A Local Gauge Description of the Interaction Between Magnetization and Electric Field in a Ferromagnet

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We apply a non-abelian gauge field approach to generalize the micromagnetic energy description of a ferromagnet. This approach, without further assumption, takes into account three different energy terms: the well-known exchange term, chiral ones, and an intrinsic anisotropy term. In the non-abelian gauge field approach, the covariant gauge derivative plays a key role. The result is the emergence of a Dzyaloshinskii–Moriya-like energy term under two conditions: the first one is the pure gauge field background and the second one is the presence of a static electric field. Moreover, this approach allows one to reach a more deep understanding of the micromagnetics theory if rewritten in a gauge-invariant formulation. In this article, clearly emerges the interpretation of the voltage-controlled magnetic anisotropy (VCMA) mechanism.

Index Terms— **Effective non-abelian gauge field, micromagnetics, voltage controlled magnetic anisotropy (VCMA).**

I. Introduction

It is well established that non-trivial topological magnetic configurations are caused by the skew-symmetric interactions such as the Dzyaloshinskii–Moriya interaction (DMI). The DMI-like energy terms are caused not only by the intrinsic spin–orbit interaction but also by the magnetization field on a curved space, then inducing a so-called "curvature induced" effective DMI [1]. Effects analogous to the real curvature are also obtained in the presence of an effective gauge field [2], [3]. A simple example of an abelian gauge field in magnetism is provided by taking the Hamiltonian of a point particle with mass m and magnetic moment $\boldsymbol{\mu}$ moving at velocity $\boldsymbol{v}$ in an electric field $\boldsymbol{E}$

$$\mathcal{H} = \frac{1}{2m}\left(\boldsymbol{\pi} + \frac{1}{c^2}\boldsymbol{E}\times\boldsymbol{\mu}\right)^2. \tag{1}$$

At low velocity, the canonical momentum $\boldsymbol{\pi} = \boldsymbol{p} - c^{-2}\boldsymbol{E}\times\boldsymbol{\mu}$ is given by the sum of the kinetic momentum $\boldsymbol{p} = m\boldsymbol{v}$ and the electromagnetic momentum $-c^{-2}\boldsymbol{E}\times\boldsymbol{\mu}$. The presence of the electromagnetic momentum is important in quantum mechanics, in which, the canonical momentum $\boldsymbol{\pi}$ becomes an operator and the electric field-dependent term assumes a meaning of an effective abelian gauge potential. In terms of the potential, the Hamiltonian is written as

$$\mathcal{H} = \frac{1}{2m}(\boldsymbol{\pi} + \mu\boldsymbol{\mathcal{A}})^2 \tag{2}$$

where μ is the quantum of the transported magnetic moment and $\boldsymbol{\mathcal{A}}$ is the abelian gauge potential. In the case of the magnons, the quanta of the spin waves, the transported moment is $\mu = -2\mu_B$ and $\boldsymbol{\mathcal{A}} = -c^{-2}\boldsymbol{E}\times\boldsymbol{e}_z$, where $\boldsymbol{e}_z$ is the direction of the polarization of the magnon. The presence of such an abelian gauge field introduces an additional phase in the corresponding Schrödinger wave function. This effect is known in the literature as the Aharonov–Casher's effect [4]. The same effect can also be demonstrated by starting from a spin-wave carrying a magnetic moment [5]. When the spin-wave propagates in an electric field it acquires an additional phase. This effect is rather small, but it can be experimentally observed [6].

