The role of influence coefficients in hardness measurements: A case study in Rockwell hardness measurements

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Abstract

In the field of hardness measurements, a problem arises when trying to understand the effect of different measurement parameters (i.e. speed of the indenter, force, thermal drift) on the measurement itself. Since the mathematical models defining hardness scales do not consider such factors, additional influence parameters are introduced linearly via influence coefficients, obtained experimentally and thus characterized by uncertainties. However, uncertainties of the influence coefficients have never been considered in the evaluation of the combined standard uncertainty of hardness measurements. In this paper, the law of propagation of uncertainty is applied taking into account the uncertainty contributions of the influence coefficients. We apply such a procedure to a case study that shows how the presented method can be used to determine the tolerance limits of the testing cycle parameters given in the related standards and verify that the actual tolerances assure hardness variations inside the expected uncertainty of the method.

Keywords: Influence coefficients, Uncertainty propagation, Hardness measurements, Law of Propagation of Uncertainty

1. Introduction

When dealing with the uncertainty analysis of a given model, the law of propagation of uncertainty (as stated in the GUM [11]) is applied: one defines a measurand Y and $\{X_i\}_{i=1}^{N}$ input quantities via a functional relationship f. Such a functional relationship can be derived analytically and/or experimentally with regard to some (or all) of its input quantities. In the latter case, experiments are performed to study how a certain input quantity affects the measurement output. This situation is particularly interesting when a pre-existing model has to be modified to include the effects of *additional* parameters. As a very first approximation (as is the case in most practical scenarios), the relationship between the measurement output and the additional input quantities can be assumed to be linear: in this case, the pre-existing mathematical model can be modified by introducing the additional parameters linearly via *influence coefficients*. Most of the time, the difficulty (or even impossibility) of deriving a (new) complete mathematical model considering the additional parameters leads to the determination of an experimental model, where the influence coefficients are determined experimentally and thus characterized by uncertainties.
Such a scenario has been the central topic of research in the field of hardness measurements. In the last couple of decades, scientists have been trying to quantify- via experimentally determined influence coefficients- how

parameters such as operating temperature, speed of indenter, force, and other factors influence the hardness measurements [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. For each parameter, a *reference value* has been prescribed in the international definitions [16] to standardize the hardness measurement procedures and assure hardness variations inside a common unified uncertainty of the method. However, for practical purposes, the standards define reference values with an associated interval, which represents the admissible range of values for such parameters. In this spirit, experimental campaigns carried out by National Metrology Institutes (NMIs) in the field of hardness measurement led to the experimental determination of influence coefficients. As stated in [12], while it is true that NMIs have published papers regarding the determination of some of such influence coefficients, nearly none of them evaluated their uncertainties and, when such uncertainties were published, no clear explanation was given of how they were obtained. Even further, to the best of our knowledge, no published paper has dealt with the consequence of considering the uncertainties of the influence coefficients in the overall uncertainty of the model.

In this paper, it is shown how considering the influence coefficients and their uncertainties allows us to correct the hardness measurements and more accurately estimate the overall uncertainty. Such considerations are of fundamental importance for the CCM Working Group on Hardness of Consultative Committee of Mass and Related Quantities of the CIPM (CCM-WGH) when establishing international definitions or during international comparisons between NMIs or accredited laboratories. As an explanatory example, if two 'identical' NMIs perform some measurements on perfectly homogeneous and identical hardness reference blocks and one performs all the measurements at exactly the reference values (improbable) and the other does not but it still is in accordance with the standards and we suppose that all the other sources of uncertainties are the same (improbable again), according to the current procedures the two measurements should have the same uncertainty budget. Instead, it is expected that the first NMI has a lower uncertainty budget than the other one. Even further, we may ask ourselves how to choose a reference value for a possible new influence parameter.

In this paper, we investigate how the uncertainties of the influence coefficients contribute to the overall standard uncertainty of the measurement and how considering such uncertainty contributions proposes a solution to the problems presented above.

