Small electric currents

Luca Callegaro
Istituto Nazionale di Ricerca Metrologica
Strada delle Cacce, 91 - 10135 Torino, Italy

The values of electrical quantities span over an incredibly wide range. In the same laboratory one can manage the measurement of, say, both small 1 pF capacitors and of 1000 F supercaps for energy storage. Conductors ask for measurement of resistances in the milliohm; insulator resistance can go up to several teraohm. A range of 15 orders of magnitude seems normal – until one considers that the same range, for a length measurement for example, would go from the size of a bacterium to a journey to the moon. Technological development pushes the need for accurate measurements of electrical quantities at the extremes of these ranges.

The need for measuring small electric currents - and the corresponding integral quantity, the electric charge - exists since long. From the ancient era until the eighteenth century, the natural electrical phenomenon that had a chance to be investigated in the laboratory was triboelectricity. Rubbing an amber - *elektron* in greek - rod gives it a positive charge in the nanocoulomb range. Charles-Augustin de Coulomb and Henry Cavendish, with charges of such small magnitude, were able to perform quantitative experiments and derive the fundamental laws of electrostatics.

The interest in small currents and charges increased strongly with the discovery of ionizing radiation. Alpha and beta ionizing radiation is made of charged particles; ultraviolet, X-ray, gamma, and neutron radiation are not charged themselves, but can split charges in matter. A measurement of the collected current or charge is thus an indirect measurement of the radiation intensity. The primary meter of radiation activity of the Marie Curie Chemistry Laboratory was built around a delicate torsion balance electrometer used as zero detector and, as a variable charge generator, a piezoelectric crystal loaded with small weights [1]. The charge measured was in the picocoulomb range. Figure 1 shows a 1923 certificate claiming a 1 % accuracy in the measurement of the sample activity.

Small current measurements are nowadays widespread in semiconductor industry, material science, chemistry and biology. The need for improved accuracy is continuously expanding. An example is from photometry: predictable quantum-efficiency detectors (PQED) are a new class of photodiodes having a calculable (with an accuracy in the 10^{-5} range) photon rate to electric current conversion factor, and output currents in the pA to µA range. Another is from air pollution monitoring: ultrafine particles (smaller than 100 nm in diameter) can be "counted" with Faraday-cup electrometers, having a current output proportional to the particle number in the fA to pA range.

The ultimate sensitivity for charge measurement, that of detection of individual elementary charges, was first reached in Millikan’s famous 1909 experiment [3]. Individual charge carriers were deposited on oil droplets, moved in air by a vertical electrostatic field balancing their weight. Millikan assigned to the elementary charge a value lower than the real one because he relied on a wrong value of the viscosity of air. The experiment is not only a milestone in physics, but also a textbook case of bias in scientific development: for several years, other experiments gave a value compatible with Millikan's. Only in the 1930s the new values settled around the one presently accepted.

The existence of an elementary charge and its naming as *electron* predates Millikan's experiment and was first stated by George Johnstone Stoney in 1881, based on an analysis of electrolytic phenomena: his estimate was 10 zC, about 1/16 of the correct value. Stoney also recognized the elementary charge as a fundamental constant of nature. The accuracy of its determination steadily improved during the years. In 2019 the International System of units was deeply revised [4], and nowadays electromagnetic units are based on a value, fixed forever and exact, i.e., with uncertainty equal to zero, of the elementary charge $e = 1.60217663\times10^{-19}$ C.

The 2019 revision of the SI gives freedom about how to realise the unit of electromagnetic quantities and perform traceable generation and measurement of small electric currents. Any experiment linking, directly or indirectly, the generated or measured current or charge to the fixed value of e (and to the SI unit of time, the second, which is also related to a physical constant) can be considered a *realisation* of the unit of electric current in the specific current range of interest.

As said, the link of the experiment to e can be more or less direct. In the following, I will outline some experimental designs that are being pursued in the last decades by electrical metrologists worldwide.

Generation and measurement of small currents – the classical way

At the core of any measurement is the comparison of the quantity of interest with a known one, and small-current measurements make no exception. A small-current measuring setup includes a variable current generator traceable to other electrical quantities (voltage, current or capacitance, see below) and a sensitive zero detector; a feedback loop equates the current being measured to that generated at the detection point.

