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30 June 2024

Bilateral Comparison of 1.018 V and 10 V Standards

between the INRIM (Italy) and the BIPM,

November to December 2023

(part of the ongoing BIPM key comparison BIPM.EM-K11.a and b)

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Introduction

As part of the ongoing BIPM key comparison BIPM.EM-K11.a and b, a comparison of the 1.018 V and 10 V voltage reference standards of the BIPM and the *Istituto Nazionale di Ricerca Metrologica* (INRIM), Torino, Italy, was carried out from November to December 2023. Two BIPM Zener diode-based travelling standards (Fluke 732B), BIPM_7 (Z7) and BIPM_9 (Z9), were transported by freight to INRIM and back to BIPM. In order to keep the Zeners powered during their transportation phase, an additional battery was connected in parallel to the internal battery.

At INRIM, the reference standard for DC voltage is a Programmable Josephson Voltage Standard (PJVS). The output electromotive force (EMF) of each travelling standard was measured by direct comparison with the primary standard.

At the BIPM, the output EMF of each travelling standard was calibrated before and after the measurements at INRIM against the PJVS developed at the BIPM around a PTB programmable SNS (Superconductor/Normal Metal/Superconductor) array.

Results of all measurements were corrected by the BIPM for the dependence of the output voltages of the Zener standards on internal temperature and ambient atmospheric pressure.

Outline of the measuring method

INRIM 1.018 V and 10 V measurements

At INRIM, the reference standard for DC voltage is a 10 V Programmable Josephson Voltage Standard (PJVS) [1] fabricated by *Supracon AG* and computer-controlled with the

AC *SupraVOLT* control software [2] to verify the array performances and to generate the programmable DC quantum voltages.

The 1.018 V and 10 V electromotive force (EMF) outputs of each travelling standard were measured by direct comparison with the PJVS. Each output terminal of the travelling standards was connected in series opposition to the PJVS array using a manual low thermal EMF reversing switch. The EMF differences between the standard and the PJVS array are measured using a digital nanovoltmeter *Keithley 2182A* operated on its 10 mV range and processed in real-time with a custom data acquisition software developed by INRIM.

Eight data points were taken consecutively - with the simple mean value being considered as the result of the day. Each individual data point represents the mean of 40 measurements (20 in positive and 20 in negative polarity). The nanovoltmeter input was shorted before each polarity reversal and restored right after. Each data point lasted about 10 minutes.

Frequency and power of the microwave irradiating the PJVS array were set to maximize the quantum operating margins and were kept fixed throughout the whole measurement. The PJVS array was programmed to generate the closest quantum voltage level to the output voltage of the standard under measurement. Since the voltage resolution of the PJVS array is approximately equal to 144 µV, the magnitude of the voltage recorded by the nanovoltmeter was always lower than 75 µV.

The two standards were placed in a *Kambic TK-190 US* temperature chamber set to 23.0 °C more than 24 hours before the first day of measurement and have been kept within the chamber throughout the entire measurement session. Each measurement has been performed with the standards disconnected from the mains power for at least two hours. The "GUARD" and "CHASSIS" binding posts were jointly connected to the common ground point of the setup.

The internal thermistor resistance of the standards was measured with an *Agilent 3458A* multimeter operated on its 1 MΩ range, supplying a current of 5 µA to avoid self-heating effects. The thermistor resistance was recorded prior to the start and after the end of each measurement, and the average was taken as reference value for the measurement. The same procedure has been applied to measure the barometric pressure by means of a portable pressure gauge (model *Ruska 6220*).

BIPM Measurements for 1.018 V and 10 V

The output voltage of the Zener standard to be measured was connected in series opposition to the BIPM Josephson Voltage Standard - PTB 10 V SNS array (S/N: 2013-02/4a) [3], through a low thermal EMF multiplexer [4-5]. The binding post terminals "GUARD" and "CHASSIS" of the Zener standard were connected together and connected to a single point which is the grounding reference point of the measurement setup.

The measurements started at least two hours after the mains plug at the rear of the Zeners had been disconnected in order for the Zener internal temperature to stabilize.

In this comparison, the BIPM detector was a digital nanovoltmeter *Keithley 2182A* operated on its 10 mV range. A computer was used to monitor, record the measurements, acquire the data, correct for temperature and pressure dependence, and calculate results.

The BIPM array biasing frequency was adjusted in such a way that the voltage difference between the primary and the secondary voltage standards was always below 1 µV for both nominal voltages.

One individual measurement point was acquired according to the following:

1- The Zener and the BIPM array are set in their positive polarity, connected in series opposition and the detector data reading sequence starts;

2- The polarity of the detector is reversed and a reading sequence is carried out. The number of measurements is twice the number acquired in step 1;

3- The polarity of the detector is reversed again to match the conditions of step 1 and the reading sequence restarts;

4- The Zener and the BIPM array are set in their negative polarity, connected in series opposition and the detector data reading sequence starts;

5- The polarity of the detector is reversed and a reading sequence is carried out. The number of measurements is twice the number acquired in step 4;

6- The polarity of the detector is reversed again to match the conditions of step 4 and the reading sequence restarts.

The reversal of the array polarity (by reversing the bias current) is always accompanied by a reversal of the Zener voltage standard using the multiplexer. The reversal of the detector polarity is done to cancel out any internal detector thermal EMF with a constant drift rate. Each data acquisition step consists of 50 preliminary measurements followed by 100 measurements. Each of these should not differ from the mean of the preliminary measurements by more than four times their standard deviation. If so, the software warns the operator with a beep. If too many beeps occur, the operator can restart the "Data Acquisition" step in progress. The procedure to acquire one individual measurement point is repeated five times in a row and the mean value corresponds to one result on the graph (cf. Fig. 1, 2, 3, and 4).