Having highlighted the electric field effect on spin waves, it is of interest to extend to a general micromagnetics description of a ferromagnet. In this article, we set this problem by linking the electric field to an effective non-abelian gauge potential $\mathcal{A}_\alpha$. For each fixed $\alpha = 1, 2, 3$ the non-abelian gauge field $\mathcal{A}_\alpha$ is a second rank tensor in the Euclidean space. It can be canonically decomposed into three parts: an isotropic term, a skew-symmetric, and a symmetric traceless part. By restricting the attention to the skew-symmetric part, one can further express $\mathcal{A}_\alpha$ in terms of a vector $1/2(\mathcal{A}^{\kappa}_{\beta\alpha} - \mathcal{A}^{\beta}_{\kappa\alpha}) \to \boldsymbol{\mathcal{A}}_\alpha$. The key point is to substitute, in the micromagnetic energy, the partial derivative ∂_α by the following expression: $\partial_\alpha \to D_\alpha = \partial_\alpha + \boldsymbol{\mathcal{A}}_\alpha\times$ [7]. The new spatial derivative operator is called *gauge covariant derivative*. The gauge field is a function of the electric field $\boldsymbol{E}$ and the spatial derivative operator becomes $D_\alpha m^\rho = \partial_\alpha m^\rho + \gamma_l c^{-2}(E^\rho m_\alpha - \delta^\rho_\alpha \mathbf{E}\cdot\mathbf{m})$. The gauge covariant derivative affects the exchange interaction term that, in cartesian coordinates, is proportional to this scalar product $A\partial_\alpha\boldsymbol{m}\cdot\partial^\alpha\boldsymbol{m} = A(\nabla\boldsymbol{m})^2$. In the presence of the electric field the exchange term becomes $AD_\alpha\boldsymbol{m}D^\alpha\boldsymbol{m}$. If this term is expanded to recover the original derivative, one finds two additional energy terms. These terms consist of a DMI-like term, which is proportional to the electric field [8] and an anisotropy-like term corresponding to an easy plane perpendicular to the direction of the electric field and proportional to E^2. Such a term has

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the same expression as the typical terms introduced to describe the voltage-controlled magnetic anisotropy (VCMA) [9], [10]. It is clear that in a real system there will be VCMA effects dependent on the specific material that may play a relevant role. The present theory has the merit to evidence an effect that is a direct consequence of the coupling of the magnetization with the electric fields. Therefore, it should be relevant in YIG ferrite that does not exhibit spontaneous electric polarization. Finally, we briefly suggest and discuss a possible application of the presented theory to electrically assisted magnetization switching, which is relevant for the realization of low power memories [10].

II. Theory

The magnetization of a ferromagnet is described by a unit vector field $\boldsymbol{m} = M_s^{-1}\boldsymbol{M}$ where $\boldsymbol{M}$ is the magnetization and M_s its saturation value. The magnetization in a group theoretical formalism is a field configuration which minimizes the micromagnetic energy density (i.e., Hamiltonian energy density). Moreover, it is known that the micromagnetic energy density is invariant with respect to a global rotation (i.e., acting with the SO(3) rotation group on the whole ferromagnetic body) [11] and the magnetization vector $\boldsymbol{m}(\boldsymbol{x}, t)$ is invariant under a rotation around itself. These assumptions are equivalent to investigate the motion of a representative point on the surface of unit sphere S^2 parametrized, for example, by $\boldsymbol{m}(\boldsymbol{x}, t) = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)^t$. Within this interpretation, the magnetization vector lies on the coset space SO(3)/SO(2) equivalent to S^2 consisting of those elements in SO(3) that leave unchanged a given $\boldsymbol{m}(\boldsymbol{x}, t)$, so it is just a rotation around $\boldsymbol{m}(\boldsymbol{x}, t)$ itself, and consequently, it is isomorphic to SO(2). However, the micromagnetic energy density considered here contains two terms: the exchange energy and the crystalline anisotropy

$$u(\boldsymbol{m}, \partial_\alpha \boldsymbol{m}) = A\partial_\alpha \boldsymbol{m} \cdot \partial^\alpha \boldsymbol{m} + f_{\boldsymbol{n}}(\boldsymbol{m}). \tag{3}$$

In (3), the first term is proportional to the exchange stiffness A, and the anisotropy term is $f_{\boldsymbol{n}}(\boldsymbol{m})$, which depends on some local direction $\boldsymbol{n}$. Considering that $\boldsymbol{m}(\boldsymbol{x}, t)$ transforms according to a representation of a certain element of the SO(3) non-abelian Lie group, we get $\boldsymbol{m}'(x', t) = \mathcal{R}\boldsymbol{m}(\mathcal{R}^{-1}x', t)$. The matrix $\mathcal{R}$ is written using the exponential form $\mathcal{R}(\boldsymbol{l}, \psi) = \exp(-\boldsymbol{\psi} \cdot \boldsymbol{J})$ and it is a rotation around the unit vector $\boldsymbol{l}$ of an amount equal to ψ, hence, $\boldsymbol{\psi} = \psi\boldsymbol{l}$. In this expression, $\boldsymbol{J} = (\boldsymbol{J}_1, \boldsymbol{J}_2, \boldsymbol{J}_3)$ is a vector whose elements are the infinitesimal generators of the Lie-algebra SO(3) of the group SO(3) satisfying the following commutation rule $[\boldsymbol{J}_\alpha, \boldsymbol{J}_\beta] = \varepsilon_{\alpha\beta}{}^{\sigma}\boldsymbol{J}_\sigma$ the coefficients $\varepsilon_{\alpha\beta}{}^{\sigma}$ is the Levi–Civita symbol (i.e., the structure constants of the SO(3)-algebra). The infinitesimal generator of the SO(3)-group consists in the 3×3 real skew-symmetric matrices, and the element of the SO(3)-group are the orthogonal matrices such that $\mathcal{R}^t = \mathcal{R}^{-1}$. The local gauge symmetry (i.e., local gauge redundancy) has a counterpart formally introduced by the definition of the non-abelian gauge covariant derivative acting on the magnetization vector lying in the Euclidean space [3], [7], [12]. The gauge covariant derivative of the magnetization field $\boldsymbol{m}(x, t)$ has the following expression:

$$D_\alpha \boldsymbol{m} = \partial_\alpha \boldsymbol{m} + \boldsymbol{\mathcal{A}}_\alpha \times \boldsymbol{m}. \tag{4}$$

Now, it is possible to show that the micromagnetic energy is gauge invariant after the substitution of the partial derivative of the magnetization $\partial_\alpha \boldsymbol{m}$ with the gauge covariant derivative of the magnetization $D_\alpha \boldsymbol{m}$. This can be shown in (3) by applying a local rotation $\boldsymbol{m}' = \mathcal{R}(x)\boldsymbol{m}$ and also modifying the non-abelian gauge field $\mathcal{A}'_\alpha = \mathcal{R}\mathcal{A}_\alpha\mathcal{R}^t + (\partial_\alpha\mathcal{R})\mathcal{R}^t$, by using the correspondence that at fixed index α any skew-symmetric non-abelain gauge $\mathcal{A}_\alpha$ can be expressed in terms of a vector $\boldsymbol{\mathcal{A}}_\alpha$. This means that a particular orientation of a local reference basis has no physical relevance and in our description, the new local vector basis is expressed as a linear combination with respect to the standard Euclidean vector basis $\boldsymbol{e}_\alpha$ as $\boldsymbol{f}_\beta(x) = \mathcal{R}_{\beta\alpha}[\boldsymbol{\psi}(x)]\boldsymbol{e}_\alpha$. Hence, the local vector basis cannot be uniquely defined by the geometry of the space but it has to be specified according to the presence of an electric field as shown in Section III. Moreover, (4) is also useful in order to define the parallel transport of the magnetization vector in our gauged micromagnetics field theory. Now, the gauge invariant micromagnetic energy is defined as

$$\tilde{u}(\boldsymbol{m}, D_\alpha \boldsymbol{m}, \boldsymbol{\mathcal{A}}_\alpha) = AD_\alpha \boldsymbol{m} \cdot D^\alpha \boldsymbol{m} + f_{\boldsymbol{n}}(\boldsymbol{m}). \tag{5}$$

Subsequently, we consider the effect of a pure gauge field alone $\mathcal{A}_\alpha = (\partial_\alpha\mathcal{R})\mathcal{R}^t$. The gauge covariant derivative now assumes the following form:

$$D_\alpha \boldsymbol{m} = \partial_\alpha \boldsymbol{m} - \partial_\alpha \boldsymbol{\psi} \times \boldsymbol{m}. \tag{6}$$

Finally, as a result, this method gives three different contributions to the generalized exchange energy: a symmetric exchange term, an antisymmetric exchange term [i.e., Dzyaloshinskii–Moriya (DM)-like term], and an effective anisotropy contribution

$$\begin{aligned}\tilde{u}(&\boldsymbol{m}, D_\alpha \boldsymbol{m}, \partial_\alpha \boldsymbol{\psi}) \\ &= A[\partial_\alpha \boldsymbol{m} \cdot \partial^\alpha \boldsymbol{m} + -2(\partial_\alpha \boldsymbol{\psi} \times \boldsymbol{m}) \cdot \partial^\alpha \boldsymbol{m} \\ &\qquad + (\partial_\alpha \boldsymbol{\psi} \times \boldsymbol{m}) \cdot (\partial^\alpha \boldsymbol{\psi} \times \boldsymbol{m})] + f_{\boldsymbol{n}}(\boldsymbol{m}).\end{aligned} \tag{7}$$

