Electric Properties Tomography via Green's Integral Identity

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Electric Properties Tomography (EPT) is a quantitative imaging technique performed starting from data acquired during a Magnetic Resonance Imaging (MRI) exam [Leijsen (2021)]. EPT aims at measuring, in each voxel/pixel of the tomographic image, the electric conductivity and, when possible, also the dielectric permittivity of the scanned biological tissues, to be used as objective biomarkers.

In recent years, a plethora of EPT methods were proposed, but the creation of reliable EPT images is still challenging. Typical problems in EPT are represented by the amplification of the input measurement noise and the presence of image artifacts at the boundary between different tissues. To overcome these issues, here we propose an EPT approach based on Green's integral identity. The starting point is the complex Helmholtz equation describing the propagation of the B^+ component of the magnetic flux density (which rotates with angular frequency ω) through a body with conductivity σ and permittivity ε (that may change from point to point):

$$\nabla^2 B^+ = j\omega\mu_0 \left(\sigma + j\omega\varepsilon\right) B^+ \tag{1}$$

where j is the imaginary unit and μ_0 is the magnetic permeability of vacuum.

If the right-hand side of (1) were a known term, we would have a Poisson equation, whose solution, exploiting Green's integral identity, would take the form:

$$B^+\left(P\right) = \oint_{\partial\Omega} \left[\Psi \frac{dB^+}{dn} - B^+ \frac{d\Psi}{dn} \right] ds + j\omega\mu_0 \int_{\Omega} \left(\sigma + j\omega\varepsilon\right) B^+ \Psi dv \tag{2}$$

where P is a generic point within a region Ω surrounded by a surface $\partial\Omega$ (oriented according to an outward normal direction n) and Ψ is the Green function for 3D elliptic problems [Morse (1953)].

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During MRI, the spatial distribution of B^+ can be sampled in a regular grid of N_V voxels. If we assume that each i-th voxel is homogeneous, (2) can be rewritten as

$$B^+(P) = \oint_{\partial\Omega} \left[\Psi \frac{dB^+}{dn} - B^+ \frac{d\Psi}{dn} \right] ds + j\omega\mu_0 \sum_{i=1}^{N_V} (\sigma + j\omega\varepsilon)_i \int_{\Omega_i} B^+ \Psi dv \qquad (3)$$

In a general case, (3) is a complex equation involving N_V complex unknowns (the complex conductivities $\sigma + j\omega\varepsilon$ of the N_V voxels). By placing the reference point P at the barycentre of each voxel in turn, a system of N_V independent equations is obtained, from which the conductivity and permittivity can be calculated.

The proposed formulation has been specifically conceived to model heterogeneous regions (including discontinuities in the spatial distribution of the target parameters), avoiding image artifacts at the boundary between different tissues. Moreover, the use of the integrals in the right-hand side of (3) introduces a spontaneous compensation of the noise that affects the B^+ values used in the integrals themselves. Thus, the size of the region Ω in which the EPT formulation is applied must be chosen as a trade-off between the computational burden (the larger N_V, the bigger the algebraic system to solve) and the need for noise mitigation.

A residual issue resides in the noise that affects the reference value $B^+(P)$ in each equation of the system. To make the procedure more robust with respect to it, the unknowns in (3) can be grouped based on the tissue they belong to. This can be done exploiting the contrast image produced by the traditional MRI exam, which allows differentiating the different tissues in the scanned region. In this case, the number of unknowns becomes lower than N_V and the application of (3) produces a rectangular system of equations, which can be solved in the least-square sense.

If the magnitude of B^+ is quite homogeneous (a common condition in clinical MRI scanners), it can be shown that a phase-based version of the proposed formulation holds, which allows calculating the conductivity σ from the knowledge of the phase of B^+ only, hence simplifying the acquisition stage.

At the conference, the performances of the proposed method will be illustrated and compared to those of other state-of-the-art EPT techniques.

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