2. Hardness measurements and the problem of traceability

Most of the mathematical models defining hardness scales do not directly include factors (such as the speed of the indenter, the force, maximum displacement, thermal drift, etc.) that still have to be taken into account to follow the standard measuring procedures. For example, the Rockwell hardness model [13, 14]:

$$HR = N - \frac{h}{S} \tag{1}$$

(where N and S are constants as defined in the related standards [13, 14, 15]), simply takes as an input variable the indentation depth *h*, but does not state how the force intensity, speed of the indenter, force application dwell times, contact area, or other potential key factors influence the hardness measurement *HR*. On the other hand, ISO 6508 series [13, 14, 15] state the parameters (and their admissible ranges) to be controlled during hardness measurements. Those are, for example:

- laboratory temperature T_{lab};
- application and duration times of the different applied forces;
- velocity of the indenter;

- depth-measurement systems;
- machine hysteresis.

Therefore, it is important to study the effect of such additional variables on the measurement result. The steps to be taken are schematized as follows:

1) As a first approximation, a linear model can be assumed to take into consideration the additional variables and experiments are needed to establish their influence. Once determined, a numerical reference value is chosen for the international definitions [14, 15,16].

2) In order to choose such a reference number and to evaluate the associated uncertainty contribution, influence coefficients must be determined experimentally and their uncertainty contribution has to be propagated via the law of propagation of uncertainty [11] to evaluate the combined standard uncertainty of the hardness measurement. Knowledge of the influence coefficients is fundamental to properly assess tolerance intervals as prescriptions in the related standards [14, 15], in order to obtain a predetermined maximum uncertainty value.

To introduce the N additional influence parameters, the following linearized mathematical model can be postulated:

$$HR = N - \frac{h}{S} + \sum_{i=1}^{N} c_i X_i \qquad (2)$$

where X_i is the value of the i-th influence parameter and c_i is its influence coefficient. From a careful experimental design (as carried out in [12]), one can obtain statistical information (e.g. sample mean and standard deviation) of both c_i and X_i. Once this has been done, it is possible to evaluate the combined standard uncertainty of the hardness measurement, considering the contribution of the influence coefficients c_i, which are usually neglected in international comparisons [2, 12].
As briefly described in the introduction, international definitions of hardness scales define the influence parameters and prescribe exact reference values for each of them. For instance, according to the definition of the Rockwell Hardness HR45N scale [16], the reference values for the total and recovery dwell times are respectively 5 s and 4 s, for the temperature of the test is 23°C, for the mean indentation velocity is 30 μm s^{-1}, etc. However, in the related standards, the reference values are stated with an associated interval, which represents the admissible values for such parameters. Such intervals are given due to both technical and physical reasons and to meet industrial practical needs. For instance, the temperature of the test can change the viscosity of the hardness block and the hardness measurement can be consequently affected (physical reason); in addition, it is common in most laboratories to work within a temperature range around 20 °C, and related quantities (i.e. length standards) are defined around this temperature, hence a reference value can be chosen to be 23 °C (technical reason).
However, a recurrent problem is often encountered in international comparisons between NMIs. In this case, being in accordance with the standards does not convey the full picture of the measurement results. The problem presented in the introduction section is a common scenario. Two identical NMIs with the same uncertainty budget carry out some hardness measurements: the first one performs all the measurements at exactly the

reference values, while the other one does not. In this way, the measurement results and uncertainty analysis are affected by the two different measurement procedures, although both meet standard requirements. To overcome the problem, it is proposed to apply a correction to the generalized model using the previously evaluated influence coefficients in order to refer hardness measurements to the reference values (HR^{ref}):

$$HR^{\mathrm{ref}} = N - \frac{h}{S} + \sum_{i=1}^{N} \Delta HR_i = N - \frac{h}{S} + \sum_{i=1}^{N} c_i \underbrace{\left(X_i - X_i^{\mathrm{ref}}\right)}_{\Delta X_i} \quad (3)$$