Results at 10 V

Figure 1 shows the measured values obtained for the two standards by the two laboratories at 10 V. Figure 2 presents the voltage evolution of the simple mean of the two standards which is used to compute the final result at 10 V. A linear least squares fit is applied to all of the individual BIPM results, and to the mean value of both transfer standards. The comparison result is the voltage difference between the BIPM fitted value at the mean date of the INRIM measurements (01/12/2023) and the mean value of the INRIM measurements, and the related uncertainties.
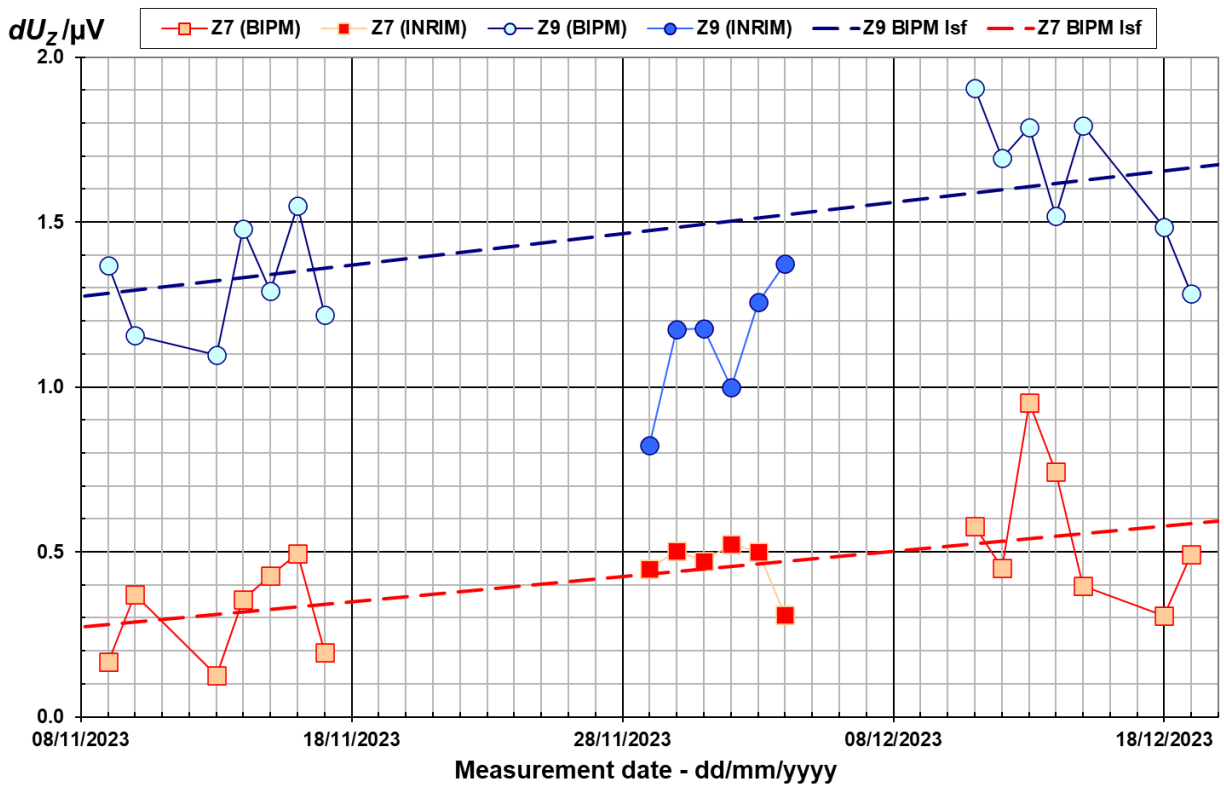
Figure 1: Voltage of Z7 (squares) and Z9 (disks) at 10 V measured at both institutes (light markers for BIPM and dark markers for INRIM), referred to an arbitrary offset, as a function of the measurement date with a linear least-squares fit (lsf) to the BIPM measurements.
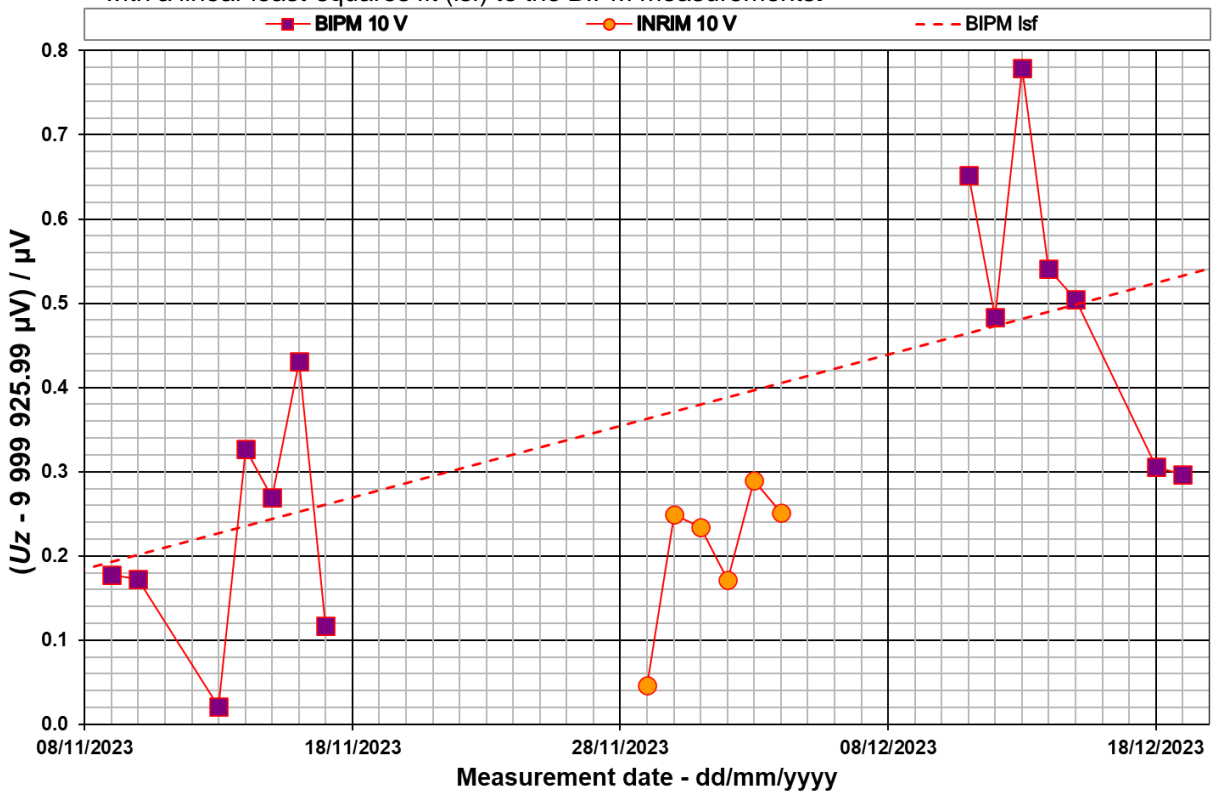
Figure 2: Voltage evolution of the arithmetic mean of the two standards at 10 V. INRIM measurements are represented by disks and BIPM measurements by squares. A least-squares fit is applied to the BIPM measurements.

