Determination and Uncertainty Propagation of Sensitivity Coefficients in Rockwell Hardness Measurements

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Abstract

In the field of hardness measurements, a problem arises when trying to understand how different measurement parameters (such as speed of the indenter, force, maximum displacement, thermal drift, etc.) affect the outcome of the measurement itself. Because the mathematical model defining hardness scales do not consider such factors, the simplest way to include additional influence parameters in the mathematical model is to introduce them linearly via sensitivity coefficients, which are obtained experimentally and thus characterized by uncertainties. However, uncertainties of the sensitivity coefficients are in general not considered in the evaluation of the combined standard uncertainty of the hardness measurements.
In this paper a general procedure is presented and applied to practical case-studies on HRA and HRC hardness measurements. In this field, the international definitions (standards) state each sensitivity coefficient for influence parameters about specific reference values: additional (generally neglected) uncertainty contributions reflect how much the measurements fail to be performed at exactly the prescribed reference values. Indeed, in the second case-study presented in this paper, we have found that the expanded uncertainty obtained via the proposed method is about 50 % larger than the one reported in the example of the international guidelines. This result highlights the importance of considering the uncertainties of the sensitivity coefficients when evaluating the combined standard uncertainty of any measurement.

Keywords: Sensitivity coefficients, Uncertainty propagation, Hardness measurements, Law of Propagation of Uncertainty.

1. Introduction

When dealing with the uncertainty analysis of a given model, the *law of propagation of uncertainty* (as stated in the GUM [1]) is applied: one defines a measurand Y and $\{X_i\}_{i=1}^N$ input quantities via a *functional* relationship f. Such functional relationship can be derived analytically and/or experimentally with regards to some (or all) of its input quantities. In the latter case, the easiest way to consider the effect of the input quantities in the measurement result is to introduce such input parameters linearly in the mathematical measurement model via *sensitivity coefficients*. Indeed, as stated in the GUM, such coefficients '[...] describe how the output estimate Y varies with changes in the values of the input estimates' (quote 5.1.3 of [1]) and can be '*[...] determined experimentally [...]. In this case, the knowledge of the function f [...] is accordingly reduced to an empirical first-order Taylor series expansion based on the measured sensitivity coefficients*' (quote 5.1.4 of [1]). This quote has to be interpreted in the *broader sense*: according to quote 4.1.2 of the GUM [1] the function f is '*that function which contains every quantity, including all corrections and correction factors, that can contribute a significant component of uncertainty to the measurement result*'. Therefore, when the sensitivity coefficients are associated with a non-negligible uncertainty contribution, they must be treated as any other *input quantity*.
The considerations above are of practical interest in the field of hardness measurement, where researchers have been trying to quantify- via experimentally determined sensitivity coefficients- how factors such as operating temperature, speed of indenter, force, etc. influence the hardness measurement itself [2, 3, 4, 5, 6, 7, 8, 9].
In this paper a Monte Carlo method applied to linear regression [10, 11] is used to deal with the determination of

sensitivity factors and their related uncertainties; then we will investigate how such uncertainties contribute to the combined standard uncertainty of the measurement. A general procedure is presented and applied to practical case studies on the HRA and HRC hardness measurements [4, 5, 12, 6, 9].

2. Evaluation and propagation of uncertainty

2.1. Law of propagation of uncertainty

Consider $\{X_i\}_{i=1}^{N}$ the set of N directly measurable *input* quantities and suppose we establish the following mathematical model for a generic measurand Y:

$$Y = \mathcal{F}(X_1, X_2, \ \dots \ , X_N). \tag{1}$$

According to the GUM framework, uncertainties of the input variables *propagate* following the *law of propagation of uncertainty* [1]: the combined uncertainty of the measurement can be calculated as:

$$\begin{gathered} u_Y^2 = \sum_{i=1}^{N} \left(\frac{\partial \mathcal{F}}{\partial X_i}\bigg|_{X_0} \right)^2 u^2(X_i) + \\ 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial \mathcal{F}}{\partial X_i}\bigg|_{X_0} \frac{\partial \mathcal{F}}{\partial X_j}\bigg|_{X_0} u(X_i, X_j), \end{gathered} \tag{2}$$