where ΔX_i is the difference between the measured input quantity X_i and its reference value X_i^{ref} of the i-th influence parameter. In the limiting case, when all parameters coincide with their reference values, the correction is null and the usual classical model is obtained (Eq. (1)).
Regarding the uncertainty of the above-mentioned model, considering the linearized modified model, neglecting the covariance terms and higher order terms*, we get:

$$u^2(HR^{\mathrm{ref}}) = \frac{u^2(h)}{S^2} + \sum_{i=1}^{N}\left(c_i^2 u^2(\Delta X_i) + \Delta X_i^2 u^2(c_i)\right) = \frac{u^2(h)}{S^2} + c_1^2 u^2(\Delta X_1) + \cdots + c_N^2 u^2(\Delta X_N) + \\ + \underbrace{\Delta X_1^2 u^2(c_1) + \cdots + \Delta X_N^2 u^2(c_N)}_{u_{\mathrm{add}}^2} \quad (4)$$

Up to now, to the best of our knowledge, the last terms on the right-hand-side of equations (3) and (4) have always been neglected as also shown in EURAMET cg-16 Version 2.0 document about "Guidelines on the Estimation of Uncertainty in Hardness Measurements" [2]. However, once the influence coefficients are stated with their standard uncertainties by the NMIs, such terms cannot be neglected. One can observe the matter more generally regarding the correction procedure as itself associated with uncertainty (which is true if the influence coefficients are experimentally determined). Hence, when different NMIs report the corrected hardness measurements, then the additional uncertainty contributions u_{add}^2 come into play. These additional uncertainty contributions reflect how much the measurements failed to be performed at exactly the reference values.
From the above consideration, it can be seen the importance of a unified procedure for both the evaluation of influence coefficients and, most of all, their uncertainties.

3. A Case study on superficial Rockwell Hardness HR45N

To show the proposed method, we consider the case of HR45N superficial Rockwell Hardness measurement. The international definition for HR45N [16] defines the reference value of the mean indentation velocity of final additional test force $V_{\mathrm{fa}}^{\mathrm{ref}} = 30\ \mu\mathrm{m}\cdot\mathrm{s}^{-1}$ and prescribes the application time of the additional test force

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* The problem of the covariance terms deserves an entire paper which is out of the scope of the current paper, which is to present a general conceptual picture. The Authors will deal with the issue in a more formal way in a future publication and with more collected data from NMIs.

$t_{aa}{}^{ref}$ to be $\leq$ 4 s. The prescribed conditions appear to be less stringent for t_{aa} rather than for V_{fa}, making the latter parameter the candidate for our case study. Indeed, small variations of the additional time t_{aa} do not sensibly affect the measurement, as far as the additional force velocity V_{fa} is controlled about its reference value [3]. Now, the proposed corrected linear model, considering only the velocity V_{fa} as an additional influence parameter is:

$$HR45N^{ref} = N - \frac{h}{S} + c_{V_{fa}}(V_{fa} - V_{fa}^{ref}) \tag{5}$$

The proposed combined uncertainty (again neglecting covariance terms and higher order terms) is evaluated as:

$$u^2(HR45N^{ref}) = \frac{u^2(h)}{S^2} + c_{V_{fa}}^2 u^2(\Delta V_{fa}) + (\Delta V_{fa})^2\, u^2\left(c_{V_{fa}}\right) \tag{6}$$