Uncertainty Budgets at 10 V

BIPM uncertainty budget at 10 V

Table 1 summarizes the uncertainties related to the calibration of a Zener diode against the Josephson array voltage standard at the BIPM at the level of 10 V.

Experience has shown that flicker or $1/f$ noise ultimately limits the stability characteristics of Zener diode standards and it is not appropriate to use the standard deviation divided by the square root of the number of observations to characterize the dispersion of measured values. For the present standards, the relative value of the voltage noise floor due to flicker noise is about 1.5 parts in 10^8 [6]. The Type A standard uncertainty in the Table 1 therefore has a lower limit of 150 nV. However, if the standard deviation of the measurements at the mean date of the participant is larger than the flicker noise floor, it is this standard deviation which is considered to be the Type A standard uncertainty.

JVS & detector uncertainty components	Uncertainty (nV)
Noise of the measurement loop that includes the residual thermal EMF including the residual EMF of the reversing switch (Type A)	2
Detector gain (Type B)	negligible
Leakage resistance (Type B)	4
Frequency (Type B)	0.1
Zener noise (Type A)	Not lower than the $1/f$ noise estimated as 150 nV, included in the comparison uncertainty budget (Table 3)
Zener pressure and temperature correction	Included in the comparison uncertainty budget (Table 3)

Table 1: Estimated standard uncertainties arising from the JVS and the measurement setup for Zener calibrations with the BIPM equipment at the level of 10 V.

INRIM uncertainty budget at 10 V

Tables 2a and 2b lists the uncertainties related to the calibration of the Zeners at INRIM for Z7 and Z9, respectively.

Note that the uncertainty of the temperature and pressure corrections (in italic) are given as an indication only and do not contribute to the final uncertainty budget used for this comparison as they are applied by the BIPM and included in the comparison uncertainty budget (Table 3).

Quantity	Estimate	Type	Dist.	Standard uncertainty	Sensitivity	Uncertainty contribution	Degree of freedom
Difference voltage measured by the nanovoltmeter	28.56 µV	A	Norm.	50.5 nV	1	50.5 nV	5
Microwave Reference frequency [*]	69.649999941 GHz	B	Rect.	4.0 Hz	144 pV/Hz	0.6 nV	∞
Voltage due to leakage current	0 V	B	Rect.	2.9×10^{-11}	10 V	0.3 nV	∞
Voltage due to gain error of the nanovoltmeter	0 V	B	Rect.	5.8×10^{-5}	28.56 µV	1.7 nV	∞
Voltage due to the non-linearity of the nanovoltmeter	0 V	B	Rect.	17.3 nV	1	17.3 nV	∞
Non-compensated EMF of the measurement circuit	0 V	B	Rect.	11.5 nV	1	11.5 nV	∞
Temperature coefficient of the Zener	*38.606 kΩ*	*B*	*Rect.*	*5 Ω*	*2.51 nV/Ω*	*12.6 nV*	*10*
Pressure coefficient of the Zener	*981.1 hPa*	*B*	*Rect.*	*0.5 hPa*	*18.74 nV/hPa*	*9.4 nV*	*14*

[7]

Combined uncertainty……………………………………………. $u(U_z)$ = 55 nV

Relative combined uncertainty…………………...………… $u(U_z)$ / U_z = 5.5 nV/V

Effective degrees of freedom [†]………………..…… ………………………..v_{eff} = 7

Coverage factor [‡]……………………………………...……….…………… $k_{0.95}$ = 2.36

Expanded uncertainty (95%)………...……………$U(U_Z) = k_{0.95} \times u(U_Z)$ = 130 nV

Relative expanded uncertainty…………………………….. $U(U_Z)$ / U_Z = 13.0 nV/V

Table 2a: Estimated standard uncertainties of U_z for a Zener calibration with the INRIM equipment at the level of 10 V for Zener Z7.

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[*] Type A uncertainty component of the applied microwave frequency *f* is already included in the measured voltage difference.

[†] Effective degrees of freedom v_{eff} are calculated with Welch-Satterthwaite formula.