where $u(X_i)$ and $u(X_i, X_j)$ are respectively the standard uncertainty of X_i and the covariance term related to X_i, X_j. The partial derivatives $\partial\mathcal{F}/\partial X_i$, evaluated at the expectations $\boldsymbol{X}_0$ of the point $\boldsymbol{X}$, are called *sensitivity coefficients* [1] and denoted with c_i; (2) can be rewritten in the form:

$$u_Y^2 = \sum_{i=1}^{N} c_i^2 u^2(X_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_i \, c_j u(X_i, X_j) \tag{3}$$

2.2. Sensitivity coefficients with non-negligible uncertainties

If the sensitivity coefficients are calculated from the mathematical model, they are stated as numerical values with no associated uncertainty. However, when obtained experimentally (quote 5.1.4 of [1]), such coefficient has a variability and therefore has to be treated as a *random variable*. As stated in Section 1 of this paper, according to quote 4.1.2 of the GUM [1], if the uncertainty of the sensitivity coefficient contributes significantly to the uncertainty of the measurement result, then such coefficient has to be dealt as an *input quantity*. With that in mind, and underlining the conceptual difference between the sensitivity coefficient determined from the mathematical model (Section 2.1) and the experimentally determined sensitivity coefficient with non-negligible uncertainty, (1) is rewritten as:

$$Y = \widetilde{\mathcal{F}}(c_1, X_1; \dots ; c_N, X_N) \tag{4}$$

In this case, the law of propagation of uncertainty[1] also includes the c_i uncertainty terms, namely:

$$\begin{gathered} \sum_{i=1}^{N} \left(\frac{\partial \mathcal{F}}{\partial c_i}\bigg|_{\boldsymbol{c}_0, \boldsymbol{X}_0} \right)^2 u^2(c_i) + \\ +2 \sum_{i=1}^{N-1} \sum_{j>i}^{N} \frac{\partial \mathcal{F}}{\partial c_i}\bigg|_{\boldsymbol{c}_0, \boldsymbol{X}_0} \frac{\partial \mathcal{F}}{\partial c_j}\bigg|_{\boldsymbol{c}_0, \boldsymbol{X}_0} u(c_i, c_j) + \\ + \sum_{i,j=1}^{N} \frac{\partial \mathcal{F}}{\partial c_i}\bigg|_{\boldsymbol{c}_0, \boldsymbol{X}_0} \frac{\partial \mathcal{F}}{\partial X_j}\bigg|_{\boldsymbol{c}_0, \boldsymbol{X}_0} u(c_i, X_j), \end{gathered} \tag{5}$$

where c_{i0} is the expectation of the sensitivity coefficients c_i.
Such an analysis is important in many practical applications (as will be seen in the next section): for instance, in

the *international definitions* (standards), each sensitivity coefficient is stated for influence parameters about specific *reference values*. Variations δX_i of such parameters from their reference values X_i^{ref} must be taken into consideration in the evaluation of the combined standard uncertainty. Indeed, in (5) the partial derivatives $\partial\mathcal{F}/\partial c_i$ are evaluated at

$$\begin{aligned} \boldsymbol{c} &= \boldsymbol{c}(X_1^{\mathrm{ref}}, \dots, X_N^{\mathrm{ref}}), \\ \boldsymbol{X}_0 &= (\delta X_1 + X_1^{\mathrm{ref}}, \dots, \delta X_N + X_N^{\mathrm{ref}}) \end{aligned} \tag{6}$$

Therefore, the expectations of the variations δX_i from the reference values X_i^{ref} account for additional uncertainty contributions, which reflect how much the measurement fails to be performed at exactly the prescribed reference values.