By exploiting Ohm's law, $I=V/R$, a current can be generated by applying a known voltage V to a resistor R. Figure 2 shows a current meter based on a transresistance amplifier. The current is generated by the feedback resistor, the amplifier acts as both the zero detector and the feedback loop control. Traceability is provided by the calibration of R and of the voltmeter V.

The generation of a small current, *e.g.* of 1 pA, asks for a high-valued resistor: for example, if $V = 1$ V, $R = 1$ TΩ. Resistors of this value or larger are available, but typically they are sensitive to environmental temperature and humidity, and not very stable versus time. Their calibration is challenging, asking for dedicated instruments and traceability. With special resistor manufacturing techniques, however, high accuracy implementations have been achieved.

The C-V method is based on charging of a capacitor C with a voltage ramp $V(t)$, where t is the elapsed time. In an ideal capacitor, $Q = C\,V$, hence $I = \mathrm{d}Q/\mathrm{d}t = C\,\mathrm{d}V/\mathrm{d}t$. With a linear voltage ramp a constant current can be generated… until V becomes too high and the generator must be reset. The beauty of the C-V method is that, with capacitances in the "normal" pF to nF range, and slow (10-100 mV/s) voltage ramps—easy to achieve and measure with an electronic circuit—electric currents in the pA to fA range can be generated [5]. Sealed gas-dielectric capacitors are excellent standards, insensitive to environmental parameters and with drifts in the range of parts per million per year. They can be calibrated to high accuracy with commercial instruments. A further benefit is that, since the current depends on the voltage ramp rate, not on its absolute value, the burden voltage of the meter being calibrated does not affect the measurement. Metrology-grade implementations can reach relative uncertainties in the 10^{-4} to 10^{-5} range for a current of 1 pA. A fundamental accuracy limit is given by the knowledge of the frequency dependence of the capacitor—which is calibrated in ac, but employed in dc. The dependence affects even gas-dielectric capacitors, although the underlying physical mechanism is poorly understood.

Generation and measurement of small currents – the quantum way

Quantum Ohm's law

Both for Ohm's law and *C-V* methods described above, the traceability of the measured or generated electric current is achieved by a proper calibration of the standards employed, traceable to primary realisations of these quantities. The link to the defining constants of the SI is ultimately achieved with quantum experiments in solid-state devices, in which electrical quantities have values that are related to the defining constants of the SI by a physical law. The experiments of interest are

- the *Josephson effect*. It occurs in superconducting tunnel junction chips, biased with a radiofrequency. A quantized voltage $V = n\ K_J\ f$ is generated, where n is the number of junctions in series, f is the bias frequency, and $K_J = 2\ e\ /\ h$ is the *Josephson constant* – which includes e and the Planck constant h, both fixed with no uncertainty in the SI.

- the *quantum Hall effect*. It occurs in solid-state devices where a two-dimensional conducting layer is present. Under a strong perpendicular magnetic field and low temperatures, the Hall resistance (the ratio between a longitudinal current flowing in the device and the transverse Hall voltage given by the magnetic field) becomes quantised, its value R_H no longer being dependent on the device material or the temperature. $R_H = R_K\ /\ i$ is a simple fraction, with a small denominator i, of the resistance quantum $R_K = h\ /\ e^2$, the *von Klitzing constant*.

In a broader sense, therefore, any traceable electrical measurement of small currents—indeed, of any electrical quantity—has a quantum foundation. Each calibration step occurring in the traceability chain degrades to some extent the overall measurement accuracy.

Quantum current generators get rid of the traceability chain of calibrations by combining the Josephson and the quantum Hall experiments in one, for a quantum implementation of Ohm's law.

One of these experiments is the “Programmable Quantum Current Generator” [6], sketched in Figure 3. A Josephson system and a quantum Hall system are electrically combined to generate a current through the Ohm’s law. The current can be scaled up or down by a third experiment, a cryogenic current comparator, which acts as a ‘perfect’ DC current transformer. The approach is complicated by the different magnetic field conditions in which the Josephson and the quantum Hall experiments can operate: the former asks for a field below the mT, the latter requires fields of several T. Efforts are ongoing to combine the experiments in a single, specially designed, cryostat.