[‡] Coverage factor $k_{0.95}$ is evaluated assuming a *t*-Student distribution with v_{eff} degrees of freedom.

Quantity	Estimate	Type	Dist.	Standard uncertainty	Sensitivity	Uncertainty contribution	Degree of freedom
Difference voltage measured by the nanovoltmeter	13.32 µV	A	Norm.	135.4 nV	1	135.4 nV	5
Microwave Reference frequency [*]	69.649999941 GHz	B	Rect.	4.0 Hz	144 pV/Hz	0.6 nV	∞
Voltage due to leakage current	0 V	B	Rect.	2.9×10^{-11}	10 V	0.3 nV	∞
Voltage due to gain error of the nanovoltmeter	0 V	B	Rect.	5.8×10^{-5}	13.32 µV	0.8 nV	∞
Voltage due to the non-linearity of the nanovoltmeter	0 V	B	Rect.	17.3 nV	1	17.3 nV	∞
Non-compensated EMF of the measurement circuit	0 V	B	Rect.	11.5 nV	1	11.5 nV	∞
Temperature coefficient of the Zener	*39.228 kΩ*	*B*	*Rect.*	*5 Ω*	*-0.03 nV/Ω*	*0.2 nV*	*10*
Pressure coefficient of the Zener	*980.7 hPa*	*B*	*Rect.*	*0.5 hPa*	*19.31 nV/hPa*	*9.7 nV*	*14*
[7]	Combined uncertainty………………………………....……. $u(U_z)$ = 137 nV						
	Relative combined uncertainty…………………………… $u(U_z) / U_z$ = 13.7 nV/V						
	Effective degrees of freedom [†]………………..…………….………………. v_{eff} = 5						
	Coverage factor [‡]………………….………………………..……………… $k_{0.95}$ = 2.57						
	Expanded uncertainty (95%)………..……………$U(U_Z) = k_{0.95} \times u(U_z)$ = 352 nV						
	Relative expanded uncertainty……………………………... $U(U_Z) / U_Z$ = 35.2 nV/V						

Table 2b: Estimated standard uncertainties of U_z for a Zener calibration with the INRIM equipment at the level of 10 V for Zener Z9.

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[*] Type A uncertainty component of the applied microwave frequency *f* is already included in the measured voltage difference.
[†] Effective degrees of freedom v_{eff} are calculated with Welch-Satterthwaite formula.
[‡] Coverage factor $k_{0.95}$ is evaluated assuming a *t*-Student distribution with v_{eff} degrees of freedom.

Uncertainty contributions for the comparison INRIM/BIPM at 10 V

Table 3 lists the results and the uncertainty contributions for the comparison INRIM/BIPM at 10 V.

		Results/µV		Uncertainty/µV	
		Z7	**Z9**	**Z7**	**Z9**
1	INRIM (U_{INRIM} – 10 V)	-52.84	-94.77		
2	Type A uncertainty			0.051	0.135
3	correlated (Type B) unc.			0.021	
4	BIPM (U_{BIPM} – 10 V)	-52.85	-94.40		
5	Type A uncertainty			0.15	0.15
6	correlated (Type B) unc.			<0.005	
7	pressure and temperature correction uncertainty			0.013	0.026
8	(U_{INRIM} – U_{BIPM})	0.01	-0.37		
9	Total uncorrelated uncertainty			0.159	0.203
10	Total correlated uncertainty			0.021	
11	< U_{INRIM} – U_{BIPM} >	**-0.18**			
12	*a priori* uncertainty			0.129	
13	*a posteriori* uncertainty			0.190	
14	**comparison total standard uncertainty/µV**			**0.19**	

Table 3: Results and uncertainties of INRIM (Italy)/BIPM bilateral comparison of 10 V standards using two Zener travelling standards: reference date 1 December 2023. Standard uncertainties are used throughout.

In Table 3, the following elements are listed:

(1) the value attributed by INRIM to each Zener, U_{INRIM}, computed as the simple mean of all data from INRIM and corrected for temperature and pressure differences between both laboratories by the BIPM.

(2) INRIM combined Type A uncertainty (cf. Tables 2a and 2b).

(3) the uncertainty component arising from the realization and maintenance of the volt at INRIM: it is the quadratic combination of the Type B components of the participant uncertainty budget listed in Tables 2a and 2b. This uncertainty is completely correlated between the different Zeners used for the comparison.

(4-6) the corresponding quantities for the BIPM referenced to the mean date of the INRIM measurements. In this case, the Type A uncertainty is limited by the flicker noise level of 150 nV.

(7) the uncertainty due to the combined effects of the pressure and temperature coefficients [8, 9] and to the differences of the mean pressures and temperatures in the participating laboratories is calculated as follows:

The uncertainty of the temperature correction $u_{T,i}$ of Zener i is determined for the difference ΔR_i between the mean values of the thermistor resistances measured at both institutes which is then multiplied by the uncertainty $u(c_{T,i})$ of the relative temperature coefficients of each Zener standard:

$$u_{T,i} = U \times u(c_{T,i}) \times \Delta R_i$$

where U = 10 V, $u(c_{T,Z7})$ = 0.216 × 10^{-7} / kΩ, $u(c_{T,Z9})$ = 0.231 × 10^{-7} / kΩ, ΔR_{Z7} = 0.024 kΩ and ΔR_{Z9} = 0.097 kΩ.

The same procedure is applied for the uncertainty $u_{P,i}$ of the pressure correction for the difference ΔP_i between the mean values of the pressure measured at both institutes:

$$u_{P,i} = U \times u(c_{P,i}) \times \Delta P_i$$

where U = 10 V, $u(c_{P,Z7})$ = 0.045 × 10^{-9} / hPa, $u(c_{P,Z9})$ = 0.052 × 10^{-9} / hPa, ΔP_{Z7} = 26.0 hPa and ΔP_{Z9} = 26.4 hPa.