3. Case study: Hardness measurements and the problem of traceability

One application of the problems introduced in the previous section is related to the field of hardness measurements. In particular, we will focus on the measuring methods related to the assessment of metals hardness where an indentation is made on a test piece and, according to the model used (i.e. Rockwell, Brinell, Vickers, Knoop), some characteristic dimensions have to be determined. A problem arises when trying to understand how different measurement parameters affect the outcome of the measurement itself [4, 13, 14]. Understanding these issues is of fundamental importance for the CCM Working Group on Hardness of Consultative Committee of Mass and Related Quantities of the CIPM (CCM-WGH) when establishing the *international definitions* to be applied by National Metrology Institutes (NMIs) [15].
Most of the *mathematical* models defining hardness scales do not directly include factors (such as the speed of the indenter, the force, maximum displacement, thermal drift, etc.) that *still* have to be taken into account in order to follow the standard measuring procedures. For example, the Rockwell hardness model [16, 12]:

$$\mathrm{HR} = N - \frac{h}{S} \tag{7}$$

(where N and S are constants), simply takes as an input variable the indentation depth h, but does *not* state how the force intensity, speed of the indenter, force application dwell times, contact area or other potential key factors influence the hardness measurement HR. On the other hand, ISO 6508 [12, 17, 18] states some standard prescriptions to be followed during the measurements; for example:

- temperature T_{lab} conditions: if T_{lab} is not in the appropriate range, then the laboratory has to state how temperature affects the measurement
- timing of the different moments of the force applications: this aspect is related to the plasticity of the material under test (indentation creep, material recovery, etc.)
- velocity of the indenter
- depth-measurement systems
- machine hysteresis

Therefore, it is important to study the effect of *additional variables* to identify which parameters are significant in the measurement result. As a first approximation, a linear model can be assumed to take into consideration the additional variables and experiments are needed to establish if such parameters are of influence. Once each parameter has been determined, a *reference numerical value* has to be chosen for the international definition [4, 12]. In order to choose such a reference number and to evaluate the associated uncertainty contribution, sensitivity coefficients must be determined experimentally. In addition, knowledge of the sensitivity coefficients is fundamental to properly assess tolerance intervals as prescriptions in the related standards [17, 18], in order to obtain a *predetermined* maximum

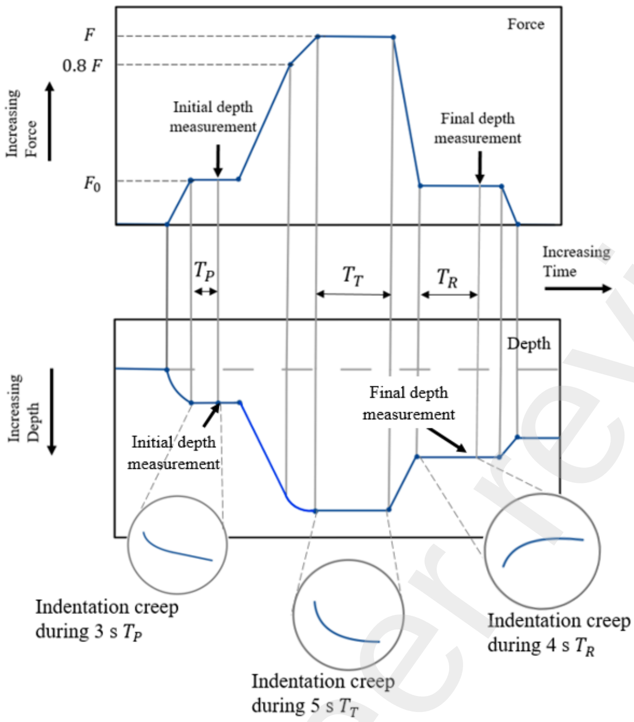

Figure 1. Rockwell hardness testing cycle: due to material creep and material recovery (shown in the close-ups), the effects of different force dwell times have to be investigated.

uncertainty value.

To introduce all the influence parameters, from (7) we can postulate the following generalized mathematical model:

$$\mathrm{HR} = N - \frac{h}{S} + \sum_{i=1}^{N} c_i\, X_i, \tag{8}$$

From a careful experimental design, one can obtain the (experimental) sensitivity coefficients c_i related to variables X_i and the combined standard uncertainty of the hardness measurement can be calculated, also taking into account the uncertainty of the sensitivity coefficients.