Now, as an example, we consider the case of an inter-laboratory comparison where two NMIs (NMI_1 and NMI_2) perform hardness measurements and report the same indentation depth h with its standard uncertainty $u(h)$, the same influence coefficient $c_{V_{fa}}$ with its standard uncertainty $u(c_{V_{fa}})$, but different mean indentation velocities of final additional test force V_{fa} with same uncertainty $u(V_{fa})$. At present, both NMIs would get the same results and uncertainties using Eq. (3) and Eq. (4) without the last terms, respectively. However, it can be seen how performing measurements at different values of the velocity V_{fa} (even with the same uncertainties) allowed by the ISO 6508:2014 standard (i.e. $15\ \mu\mathrm{m} \cdot \mathrm{s}^{-1} \leq V_{fa} \leq 40\ \mu\mathrm{m} \cdot \mathrm{s}^{-1}$), yields different measurement results and different combined standard uncertainties. From [12], two institutes (INRiM and NPL) published mean values and uncertainties of the influence coefficient V_{fa}. In that work, the experimental and data analysis procedures are carefully described in detail. They found values in the order $10^{-3} - 10^{-2} HR\ s\ \mu\mathrm{m}^{-1}$. For the sake of simplicity, we can choose INRiM values for $c_{V_{fa}}$ and $u(c_{V_{fa}})$ and evaluate the quantity $u(HR45N^{ref})$ for different values of ΔV_{fa} and $u(\Delta V_{fa})$.

As a limiting and simplified example, we set $h_{NMI_1} = h_{NMI_2} = 0.05\ mm$ with $u\left(h_{NMI_1}\right) = u\left(h_{NMI_2}\right) =$ 0.15 µm, $V_{\mathrm{fa,NMI_1}} = 30\ \mu\mathrm{m\ s}^{-1}$ with $u\left(V_{\mathrm{fa,NMI_1}}\right) = 0.25\ \mu\mathrm{m\ s}^{-1}$, $c_{V_{\mathrm{fa}},\mathrm{NMI_1}} = c_{V_{\mathrm{fa}},\mathrm{NMI_2}} = -0.006$ HR s µm^{-1} with $u\left(c_{V_{\mathrm{fa}},\mathrm{NMI_1}}\right) = u\left(c_{V_{\mathrm{fa}},\mathrm{NMI_2}}\right) = 0.021$ HR s µm^{-1}, but consider, for NMI_2, two different scenarios for the mean value of the velocity $V_{\mathrm{fa,NMI_2}}$:

1) $V_{\mathrm{fa,NMI_2,1}} = 40$ HR s µm^{-1}
2) $V_{\mathrm{fa,NMI_2,2}} = 15$ HR s µm^{-1}

both with an uncertainty equal to NMI_1, $u\left(V_{\mathrm{fa,NMI_2}}\right) = u\left(V_{\mathrm{fa,NMI_1}}\right) = 0.25$ µm. In this way, the difference between NMI_1 and NMI_2 consists only of two different mean values of velocity V_{fa}.

According to the current procedures (no corrections and negligible uncertainties of influence coefficients), the mean value and standard uncertainty for the given h and $u(h)$ are respectively $HR45N = 50$ HR45N and $u(HR45N) = 0.15$ HR45N.

By applying the corrections and the combined standard uncertainties with the proposed method, different results are obtained, in this way, as shown in Table 1.

NMI	ΔV_{fa} / µm s^{-1}	$u(\Delta V_{\mathrm{fa}})$ / µm · s^{-1}	$c_{V_{\mathrm{fa}}}$ /HR s µm^{-1}	$u(c_{V_{\mathrm{fa}}})$ /HR s µm^{-1}	$HR45N^{ref}$ / HR45N	$u(HR45N^{ref})$ /HR45N
NMI_1	0	0.25	−0.006	0.021	50.00	0.15
NMI_2	−15	0.25	−0.006	0.021	50.09	0.34
	10	0.25	−0.006	0.021	49.94	0.25

Table 1: Corrected $HR45N^{ref}$ values and combined standard uncertainties with the proposed method for the case-scenario.