The uncertainties of the measurement of the temperature and the pressure are negligible.

(8) the difference ($U_{INRIM} - U_{BIPM}$) for each Zener, and (9) the uncorrelated part of the uncertainty, calculated as the quadratic sum of lines 2, 5 and 7.

(10) the correlated part of the uncertainty, calculated as the quadratic sum of lines 3 and 6, for each travelling standard.

(11) the result of the comparison is the simple mean of the differences of the calibration results for the different standards.

(12 and 13) the uncertainty related to the transfer, estimated by comparing the following uncertainties:

(12) the *a priori* uncertainty, determined as the standard uncertainty of the mean, obtained by propagating the uncorrelated uncertainties for both Zeners;

(13) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results.

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty (10) and of the larger of (12) and (13).

To estimate the uncertainty related to the stability of the standards during transportation, we have calculated the "*a priori*" uncertainty of the mean of the results obtained for the two standards (also called statistical internal consistency). It consists of the quadratic combination of the uncorrelated uncertainties of each result. We compared this component to the "*a posteriori*" uncertainty (also called statistical external consistency) which consists of the experimental standard deviation of the mean of the results from the two travelling standards[*].

If the "*a posteriori*" uncertainty is significantly larger than the "*a priori*" uncertainty, we assume that a standard has changed in an unusual way, probably during their transportation. This is the case in the present comparison and the a posteriori uncertainty is used in the uncertainty budget. However, comparing the results obtained at BIPM of the Zeners before and after their return, it seems not obvious to conclude that the metrological quality of the standards was affected by their shipment.

The comparison result is presented as the difference between the value assigned to a 10 V standard by INRIM, at INRIM, U_{INRIM}, and that assigned by the BIPM, at the BIPM, U_{BIPM}, on the reference date of the 1st of December 2023:

$$U_{INRIM} - U_{BIPM} = \text{-0.18 } \mu V; \qquad u_c = 0.19 \text{ } \mu V$$

where u_c is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at INRIM, at the BIPM, and the uncertainty related to the comparison.

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[*] With only two travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself.

Results at 1.018 V

Figure 3 shows the measured values obtained for the two standards by the two laboratories at 1.018 V and Figure 4 presents the voltage evolution of the simple mean of the two standards which is used to compute the final result at 1.018 V.

