reaches 0.042 mg. The errors for all differences between the turntable positions are shown in Table 9.

The uncertainties $u(E_{i,j})$ were evaluated by the equation (12), considering the uncertainty of the difference of the position errors while accounting for correlation, which is obtained from the covariance matrix of the parameters provided by the RealMass software. In this example, the correlation between the error position parameters was approximately 0.8, and the uncertainty u_{drift} was considered zero.

4.2. Reduced measurement scheme

The results in Table 8 were obtained by considering all possible combinations of the four weights on the turntable, corresponding to the best estimate. However, is to be noted that similar result would have been obtained by considering only two weighing schemes, that is by changing the place of the four weights only once. For example, using only the measurements of schemes 3 and 4, the results are shown in Table 10. The values are very similar to those obtained with the four schemes, with variations within the balance resolution. Additionally, although the uncertainty associated with the positions has increased, this increase is negligible. This demonstrates that performing a single weight exchange and processing only two weighing schemes yields satisfactory results.

In general, this approach can be extended to comparators with more than four positions. Satisfactory results can be obtained with only two schemes by systematically exchanging the position of all weights and performing all mass differences between all weights. This reduced approach maintains the method's effectiveness while significantly decreasing measurement time and operational complexity, making it highly suitable for practical applications. In practical terms, with n weights on a comparator with n positions, the reduced approach requires $n(n-1)$ weighings ($n(n-1)/2$ differences for each scheme).

4.3. Application of position error corrections

Once the position errors have been determined, the appropriate correction can be assigned to each position according to the positions used in the weighings.

The RealMass Calibration software has the option of using this function automatically. For each balance in the database, it is possible to specify which corrections are to be applied depending on the positions used in the weighings.

For this example, considering the obtained corrections, the measurements of the individual schemes were recalculated, the additional uncertainty due to the corrections in Table 9 was taken into account by increasing the uncertainties associated with the difference in mass measured by the balance, which becomes 0.013 mg. Table 11 and Fig. 3 show the results and display the differences with respect to the correct results from Table 8. It can be seen that the maximum difference to the results in Table 8 are within 0.009 mg, thereby verifying the accuracy of the method.

This practical four-position example demonstrates the effectiveness of the proposed method for correcting position errors in automatic mass comparators.

It is important to note that the approach is scalable with the number of positions. The computational complexity increases proportionally, but remains manageable even for systems with dozens of positions and weights. The approach ensures its applicability across different configurations, with the main practical considerations being the corresponding increase in matrix dimensions and the need to adapt uncertainty evaluation procedures to reflect the specific metrological characteristics of the mass comparators and weights employed.

5. Conclusions

Position error represents a critical influence parameter in automatic mass comparators. Regular verification that measured mass differences remain independent of the positioning of the standards under test is essential, as this effect can degrade over time.

While verifying the position independence of mass differences is relatively straightforward, determining errors when they become non-negligible presents significant challenges, particularly for comparators with numerous available positions. This work has described a method for determining position error that enables correction of the weighing equation without substantially increasing the measurement uncertainty.

The method treats position errors as additional parameters in the weighing equation system, allowing for their simultaneous determination together with mass values using the weighted least squares approach. The implementation of this correction method is facilitated through the INRiM's RealMass Calibration software, which handles the complex matrix calculations efficiently.

The method demonstrates excellent practical efficiency, requiring only two measurement schemes with a single weight exchange to achieve satisfactory position error correction.

The proposed approach provides a robust solution for maintaining measurement accuracy in automatic mass comparators by systematically addressing position-dependent errors that can compromise the metrological performance.

CRediT authorship contribution statement

Andrea Malengo: Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Conceptualization. **Davide Torchio:** Validation, Methodology, Data curation.

Table 11
Results using position error correction.

Scheme	1	2	3	4	$u(m)$/mg
m_A	10 000.002 389	10 000.002 389	10 000.002 389	10 000.002 389	0.265
m_B	10 000.002 279	10 000.002 283	10 000.002 277	10 000.002 280	0.265
m_C	10 000.017 171	10 000.017 182	10 000.017 167	10 000.017 174	0.265
m_D	9 999.990 612	9 999.990 623	9 999.990 617	9 999.990 616	0.266