In the following, two case studies are presented: `CASE A` shows a procedure to identify if a given sensitivity coefficient can be considered *indicative* (according to the experiments performed) of a dependence of the measurement model on the proposed influence parameter, data from [5] will be used; `CASE B` shows the significance of considering the uncertainty of the sensitivity coefficients on the combined standard uncertainty of the measurement result; data from [4] will be used.

3.1. Case A: Evaluation of the sensitivity coefficients

In the field of Rockwell scale HRA, Low and Machado [5] determined the test cycle sensitivity factors in order to investigate how specific dwell times influence the hardness measurement of different materials (figure 1). In this case, the three *additional* variables are the preliminary-force (P) dwell time, total-force (T) dwell time and recovery-force (R) dwell time. Thus, the modified HR model becomes:

$$\mathrm{HR} = N - \frac{h}{S} + c_{\mathrm{P}}\, \delta T_{\mathrm{P}} + c_{\mathrm{T}}\, \delta T_{\mathrm{T}} + c_{\mathrm{R}}\, \delta T_{\mathrm{R}}, \tag{9}$$

where δT_i are the differences between the expectations of the actual *measured* values and the *reference values* stated in the international definitions. As in [5], we consider measurements performed on steel reference blocks at three different nominal hardness levels (63 HRA, 73 HRA, 83 HRA). To evaluate the sensitivity coefficients, multiple experiments were performed varying one variable at a time, while keeping the other variables as constant as possible. The uncertainty of each sensitivity coefficient has been evaluated via a Monte Carlo method applied to linear regression, in order to take into account both the variability on the input and output quantities: table 1 summarizes the results.
It has to be stressed that in [5], the expanded uncertainty of the hardness measurements was *simply* assigned to each sensitivity coefficient. In this case, although *evidently* non-negligible, the uncertainties of the sensitivity coefficients have not yet been used in the evaluation of the combined standard uncertainty of the measurement.

Table 1. Sensitivity coefficients and their expanded uncertainties $U_{95\%}$: reference dwell times about $\{3, 5, 4\}$ s respectively for preliminary, total and recovery dwell time.

Nominal	c_P/(HRA/s) (3s)	c_T/(HRA/s) (5s)	c_R/(HRA/s) (4s)
83 HRA	0.0004 ± 0.0098	-0.0149 ± 0.0072	0.0038 ± 0.0065
73 HRA	0.0120 ± 0.0173	-0.0210 ± 0.0102	0.0067 ± 0.0945
63 HRA	0.0347 ± 0.0301	-0.0490 ± 0.0512	0.0256 ± 0.0298

As far as the physical meaning of the results is concerned, it can be seen that few of the sensitivity coefficients are *indicative* of a significant correlation between some dwell times and the actual hardness measurement result. In figure 2, for the nominal hardness 83 HRA, the slopes of the linear fitting curves represent the sensitivity coefficients; such slopes show how the total and preliminary dwell times individually[1] affect the hardness measurement. In order to verify whether the sensitivity coefficient (i.e. the slope) is significant we proceed as follows:

1. the slope should be compared to its expanded uncertainty: if the value of the slope (absolute value) is larger than its (expanded) uncertainty, it is considered *indicative*.
2. if the total hardness estimation range ($\mathrm{HRA_{Max}} - \mathrm{HRA_{Min}}$) is larger than the mean (expanded) uncertainty of the experimental measurement results, the sensitivity coefficient is considered *indicative*.

