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ABSTRACT

A research work is presented, carried out to validate a method based on sensitivity analysis for evaluating the uncertainty of CMM measurements. The fitness for purpose of cylinder squares for verifying the uncertainty evaluation of coaxiality measurements has been confirmed. Ring gauges too were used and found useful, but only as to its diameter. A type B method for predicting the uncertainty considering the actual technical condition of the CMM is proposed.

1. Introduction

Technical specification ISO/TS 15530-1 [1] outlines three techniques for evaluating the uncertainty in coordinate measurements: use of calibrated workpieces or standards, simulation and sensitivity analysis. The first method is described in ISO 15530-3 [2]. Almost all uncertainty components are evaluated experimentally with minor prior information. The method is intended to be primary, i.e. with no need of individual validation. For this reason, it can be used even for verification of other methods. The second method is related to ISO/TS 15530 4 [3]. This Technical Specification does not describe a specific method, rather provides a way for testing specific uncertainty evaluation software (UES). The measurement of an artefact of simple design, high accuracy and ease of calibration is proposed. Specifically, a test cylinder (cylindrical square) is recommended as it provides diverse measurands suitable for validating UESs. As an example, it enables the investigation of coaxiality measurement for decreasing datum lengths and distances of the toleranced element to the datum. The uncertainties for measuring coaxiality are expected to depend on these parameters. Ring gauges show similar advantages: the uncertainty should increase [4] when the sampled arc central angle decreases.

As part of the EURAMET-founded EMPIR joint research project no. 17NRM03 EUCoM (Standards for the evaluation of the uncertainty of coordinate measurements in industry) [5], methods to propose as new Parts of the ISO 15530 series are being developed and validated. The focus is on techniques easily used in industrial conditions. One of these methods is based on a technique proposed by Płowucha [6,7], which can be classified as a sensitivity analysis technique according to Ref. [1] and as GUM uncertainty framework according to GUM [8]. This method uses the definitions of geometric characteristics found in Ref. [9], which is part of the Geometrical Product Specification (GPS) system of standards [10]. A model is considered for each characteristic based only on the (minimum number of) points setting the characteristic. The evaluation is based on equations expressing individual geometrical characteristic as a function of distances or of differences in the coordinates of point pairs. This is in line with ISO 10360-2 [11], where a CMM measures several calibrated test lengths, e.g. gauge blocks, in different locations and orientations in the CMM measuring volume.

Uncertainties in measuring distances or differences in coordinates of point pairs are evaluated solely from the $E_{L,MPE}$ equation (ISO 10360-2) and possibly from the actual measurement results in an ISO 10360-2 acceptance or reverification test. As a result, all uncertainty components are type B. The sensitivity coefficients are calculated analytically or numerically as partial derivatives of the measured characteristic with respect to the individual input quantities.

This paper describes:

- How the method estimates the standard uncertainty of the distance of point pairs or of the coordinate differences of point pairs,
- How the method derives the sensitivity coefficients in coaxiality and circle diameter measurement, and
- The results of the verification of the proposed method.

2. Uncertainty of distance measurement

It is assumed that the standard measurement uncertainty u of the distance between two points (or coordinate differences of two points) is equal to [[12], p. 8.4.5]:

$$u = E_{L,\ MPE} \cdot b \tag{1}$$

where $E_{L,\ MPE}$ is maximum permissible error (for size or distance) stated by the CMM manufacturer or user, and b is a coefficient depending on the type of probability distribution of errors. In the simplest but safe approach of, assuming uniform distribution, $b = \frac{1}{\sqrt{3}} = 0.58$.

The value of b can be used to take account of the actual technical condition of the CMM. Results of actual acceptance and/or reverification tests show that the errors are often a small fraction of the allowed $E_{L,MPE}$, particularly for new CMMs. An example of this situation is shown in Fig. 1.