A linear least squares fit is applied to the results of the BIPM, before and after the measurements at INRIM, to obtain the results for both standards and their uncertainties at the mean date of the INRIM measurements (01/12/2023).
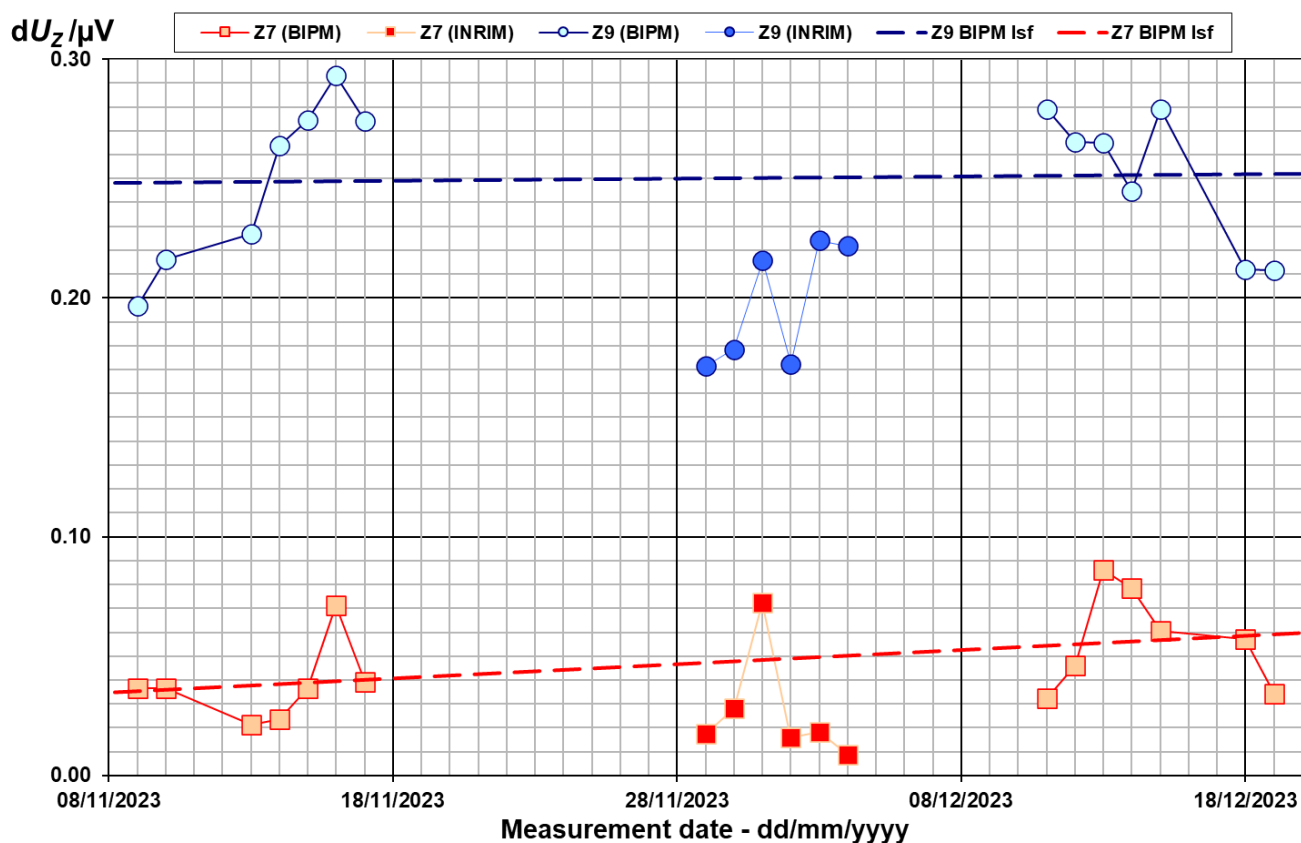
Figure 3: Voltage of Z7 (squares) and Z9 (disks) at 1.018 V measured at both institutes (light markers for BIPM and dark markers for INRIM), referred to an arbitrary offset, as a function of the measurement date with a linear least-squares fit (lsf) to the BIPM measurements.
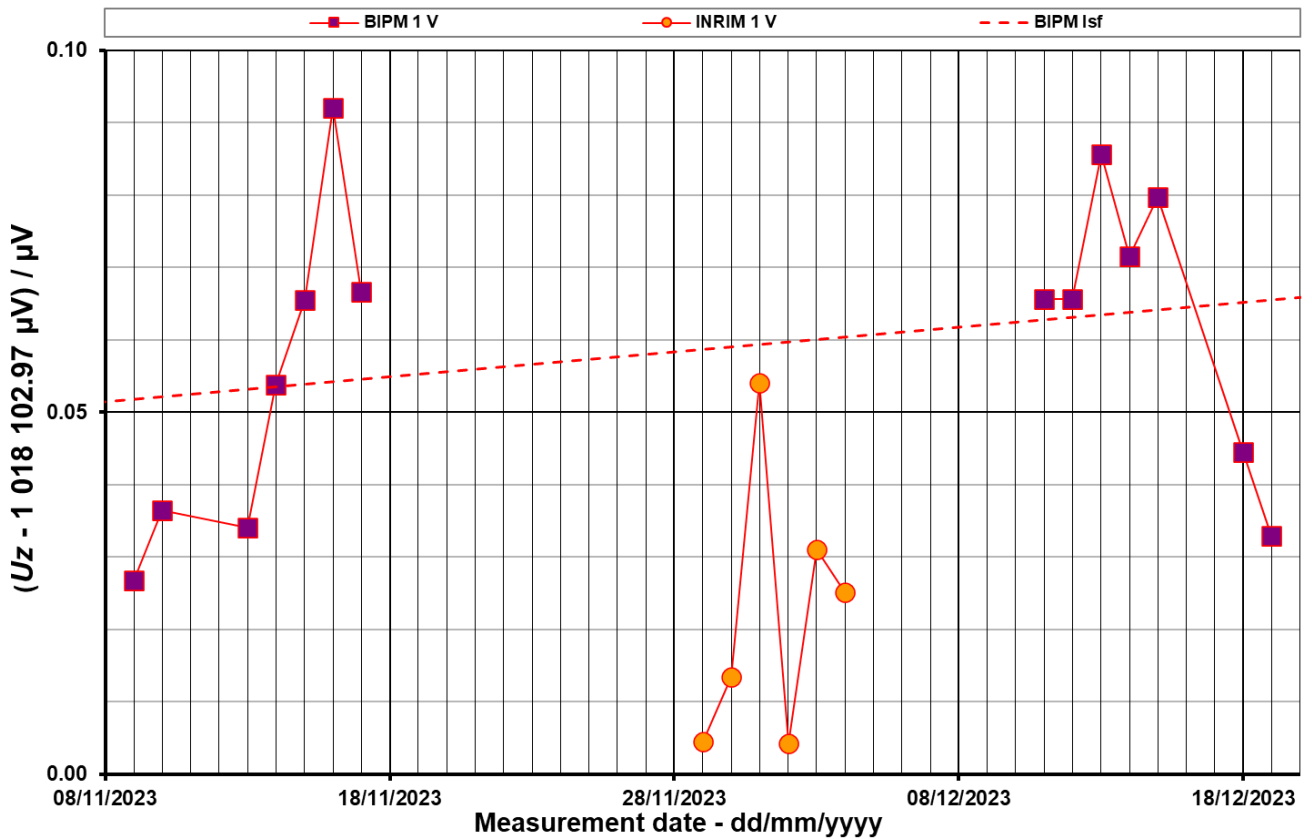
Figure 4: Voltage evolution of the arithmetic mean of the two standards at 1.018 V. INRIM measurements are represented by disks and BIPM measurements by squares. A least-squares fit is applied to the BIPM measurements.

Uncertainty Budgets at 1.018 V

BIPM uncertainty budget at 1.018 V

Table 4 summarizes the uncertainties related to the calibration of a Zener diode against the Josephson array voltage standard at the BIPM at the level of 1.018 V.

JVS & detector uncertainty components	Uncertainty (nV)
Noise of the measurement loop that includes the residual thermal EMF including the residual EMF of the reversing switch (Type A)	2
Detector gain (Type B)	negligible
Leakage resistance (Type B)	0.4
Frequency (Type B)	0.01
Zener noise (Type A)	Not lower than the $1/f$ noise estimated as 15 nV, included in the comparison uncertainty budget (Table 6)
Zener pressure and temperature correction	Included in the comparison uncertainty budget (Table 6)

Table 4: Estimated standard uncertainties arising from the JVS and the measurement setup for Zener calibrations with the BIPM equipment at the level of 1.018 V.

INRIM uncertainty budget at 1.018 V

Tables 5a and 5b list the uncertainties related to the calibration of the Zeners at INRIM for Z7 and Z9, respectively.

Note that the uncertainty of the temperature and pressure corrections (in italic) are given as an indication only and do not contribute to the final uncertainty budget used for this comparison as they are applied by the BIPM and included in the comparison uncertainty budget (Table 6).