If at least one of the two requirements above is not satisfied, then the sensitivity coefficient is *not indicative* for the given statistical risk of error (5 % for an expanded uncertainty given with a 95 % confidence level). In case of figure 2(a), the Total dwell time sensitivity coefficient seems to be indicative, because 1) the slope is $|-0.015|$ HRA/s with an expanded uncertainy of 0.007 HRA/s and 2) the total hardness estimation range is about 0.15 HRA while the mean expanded uncertainty is about 0.04 HRA; therefore, both conditions are satisfied. On the other hand, figure 2(b) shows a non indicative sensitivity coefficient, because 1) the slope is 0.0004 HRA/s with an expanded uncertainty of 0.0098 HRA/s and 2) the total hardness estimation range is about 0.002 HRA while the mean expanded uncertainty is about 0.036 HRA; thus, *both* conditions are not satisfied. The same observations can be made regarding all the other sensitivity coefficients for the different hardness measurements.

3.2. Case B: Evaluation of the expanded combined standard uncertainty

In order to see how the uncertainty contributions of the sensitivity coefficients influences the combined standard uncertainty, the data provided in the international 'Guidelines on the estimation of uncertainty in hardness measurements' [4] has been used.
In (8), the following additional parameters X_i have been considered: preliminary test force F_0, indentation velocity v, total test force F, indenter radius r, indenter angle α, preliminary test force dwell time t_0. In table 2, the sensitivity coefficients are reported with the associated uncertainties: as a first approximation, no uncertainty has been assigned to r and α, since those quantities were obtained via an ideal mathematical-physical model[2]. With such information, we

[1] Meaning: varying one of the variable, when the others are kept constant in the experiments.
[2] Such a model (for r and α) could not be fully representative of the actual physical phenomenon, so the related uncertainties can be investigated in future works.

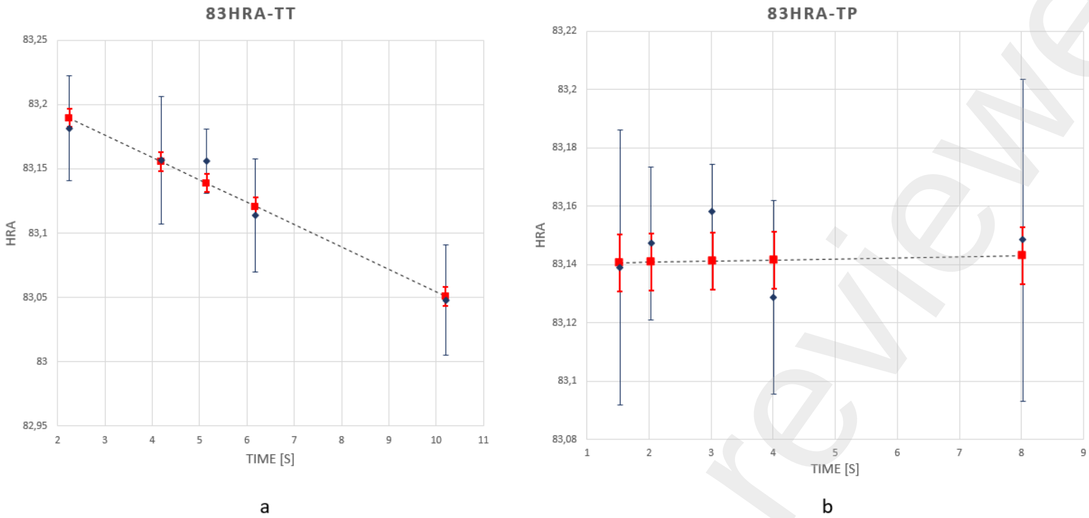

Figure 2. Nominal hardness 83 HRA, change in hardness measurement due to: **a**) Total dwell time; **b**) Preliminary dwell time. In blue the expanded uncertainties (95 % confidence level) of the experimental measurements, in red the expanded uncertainties (95 % confidence level) of the related sensitivity coefficient.