The intensity of the magnetic field required by the quantum Hall effect, which limits the integration with both Josephson and conventional electronics, can be strongly reduced in a new class of materials, called *topological insulators.* In these materials, the quantum anomalous Hall effect (QAHE) occurs: the quantization happens at fields below 1 T, and even at zero field with a magnetic doping of the material after a magnetization cycle. A current meter [7] based on the QAHE and the Josephson effect is described in Figure 4. A major drawback of QAHE is that, in present materials the quantization occurs at temperatures in the 10 mK to 1 K range. Having several applications, the research on new materials is ongoing and improvements in the operating conditions are expected [8].

Single-electron counting

Since all electrons have the same elementary charge e, if electrons are individually moved through a circuit section at a given rate f, a current $I = e\, f$ is generated. Because e is very small, currents in the pA to nA range ask for counting rates in the MHz to GHz range.

Single-electron pumps are based on the mechanism of *coulomb blockade*. Let us consider a small conductive island, electrically isolated from the environment. Cit with an individual electron modifies the electrical potential of the island (with respect to a reference) of the discrete amount $\Delta V = e/C$, where C is the island capacitance. If the island (and hence C) is sufficiently small, and the temperature sufficiently low, the ΔV given by the first electron can be large enough to prevent a second electron (which is repelled by the first) to enter the island. If the island can be periodically connected either to a source or to a drain contact (e.g., by varying the potential of gate electrodes), at each cycle an electron is transferred – "pumped" from the source to the drain.

Several types of single-electron pump devices exist, based on different materials (metal, semiconductors, superconductors). They are fabricated by nanolithography and, for accurate operation, must be operated at mK temperatures. Figure 5 shows an example of a silicon-based current pump.

Phase slip in superconducting devices. A phenomenon predicted in 2006 as a theoretical dual of the Josephson tunneling is a quantized current occurring in quasi-monodimensional superconducting structures biased by a radiofrequency voltage. Each rf cycle moves a charge quantum (in a superconductor, a Cooper pair of charge $2e$). The phenomenon was observed in ultrathin superconducting wires, and more recently in Josephson junction devices. At the moment, the observed quantisation is far from perfect (about 10^{-4} accuracy).

Outlook

The vast majority of sensors in use today convert the quantity of interest in an electrical quantity, which is then conditioned and digitized. In the majority of applications, the measurement accuracy is given by the sensor itself, by the very process of conversion from a non-electrical quantity to an electrical one. With a proper choice of the reading electronics, the uncertainty contribution given by the electrical measurement is typically negligible.

For measurements involving small electric currents this might not be the case. In the examples given in the introduction (radiation monitoring, photometry and radiometry, aerosol metrology) the accuracy of the current meter does contribute, and can dominate the overall accuracy of the measurement. Performing properly traceable calibrations of the small current meters employed, with sufficient accuracy, is a challenge.

Since 2019, electric current is quantum-defined. Quantum electrical metrology is evolving, both by probing new physical phenomena and by engineering their exploitation. Quantum generators and meters of small electric currents already exist: the expectation is that they will become more and more available for applications in the near future.

For further reading

This column is limited in space and number of references provided. The interested reader can see Ref. [10], a recent and excellent review with a large reference list.

Acknowledgments

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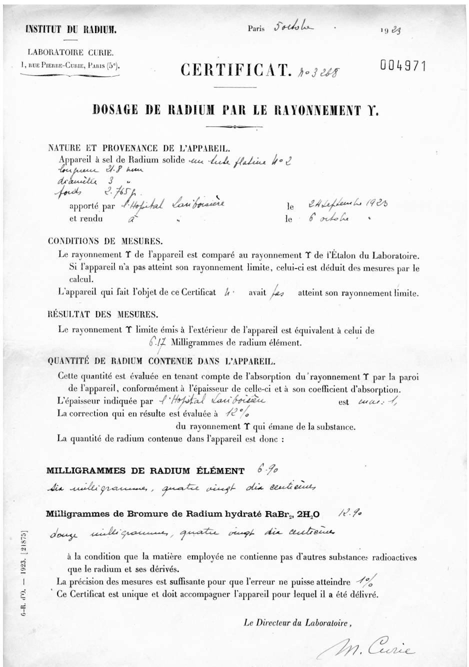

INSTITUT DU RADIUM.

LABORATOIRE CURIE.
1, RUE PIERRE-CURIE, PARIS (5e).

Paris 5 octobre 1923

CERTIFICAT. No 3268

004971

DOSAGE DE RADIUM PAR LE RAYONNEMENT Υ.