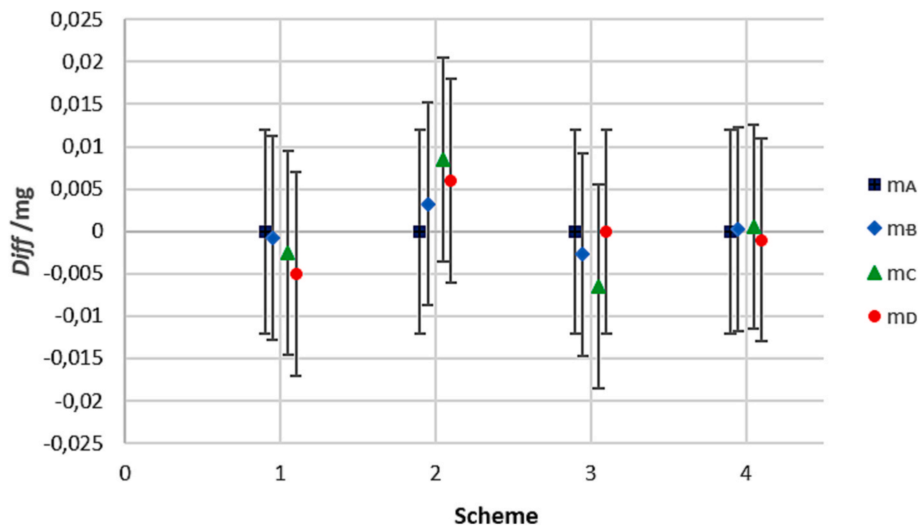

Fig. 3. Variation of the results after position error correction, with respect to the correct result; the error bars represent the standard uncertainty associated with the weighing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors would like to thank Walter Bich (INRiM) for his valuable comments and feedback on the manuscript.

Data availability

Data will be made available on request.

References

[1] S. Davidson, M. Stock, Beginning of a new phase of the dissemination of the kilogram, Metrologia 58 (2021) 033002.
[2] A. Reichmuth. A New Mass Comparator Generation for the Automatic Calibration of Weight Sets. Proceedings of the 17th International Conference on Force, Mass, Torque and Pressure Measurements, IMEKO TC3, 17-21 Sept. 2001, Istanbul, Turkey.
[3] M. Solecki, T. Szumiata, M. Rucki, Construction of a hybrid robot intended for mass standard comparison, equipped with a suspended self-centring weighing pan. IMEKO 24th Conference on the Measurement of Force, Mass and Torque. 2022. Cavtat-Dubrovnik, CROATIA.
[4] T. Fehling, T. Fröhlich, D. Heydenbluth, Design and Performance of the new Sartorius Robot-System. IMEKO 20th Conference on Measurement of Force, Mass and Torque, 2007. Merida, Mexico.
[5] X. Ren, J. Wang, R. Zhong, C. Cai, Q. Ren, Performance analysys of high capacity mass comparator AX64004. 22nd Conference on the Measurement of Force, Mass and Torque. 2014. Cape Town, SOUTH AFRICA.
[6] C. Buchner. Fully automatic mass laboratory from 1 mg up to 50 kg – Robots perform high precision mass determination. IMEKO, 20th Conference on Measurement of Force, Mass and Torque, Merida, MEXICO, 2007.
[7] Y. Ota, M. Ueki, N. Kuramoto, Evaluation of an automated mass comparator performance for mass calibration of sub-milligram weights, Measurement 172 (2021) 108841.
[8] S. Davidson, K. Fritsch, M. Grum, A. Malengo, N. Medina, G. Popa, N. Schnell, EURAMET Calibration Guide CG 18 V. 4.0. Guidelines of Calibration of Non-Automatic Weighing Instruments. 2015, EURAMET Technical Guide.
[9] M. Kochsiek, M. Gläser, Comprehensive mass metrology, Wiley-VCH Verlag Berlin GmbH, Berlin, 1999.
[10] Organisation Internationale de Métrologie Légale 2004 OIML R111-1, Paris.
[11] T.J. Quinn, The beam balance as an instrument for very precise weighing, Meas. Sci. Technol. (1992) 3 141.
[12] M. Darnieder et al. Corner loading and its influence on the tilt sensitivity of precision weighing cells. 2020 Conference: Euspen's 20th International Conference & Exhibition. Geneva, Switzerland.
[13] A. Picard, The BIPM flexure-strip balance FB-2, Metrologia 41 319 (2004).
[14] Z. Xiong, L. Cheng, Y. Su, A self-centring device designed for mass comparators, Meas.: Sens. 38 (2025) 101358.
[15] M. Stock, et al., Final report on the CCM key comparison of kilogram realizations CCM.M-K8.2021, Metrologia 60 07003 (2023).
[16] J. M. Cameron, M. C. Croarkin, R. C. Raybold. Designs for the Calibration of standards of mass. 1977 NBS Technical Note p 952 (available at: https://nvlpubs.nist.gov/nistpubs/Legacy/TN/nbstechnicalnote952.pdf).
[17] W. Bich, Bias and optimal linear estimation in comparison calibrations, Metrologia 29 15 (1992).
[18] A. Malengo, D. Torchio, Development of a comprehensive software application for realization and dissemination of the mass scale, Meas. Sci. Technol. 34 074001 (2023).
[19] W. Bich, M.G. Cox, P.M. Harris, Uncertainty modelling in mass comparisons, Metrologia 30 (1994) 495.
[20] A. Malengo and D. Torchio. RealMass Calibration Software, Release 1.1. Istituto Nazionale di Ricerca Metrologica, February 2024. Available at: https://www.inrim.it/en/services/software-and-databases/realmass-calibration-software.