To account for the actual technical condition of a CMM, we propose that the b factor is evaluated as the mean square root [[13], p. 1.14] of

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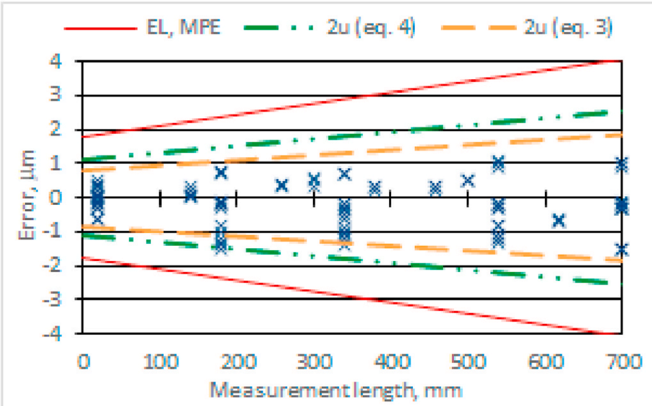

Fig. 1. Example of reverification test results for a CMM with $E_{L,\ MPE} = \pm(1.8 + L/333)$ µm (external lines); $\pm 2u$ lines are also shown for b calculated according to equations (3) and (4).

the test results (errors of indication) normalised to the range $(-1, 1)$:

$$E_{n,L,i} = \frac{E_{L,i}}{E_{L,MPE,i}} \tag{2}$$

$$b = \sqrt{\frac{1}{N}\sum_{i=1}^{N} E_{n,L,i}^2} \tag{3}$$

where N = 105 (the total number of measurements in the ISO 10360 2 test).

For the CMM of Fig. 1 $b = 0.228$. The normalised errors of indication, the $E_{L,MPE}$ values and the $\pm 2u$ values according to equations (1)–(3) are shown in Fig. 2. The equivalents of these lines are also shown in Fig. 1.

In the case of a significant asymmetry of observed measurement errors, there may be too many test results outside the range determined by $2u$. Therefore, as a more prudent solution, b can be calculated according to the formula

$$b = \frac{max(|E_{si}|)}{2} \tag{4}$$

This way, all test results are within the range of $\pm 2u$. For the analysed example $b = 0.313$. Lines corresponding to the value u calculated by applying formula (4) are shown also in Fig. 1.

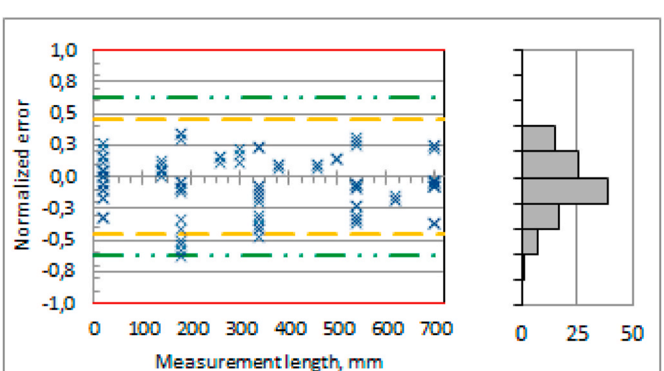

Fig. 2. Normalised errors of indication in a reverification test: plot with the $E_{L,\ MPE}$ and the $2u$ (equations (1)–(3)).

3. Measurement models

3.1. Coaxiality measurement models

Two main configurations are relevant to coaxiality measurements. In the first configuration the toleranced element (median line of the right cylinder, Fig. 3a) is separated from and external to the datum (axis of the left cylinder). In the second case the toleranced element (median line of the central cylinder, Fig. 3b) is in between two datums A and B that set the common datum A-B (common axis of the external cylinders).

The coaxiality is the smallest diameter of a cylinder sharing the axis with the datum and containing each point S on the medial line.

Let us simplify the problem by considering two points only of the datum, A and B. The coaxiality CX is then twice the distance of the point S from the straight line AB [6]:

$$CX = 2 \cdot l(S, AB) \tag{5}$$

The equation yielding the distance of a point S from a straight line AB is [[14], Table B.7]:

$$l(S, AB) = \left| \boldsymbol{BS} \times \frac{\boldsymbol{AB}}{|\boldsymbol{AB}|} \right| \tag{6}$$