III. Electric Field

In order to introduce a case of practical interest, we connect the partial derivative of the $\boldsymbol{\psi}$ vector with the applied electric field, limiting our attention only to this transformation rule

$$\partial_\alpha \psi^\kappa = -\gamma_L c^{-2} E^\sigma \varepsilon^\kappa{}_{\sigma\alpha} \tag{8}$$

and we obtain

$$[\partial_\alpha \boldsymbol{\psi} \times \boldsymbol{m}]^\rho = -\gamma_L c^{-2} E^\sigma \varepsilon^\kappa{}_{\sigma\alpha}\varepsilon_{\kappa\beta}{}^{\rho} m^\beta. \tag{9}$$

Now, we are able to express the gauge covariant derivative using (4) in terms of the Levi–Civita tensor and using (8), we remember that the product of two contracted Levi–Civita tensor can be expressed as a quadratic combination of the Kronecker delta $\varepsilon^\kappa{}_{\sigma\alpha}\varepsilon_{\kappa\beta}{}^{\rho} = \delta_{\sigma\beta}\delta^\rho_\alpha - \delta^\rho_\sigma\delta_{\beta\alpha}$. In the end, we obtain

$$D_\alpha m^\rho = \partial_\alpha m^\rho + \gamma_L c^{-2}(E^\rho m_\alpha - \delta^\rho_\alpha \mathbf{E} \cdot \mathbf{m}). \tag{10}$$

In this way, we have defined the action of this new differential operator over each component of the magnetization vector. Now, once again, if we operate the transformation $\partial_\alpha \to D_\alpha$, we find that the micromagnetic energy is expressed as a function of $D_\alpha \boldsymbol{m}$ and we can substitute it in (7). Clearly, now it is of interest to have a look at the new form of the micromagnetic energy term. Only the terms containing the partial derivatives in the micromagnetic energy are affected by this mapping. Hence, taking the general form of the exchange energy in (7) and after some algebra the exchange term assumes the following expression:

$$\begin{aligned}\tilde{u}(\boldsymbol{m}, \partial_\alpha \boldsymbol{m}, \boldsymbol{E}) = {} & A(\nabla \boldsymbol{m})^2 \\ & -2A\frac{\gamma_L}{c^2}\mathbf{E}\cdot[\mathbf{m}(\nabla\cdot\mathbf{m}) - (\mathbf{m}\cdot\nabla)\mathbf{m}] \\ & + A\left(\frac{\gamma_L}{c^2}\right)^2\left[E^2 + (\mathbf{E}\cdot\mathbf{m})^2\right]. \qquad (11)\end{aligned}$$

We summarize starting from micromagnetic gauge-invariant formulation and after the definition of the correct gauge covariant derivative affected by an effective gauge field $\boldsymbol{\mathcal{A}}_\alpha$ a DM-like interaction emerges and it is evidently driven by the presence of a static electric field $\boldsymbol{E}$.

We recognize the usual exchange term, $u_{\mathrm{EX}} = A(\nabla \boldsymbol{m})^2$, plus two additional terms that describe the coupling with the electric field. The second term on the right-hand side of (11) is the Lifshitz invariant corresponding to the DMI in continuous form and to the magneto-electric coupling [9], [13], [14]. The third term is an anisotropy energy contribution.

IV. Electric Control of the Magnetic Anisotropy and Electrically Assisted Magnetization Switching

The theory presented shows that the magnetization can be controlled by applying an electric field and it is of fundamental interest. In fact, in (11), two terms are present: the first one is $2A\gamma_L c^{-2}\boldsymbol{E}[\mathbf{m}(\nabla\cdot\mathbf{m}) - (\mathbf{m}\cdot\nabla)\mathbf{m}]$, where the term in the square bracket can be interpreted as electric polarization coupled directly to the electric field $\boldsymbol{E}$ and the second one is $A(\gamma_L/c^2)^2[E^2 + (\mathbf{E}\cdot\mathbf{m})^2]$ relevant because its meaning is that of an electrically controlled anisotropy. To this end, we focus our attention on the conservative magnetization dynamics (the damping can be neglected during fast ballistic switching) described by the Landau–Lifshitz (LL) equation [15], which in dimensionless units can be written as follows:

$$\dot{\boldsymbol{m}} = -\boldsymbol{m} \times \boldsymbol{h}_{\mathrm{eff}}. \qquad (12)$$