From Table 1, it can be seen how NMI_1 has the lowest uncertainty and the correction is exactly null. However, considering the uncertainty of such coefficient yields different combined uncertainties $u(HR45N^{ref})$. As additional evidence for intermediate values, in Figure 1, it is shown how the uncertainty $u(HR45N^{ref})$ changes for different values of ΔV_{fa} from -15 µm s^{-1} up to 10 µm s^{-1}. As expected from Eq. (6), it can be seen that hardness uncertainty linearly increases at increasing ΔV_{fa}.

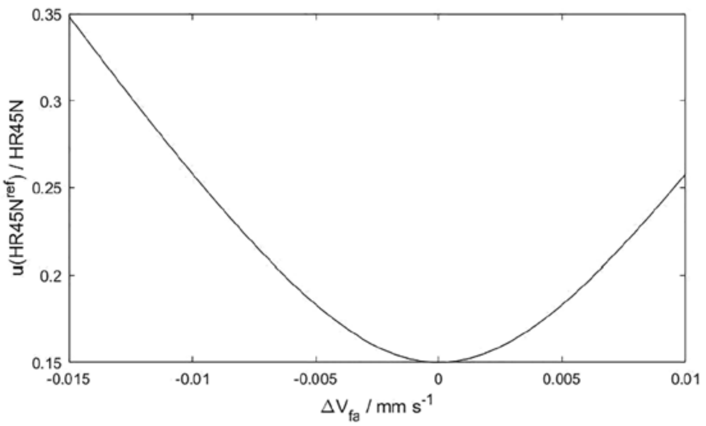

Figure 1: Uncertainty $u(HR45N^{ref})$ as function of ΔV_{fa}.

To sum up, the problems reflected by the numerical case are the following:

- When evaluating the influence coefficients, NMIs should provide both the mean and uncertainty of such coefficients: to the best of our knowledge there are still no unifying methods regarding the evaluation of the uncertainties of the influence coefficients, as pointed out in [12]. The need for such unification in this regard is paramount.
- Once the matter of a unifying procedure for the determination of the uncertainty of the influence coefficients is settled, then comparisons between different measurement conditions can be evaluated in terms of standard uncertainties.
- Finally, in the overall uncertainty evaluation, the role of the uncertainties of the influence coefficients reflects both the deviation from the standard measurement procedures (deviation from reference values) and the quality of the correction: for instance, if a laboratory states an influence coefficient with zero mean and non-null uncertainty, then even though the mean hardness measurement is not corrected, the combined standard uncertainty is still affected. We qualitatively showed that the higher the uncertainty of the influence coefficient, hence the higher the chance that such coefficient is not indicative of a linear relationship between measurement output and additional parameter, the less

accurate the correction becomes. This can be seen in the case where additional uncertainty contributions can arise even when the mean value of the influence coefficient is zero (surface b, fig.1). An experimental campaign between NMIs will certainly lead to interesting discussions and a more realistic uncertainty analysis than the one sketched in this introductory paper.

4. Conclusions

In this paper, a linear generalized mathematical model is suggested to consider the effect of additional influence parameters in hardness measurements. In particular, the role of the experimentally determined influence coefficients is analyzed. To the best of the authors' knowledge, no published paper in the field of hardness measurements has dealt with the problem of evaluating the combined standard uncertainty of the model when the influence coefficients are associated with non-negligible uncertainties. Even further, in most cases, measurements are not corrected at all, as long as the influence parameters are in the range specified by the standards. Such considerations are important as far as international comparisons are concerned. In the case study, we present a simplified scenario to qualitatively show how the uncertainty of the model varies with respect to the mean and uncertainty of the influence coefficients; in addition, it is shown how not performing the measurements at exactly the reference values, prescribed in the international definitions, yields additional uncertainty contributions. Such contributions have never been considered in the field of hardness measurements and can be applied in other measuring applications. The example shows the need for a unifying procedure when dealing with the determination of influence coefficients, in particular regarding their uncertainty.

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Conflict of interests

The Authors declare that they have no conflict of interests.

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