Quantity	Estimate	Type	Dist.	Standard uncertainty	Sensitivity	Uncertainty contribution	Degree of freedom
Difference voltage measured by the nanovoltmeter	0.126 µV	A	Norm.	7.0 nV	1	7.0 nV	5
Microwave Reference frequency [*]	69.649999941 GHz	B	Rect.	4.0 Hz	14.6 pV/Hz	0.06 nV	∞
Voltage due to leakage current	0 V	B	Rect.	2.9×10^{-11}	1.018 V	0.03 nV	∞
Voltage due to gain error of the nanovoltmeter	0 V	B	Rect.	5.8×10^{-5}	0.126 µV	0.01 nV	∞
Voltage due to the non-linearity of the nanovoltmeter	0 V	B	Rect.	17.3 nV	1	17.3 nV	∞
Non-compensated EMF of the measurement circuit	0 V	B	Rect.	11.5 nV	1	11.5 nV	∞
Temperature coefficient of the Zener	*38.597 kΩ*	*B*	*Rect.*	*5 Ω*	*-0.32 nV/Ω*	*1.6 nV*	*10*
Pressure coefficient of the Zener	*980.5 hPa*	*B*	*Rect.*	*0.5 hPa*	*1.89 nV/hPa*	*0.9 nV*	*14*

[7]

Combined uncertainty……………………………………..…….. $u(U_z)$ = 22 nV

Relative combined uncertainty………………………… $u(U_z) / U_z$ = 22 nV/V

Effective degrees of freedom [†]……………..………..……..……………….v_{eff} = 7

Coverage factor [‡]……………………………………..………………… $k_{0.95}$ = 1.96

Expanded uncertainty (95%)………...……….……$U(U_Z) = k_{0.95} \times u(U_Z)$ = 43 nV

Relative expanded uncertainty…………………..…………….. $U(U_Z) / U_Z$ = 42 nV/V

Table 5a: Estimated standard uncertainties for a Zener calibration with the INRIM equipment at the level of 1.018 V for Zener Z7.

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[*] Type A uncertainty component of the applied microwave frequency *f* is already included in the measured voltage difference.

[†] Effective degrees of freedom v_{eff} are calculated with Welch-Satterthwaite formula.

[‡] Coverage factor $k_{0.95}$ is evaluated assuming a *t*-Student distribution with v_{eff} degrees of freedom.

Quantity	Estimate	Type	Dist.	Standard uncertainty	Sensitivity	Uncertainty contribution	Degree of freedom
Difference voltage measured by the nanovoltmeter	14.189 µV	A	Norm.	17.0 nV	1	17.0 nV	5
Microwave Reference frequency [*]	69.649999941 GHz	B	Rect.	4.0 Hz	14.6 pV/Hz	0.06 nV	∞
Voltage due to leakage current	0 V	B	Rect.	2.9×10^{-11}	1.018 V	0.03 nV	∞
Voltage due to gain error of the nanovoltmeter	0 V	B	Rect.	5.8×10^{-5}	14.189 µV	0.8 nV	∞
Voltage due to the non-linearity of the nanovoltmeter	0 V	B	Rect.	17.3 nV	1	17.3 nV	∞
Non-compensated EMF of the measurement circuit	0 V	B	Rect.	11.5 nV	1	11.5 nV	∞
Temperature coefficient of the Zener	*39.224 kΩ*	*B*	*Rect.*	*5 Ω*	*-0.59 nV/Ω*	*2.9 nV*	*10*
Pressure coefficient of the Zener	*980.5 hPa*	*B*	*Rect.*	*0.5 hPa*	*2.11 nV/hPa*	*1.1 nV*	*14*

[7]	**Combined uncertainty**………………………………..…….. $u(U_z)$ **= 27 nV**
	Relative combined uncertainty…………………………… $u(U_z) / U_z$ **= 27 nV/V**
	Effective degrees of freedom [†]……………….…………..……..……….v_{eff} = 31
	Coverage factor [‡]………………………….……….……………… $k_{0.95}$ = 2.04
	Expanded uncertainty (95%)……...…………...…$U(U_Z) = k_{0.95} \times u(U_z)$ = 55 nV
	Relative expanded uncertainty………………...…………….. $U(U_Z) / U_Z$ = 54 nV/V

Table 5b: Estimated standard uncertainties for a Zener calibration with the INRIM equipment at the level of 1.018 V for Zener Z9.

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[*] Type A uncertainty component of the applied microwave frequency *f* is already included in the measured voltage difference.
[†] Effective degrees of freedom v_{eff} are calculated with Welch-Satterthwaite formula.
[‡] Coverage factor $k_{0.95}$ is evaluated assuming a *t*-Student distribution with v_{eff} degrees of freedom.

Uncertainty contributions for the comparison INRIM/BIPM at 1.018 V

Table 6 lists the results and the uncertainty contributions for the comparison INRIM/BIPM at 1.018 V.

		Results/µV		Uncertainty/µV	
		Z7	**Z9**	**Z7**	**Z9**
1	INRIM (U_{INRIM} − 1.018 V)	110.02	95.97		
2	Type A uncertainty			0.007	0.017
3	correlated (Type B) unc.			0.021	
4	BIPM (U_{BIPM} − 1.018 V)	110.04	96.02		
5	Type A uncertainty			0.015	0.015
6	correlated (Type B) unc.			<0.005	
7	pressure and temperature correction uncertainty			0.001	0.003
8	(U_{INRIM} − U_{BIPM})	-0.02	-0.05		
9	Total uncorrelated uncertainty			0.017	0.023
10	Total correlated uncertainty			0.021	
11	< U_{INRIM} − U_{BIPM} >	**-0.04**			
12	*a priori* uncertainty			0.014	
13	*a posteriori* uncertainty			0.015	
14	**comparison total standard uncertainty/µV**			**0.03**	

Table 6: Results and uncertainties of INRIM (Italy)/BIPM bilateral comparison of 1.018 V standards using two Zener travelling standards: reference date 1 December 2023. Standard uncertainties are used throughout.