The effective field $\boldsymbol{h}_{\mathrm{eff}}$ is calculated as a negative gradient of the free energy $\tilde{u}(\boldsymbol{m}, \boldsymbol{h}_a, \hat{\boldsymbol{E}})$ with the electric field $\hat{\boldsymbol{E}}$ proportional to the applied electric field $\boldsymbol{E}$, with the correct proportionality factor to be experimentally determined

$$\tilde{u}(\boldsymbol{m}, \boldsymbol{h}_a, \hat{\boldsymbol{E}}) = \frac{1}{2}\boldsymbol{m}^t N\boldsymbol{m} - \boldsymbol{m}\cdot\boldsymbol{h}_a + [\hat{E}^2 + (\hat{\boldsymbol{E}}\cdot\mathbf{m})^2] \quad (13)$$

and it has the following expression:

$$\boldsymbol{h}_{\mathrm{eff}} = -N\boldsymbol{m} + \boldsymbol{h}_a - 2(\hat{\boldsymbol{E}}\cdot\boldsymbol{m})\hat{\boldsymbol{E}} \qquad (14)$$

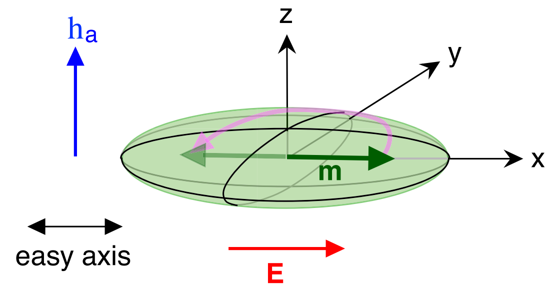

Fig. 1. $\boldsymbol{h}_a$ is the dimensionless applied magnetic-filed along the z-axis or in the y-direction, $\boldsymbol{E}$ is the electric field an $\boldsymbol{m}$ the dimensionless magnetization $\boldsymbol{m}$.

with N is the matrix of the demagnetizing factors. The LL-equation now assumes the following aspect:

$$\dot{\boldsymbol{m}} = \boldsymbol{m}\times N\boldsymbol{m} - \boldsymbol{m}\times\boldsymbol{h}_a + 2(\hat{\boldsymbol{E}}\cdot\boldsymbol{m})\boldsymbol{m}\times\hat{\boldsymbol{E}}. \qquad (15)$$

Electrically assisted precessional magnetization switching using external magnetic field pulses can be described by (15) assuming the applied field either oriented along the intermediate $\boldsymbol{h}_a = h^y\boldsymbol{e}_y$ or hard $\boldsymbol{h}_a = h^z\boldsymbol{e}_z$ axis and the electric filed aligned with the easy axis $\hat{\boldsymbol{E}} = \hat{E}\boldsymbol{e}_x$ as in Fig. 1

$$\dot{\boldsymbol{m}} = \boldsymbol{m}\times N'\cdot\boldsymbol{m} - \boldsymbol{m}\times\boldsymbol{h}_a \qquad (16)$$

with N' defined as

$$N' = \begin{pmatrix} N_x + 2\hat{E}^2 & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{pmatrix}. \qquad (17)$$

In both situations, the LL-equation (15) can be solved analytically [16], [17], finding the possible critical values and switching times for electrically assisted switching.

V. Conclusion

The exchange part of the micromagnetic energy has been rewritten in a gauge-invariant form because the reference frame orientation is unspecified with respect to a local rotation which defines an invariance property of the exchange interaction terms in the micromagnetic energy density of the magnetization $\boldsymbol{m}(\boldsymbol{x}, t)$. With the introduction of an effective gauge field $\boldsymbol{\mathcal{A}}_\alpha$ in the generalized micromagnetic energy density (11) after a formal redefinition of the partial derivative with the gauge covariant derivative, there emerges a DM-like interaction driven by a static electric field and an intrinsic anisotropy term. Moreover, the transformation group, which expresses this invariance, imposes definite restrictions on the dispersion relation linked to the micromagnetic energy that will be studied in a future publication. Therefore, our findings point to a more advanced theoretical interpretation with respect to previous discoveries [5] in order to drive experiments in manipulating spin waves and developing electrically tunable magnonic devices.

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