NATURE ET PROVENANCE DE L'APPAREIL.

Appareil à sel de Radium solide un tube platine No 2
longueur 21.8 mm
diamètre 3 "
poids 2.765 g.
apporté par l'Hopital Lariboisière le 24 septembre 1923
et rendu à " le 6 octobre "

CONDITIONS DE MESURES.

Le rayonnement Υ de l'appareil est comparé au rayonnement Υ de l'Étalon du Laboratoire. Si l'appareil n'a pas atteint son rayonnement limite, celui-ci est déduit des mesures par le calcul.

L'appareil qui fait l'objet de ce Certificat n' avait pas atteint son rayonnement limite.

RÉSULTAT DES MESURES.

Le rayonnement Υ limite émis à l'extérieur de l'appareil est équivalent à celui de 6.17 Milligrammes de radium élément.

QUANTITÉ DE RADIUM CONTENUE DANS L'APPAREIL.

Cette quantité est évaluée en tenant compte de l'absorption du rayonnement Υ par la paroi de l'appareil, conformément à l'épaisseur de celle-ci et à son coefficient d'absorption.

L'épaisseur indiquée par l'Hopital Lariboisière est mm: 1,

La correction qui en résulte est évaluée à 12% du rayonnement Υ qui émane de la substance.

La quantité de radium contenue dans l'appareil est donc :

MILLIGRAMMES DE RADIUM ÉLÉMENT 6.90

Six milligrammes, quatre vingt dix centièmes

Milligrammes de Bromure de Radium hydraté $RaBr_2, 2H_2O$ 12.90

douze milligrammes, quatre vingt dix centièmes

à la condition que la matière employée ne contienne pas d'autres substances radioactives que le radium et ses dérivés.

La précision des mesures est suffisante pour que l'erreur ne puisse atteindre 1%

Ce Certificat est unique et doit accompagner l'appareil pour lequel il a été délivré.

Le Directeur du Laboratoire,

M. Curie

6-R. d'O. — 1923. [21875]

Figure 1. Certificate of radioactivity for radium tubes of the Lariboisère Hospital of Paris, France. Close to the bottom line: "La précision des mesures est suffisante pour que l'erreur ne puisse atteindre 1%." [The precision of the measurements is such that the error does not reach 1%]. The certificate was signed by Marie Curie [2].

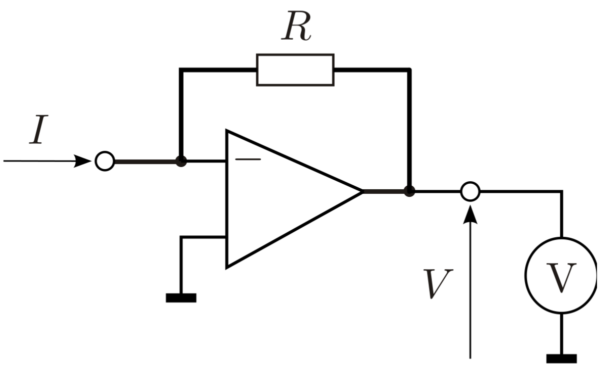

Figure 2. A transresistance amplifier as a current meter. The current I is converted to a voltage V=-RI, read by the voltmeter V.

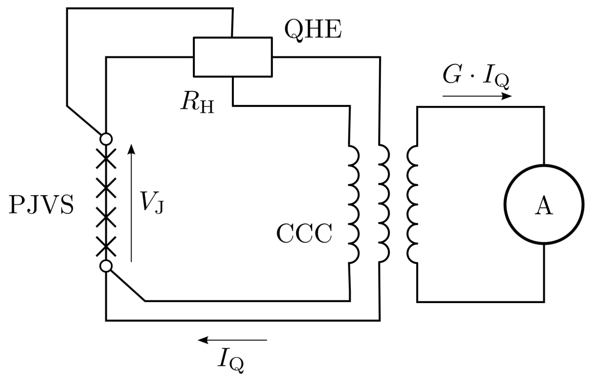

Figure 3. A simplified circuit diagram of the Programmable Quantum Current Generator (PQCG) ongoing at the Laboratoire National d'Essais (LNE), France [6]. The voltage V_J from a Programmable Josephson Voltage Generator (PJVS) applied to a quantum Hall effect resistor (QHE) of resistance R_H generates the quantized current $I_Q = V_J / R_H$. The effect of the wiring resistance is eliminated by the peculiar connection scheme. The current is scaled up or down by a turns ratio factor G with a Cryogenic Current Comparator (CCC), which acts as a DC transformer, and is available (for example) for the calibration of a meter A. The three experiments PJVS, QHE and CCC are hosted in three different cryogenic environments.