In Table 6, the following elements are listed:

(1) the value attributed by INRIM to each Zener U_{INRIM}, computed as the simple mean of all data from INRIM and corrected for temperature and pressure differences between both laboratories by the BIPM.

(2) the INRIM Type A uncertainty (cf. Tables 5a and 5b).

(3) the uncertainty component arising from the realization and maintenance of the volt at INRIM: it is the quadratic combination of the Type B components of the participant uncertainty budget listed in Tables 5a and 5b. This uncertainty is completely correlated between the different Zeners used for the comparison.

(4-6) the corresponding quantities for the BIPM referenced to the mean date of INRIM measurements. In this case, the Type A uncertainty is limited by the flicker noise level of 15 nV.

(7) the uncertainty due to the combined effects of the pressure and temperature coefficients [8, 9] and to the differences of the mean pressures and temperatures in the participating laboratories is calculated as follows:

The uncertainty of the temperature correction $u_{T,i}$ of Zener i is determined for the difference ΔR_i between the mean values of the thermistor resistances measured at both institutes which is then multiplied by the uncertainty $u(c_{T,i})$ of the relative temperature coefficients of each Zener standard:

$$u_{T,i} = U \times u(c_{T,i}) \times \Delta R_i$$

where U = 1.018 V, $u(c_{T,Z7})$ = 0.211 $\times$ 10^{-7} / kΩ, $u(c_{T,Z9})$ = 0.339 $\times$ 10^{-7} / kΩ, ΔR_{Z7} = 0.016 kΩ and ΔR_{Z9} = 0.092 kΩ.

The same procedure is applied for the uncertainty $u_{P,i}$ of the pressure correction for the difference ΔP_i between the mean values of the pressure measured at both institutes:

$$u_{P,i} = U \times u(c_{P,i}) \times \Delta P_i$$

where U = 1.018 V, $u(c_{P,Z7})$ = 0.041 $\times$ 10^{-9} / hPa, $u(c_{P,Z9})$ = 0.048 $\times$ 10^{-9} / hPa, ΔP_{Z7} = 26.4 hPa and ΔP_{Z9} = 26.2 hPa.

The uncertainties of the measurement of the temperature and the pressure are negligible.

(8) the difference ($U_{INRIM} - U_{BIPM}$) for each Zener, and (9) the uncorrelated part of the uncertainty, calculated as the quadratic sum of lines 2, 5 and 7.

(10) the correlated part of the uncertainty, calculated as the quadratic sum of lines 3 and 6, for each travelling standard.

(11) the result of the comparison is the simple mean of the differences of the calibration results for the different standards.

(12 and 13) the uncertainty related to the transfer, estimated by comparing the following uncertainties:

  (12) the *a priori* uncertainty, determined as the standard uncertainty of the mean, obtained by propagating the uncorrelated uncertainties for both Zeners;
  (13) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results.

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty (10) and of the larger of (12) and (13).

In this case the *a priori* uncertainty is comparable to the *a posteriori* uncertainty. We conclude that at 1.018 V both Zeners behaved consistently within the uncertainty of the comparison.

The result of the comparison is presented as the difference between the value assigned to a 1.018 V standard by INRIM, at INRIM, U_{INRIM}, and that assigned by the BIPM, at the BIPM, on the reference date of the 1st of December 2023:

$$U_{INRIM} - U_{BIPM} = \text{-0.04 } \mu V; \quad u_c = 0.03 \text{ } \mu V$$

where u_c is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at the BIPM, (based on K_J) and at INRIM and the uncertainty related to the comparison.

Conclusion

The final result of the comparison is presented as the difference between the values assigned to DC voltage standards by INRIM, at the level of 1.018 V and 10 V, at INRIM, U_{INRIM}, and those assigned by the BIPM, at the BIPM, U_{BIPM}, at the reference date of the 1st of December 2023.

$$U_{INRIM} - U_{BIPM} = -0.04 \ \mu V; \qquad u_c = 0.03 \ \mu V, \text{ at } 1.018 \text{ V}$$

$$U_{INRIM} - U_{BIPM} = -0.18 \ \mu V; \qquad u_c = 0.19 \ \mu V, \text{ at } 10 \text{ V}$$

where u_c is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at the BIPM and at INRIM, based on K_J, and the uncertainty related to the comparison.

These are very good results for both nominal voltages. The comparison results show that the voltages standards maintained by INRIM and the BIPM were equivalent, within their stated standard uncertainties which are consistent with the comparison result for $k = 1$.

The travelling standards behaved differently, because one of them (Z9) exhibited a voltage difference significantly larger than the difference measured for the second travelling standard (Z7), for both output nominal voltages.

- The hypothesis of a possible leakage introduced at the level of a scanner line was excluded since INRIM operates a single identical switch to measure all voltage outputs.
- The consistent behaviour of Z9 at the BIPM before and after INRIM at both voltages does not support the hypothesis of a transport-related effect.

The behavior of a zener secondary voltage standard is always questionnable in regards with its short term drift (typically during a comparison exercise) and long term drift (over the years). The tendency of these two drifts might be very different.

The two standards are equally maintained and regulary measured in the BIPM laboratories and this internal maintenance confirms their capabilities to be operated as transfer standards. Nonetheless, better results would have been achievable if standards of higher quality would exist.

The present comparison results could be improved by comparing the two PJVS directly. This option can definitively be considered in the future.

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