can re-evaluate the expanded uncertainties in [4] for a nominal hardness of about 60 HRC. The results are summarized in table 3: `rows a-b` report the expanded uncertainties with, respectively, negligible and non-negligible sensitivity coefficients, assigning to each parameter exactly its *reference value* (i.e. $\delta X_i = 0$) with a variability defined by the tolerances given in the related standards (table 4.2 in [4]); `rows c-d` report the expanded uncertainties with the same two methods as before, where δX_i are the expectations of the *actual* variations of each parameter from its *reference value*, with a variability given as the measurement uncertainty of each parameter (as in the example given in table 4.5 in [4] but applied for a nominal hardness about 60 HRC).
While for the first two rows, the expanded uncertainty is almost the same applying the two methods, last two rows show a significant difference that can be explained by considering the law of propagation of uncertainty. As a first approximation, neglecting the covariance terms, the expanded uncertainty is evaluated as:

$$u_{\mathrm{HR}}^2 = \frac{u^2(h)}{S^2} + \sum_i c_i^2 u^2(\delta X_i) + \sum_i \delta X_i^2\, u^2(c_i) \tag{10}$$

where in case of `rows c-d`, δX_i is not null. Indeed, an *additional* uncertainty contribution is obtained from the variations of the parameters from their reference values: such new terms $\delta X_i\, u(c_i)$ would have not been considered in case of negligible (or null) uncertainties of the sensitivity coefficients.
Therefore, it is shown how the results given applying the presented method can be used to

- determine the tolerance limits of the testing cycle parameters given in the related Standards, i.e. dwell times, velocities, shape of indenters, etc.
- verify that the actual tolerances assure hardness variations inside the expected uncertainty of the method

It must be noticed that, if for the sensitivity coefficients c_α, c_r non-null uncertainties were given, one would have observed a *more pronounced* difference between the results applying the two methods.
As a final remark, we underline that in [4] the modified model is:

$$\mathrm{HR} = N + c_h\, h + \sum_i c_i\, \delta X_i, \tag{11}$$

where $c_h = -1/S$; the uncertainty of c_h is indeed null, since the sensitivity coefficient is obtained from the analytical model.

Table 2. Estimate values for the input variables as in table 4.2 in [4]; related expanded uncertainties $U_{95\%}$ evaluated by experimental results [6, 13, 14].

	`Estimate`	U
c_{F_0} [HRC/N]	0.05	0.02
c_F [HRC/N]	0.02	0.003
c_α [HRC/°]	0.04	0
c_r [HRC/μm]	0.05	0
c_h [HRC/μm]	0.5	0
c_v [HRC/(μm/s)]	0.03	0.01
c_{t_0} [HRC/s]	0.004	0.003
c_t [HRC/s]	0.03	0.02

Table 3. `rows a-b`: expanded uncertainties with, respectively, negligible and non-negligible sensitivity coefficients, each parameter evaluated at its *reference value* and variability defined by the tolerance given in table 4.2 in [4]; `rows c-d`: expanded uncertainties with the same two methods, with the *actual* variations of each parameter from its *reference value* and variability given in table 4.5 in [4](for 60 HRC).

		U
a	`HRC EURAMET [4]` $\delta X_i = 0$	1.26
b	`HRC MODIFIED` $\delta X_i = 0$	1.27
c	`HRC EURAMET [4]`	0.07
d	`HRC MODIFIED`	0.11

4. Conclusions

A specific procedure for the evaluation of the combined standard uncertainty in case of sensitivity coefficients with non-negligible uncertainties has been developed in this paper. The method has been applied to the case of hardness measurements: **Case A** with experimental data of Rockwell A measurements from [5]; **Case B** with the data of Rockwell C from [4] and [13, 14].

Case B shows that neglecting and non-neglecting the uncertainties of the sensitivity coefficients yield similar results when the parameters are evaluated at their reference values with a variability defined by the tolerance given in the related *standards*. In the case where the parameters are experimentally measured (bias and its uncertainty) the proposed method results in *additional* contributions to the combined standard uncertainty of the measurement: such contributions account for how much the measurements fail to be performed at exactly the prescribed reference values stated in the international definitions (standards). In the case study, the expanded uncertainty obtained via the presented method is about 50 % larger than the one reported in the example of the international guidelines [4]. Therefore, when estimating the sensitivity coefficients of influence variables, it is important to evaluate their uncertainty and consider such contributions when evaluating the combined standard uncertainty of the measurement.

Conflict of interests

The Authors declare that they have no conflict of interests.

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