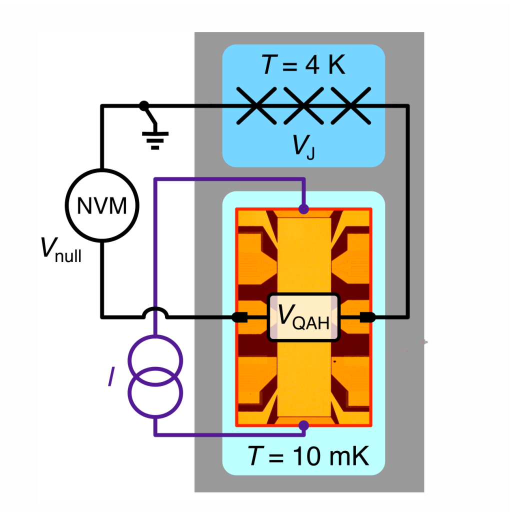

Figure 4. A quantum current meter based on the Josephson effect (JE) and the quantum anomalous Hall effect (QAHE). The QAHE device is made of a magnetically-doped topological insulator, shaped at an Hall bar and kept at T =10 mK in a dilution refrigerator. After an initial magnetization, the device can operate at zero magnetic field. The current I flowing through the QAHE device generates a Hall voltage $V_{QAH} = R_K\, I$. In the same cryostat, at T = 4 K, a Josephson junction array generates a quantized reference voltage V_J. V_{QAH} is measured by comparing it with V_J with the nanovoltmeter NVM, which measures the voltage difference $V_{null} = V_{QAH}$ - V_J. Currents from a few to several hundred nA have been generated. From Ref. [7], courtesy of the authors.

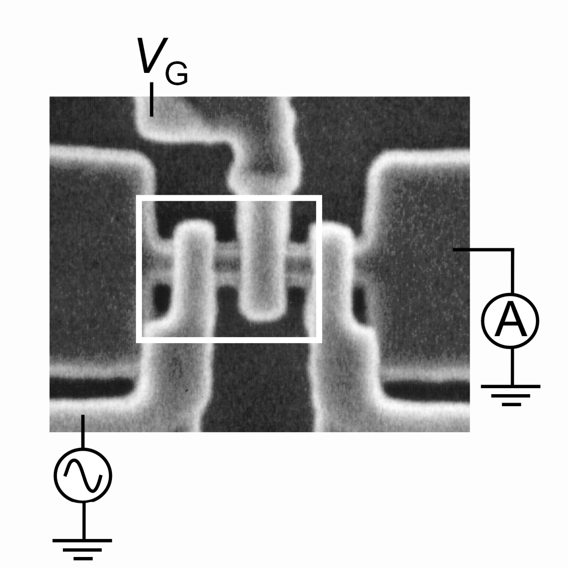

Figure 5. A single-electron pump, fabricated by MOSFET technology on silicon [9]. The current flows from the source to the drain (the two large electrodes at left and right) through a 20-nm Si nanowire, and is measured by the ammeter A. Three gate electrodes define a quantum dot on the nanowire; individual electrons are cyclically trapped on and released from the quantum dot by applying to one of the gates a periodic potential. Courtesy of Akira Fujiwara, NTT Basic Research Laboratories, Japan.

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Luca Callegaro joined the Istituto Nazionale di Ricerca Metrologica/National Institute of Metrological Research (INRIM), Turin, Italy, in 1996, where he is currently the Head of the Quantum Electronics Group. His research interests focus on electrical impedance metrology. He is responsible for the Italian National standards of electrical impedance. Dr. Callegaro is a member, and served four years as the Chair, of the Technical Committee for Electricity and Magnetism (TC-EM) of the European Association of National Metrology Institutes (EURAMET), and he is the Italian delegate to the Consultative Committee for Electricity and Magnetism (CCEM) of the International Committee for Weights and Measures (CIPM). He received the M.Sc. degree in electronic engineering and the Ph.D. degree in physics from the Politecnico di Milano, Milan, Italy, in 1992 and 1996, respectively.