Equation (6) is taken as the measurement model for the evaluation of the uncertainty. An example of budget is given in [[6], Table 4]. In the example it is assumed that the workpiece is oriented along x axis. Difference of x coordinates of the points A and B is designated as ab_1 and difference of z coordinates of points B and S as bs_3 (it's assumed that point S lies in xz-plane). Only two input quantities (ab_3 and as_3) out of the 6 (ab_1, ab_2, ab_3, bs_1, bs_2 and bs_3) are not null. Uncertainty components for bs_3 and ab_3 are included in the uncertainty budget with weights equal to 1 and to bs_1/ab_1, respectively. Therefore, a general equation for the standard uncertainty of the measurement of coaxiality is:

$$u_{CX} = 2\sqrt{1^2 + \left(\frac{L}{l}\right)^2} \cdot E_{L,\ MPE}(0) \cdot b \tag{7}$$

where L is the distance of the toleranced element from the datum ($|\mathbf{BS}|$), l the datum length ($|\mathbf{AB}|$), $E_{L,MPE}(0)$ the maximum error of indication for a null length.

If the toleranced element lies outside the datum, the ratio L/l can be greater than 1; when it is in between the datums A and B, the ratio is no larger than 0.5.

3.2. Circle diameter measurement models

Different equations are available in literature for evaluating the diameter d of a circle through three points. The simplest one is that of the radius of the circle circumscribed of a triangle:

$$d = \frac{abc}{2P} \tag{8}$$

where P is the area of the triangle and a, b, c the lengths of its sides, which and can be expressed through the differences in the coordinates of the points.

In turn, the area of the triangle P can be expressed using the Heron's equation

$$P = \frac{\sqrt{(a+b+c)(a-b+c)(a+b-c)(-a+b+c)}}{4} \tag{9}$$

or by means of a vector product of 2 sides of a triangle:

$$P = \frac{|AB \times AC|}{2} \tag{10}$$

Depending on the chosen equations, the uncertainty evaluation is based either on 3 input quantities (a, b, c for equations (8) and (9)) or 9 input quantities (coordinates of vectors $\boldsymbol{AB}$, $\boldsymbol{AC}$, $\boldsymbol{BC}$ for equations (8)

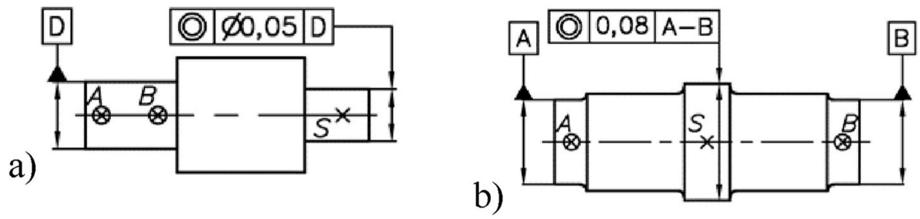

Fig. 3. Two examples of coaxiality specification; tolerated element: a) separated from and external to the datum, b) in between two datums setting a common datum.

and (10)). The sensitivity coefficients are then derived as partial derivatives, evaluated either analytically or numerically.

4. Verification

4.1. Experimental uncertainty

For the verification of the new method, two characteristics were selected for which the measurement strategy has a significant impact on the uncertainty: coaxiality and arc radius (diameter). In both cases, the experiment consisted in a 20-fold measurement of a standard object with a known value of the characteristic [2].

4.2. Uncertainty of coaxiality measurement

A cylinder square with a diameter of 80 mm was used for the tests. The centre coordinates of 17 circles at intervals of 5 mm were measured covering 80 mm of cylinder length. Measurements were carried out on the CMM with $E_{L,\ MPE} = \pm(4 + 6L/1000)$ µm.

On this basis, coaxiality was calculated for 7 different combinations of the datum length (10, 15, 20, 25, 30, 35, 70 mm) and the distances of the toleranced element from the datum. The measurements were repeated at longer time intervals as recommended in Ref. [2].

The known coaxiality value is 0. The coaxiality values (as the values of all geometrical deviations) are positive, therefore the Weibull distribution was assumed as the probability distribution of measurement errors, and the value of the expanded measurement uncertainty U was calculated as 0.95 quantile of this distribution (Fig. 4).

The comparison of the results for 2 example datum lengths are presented on Fig. 5.

The measurement uncertainty shows a strong dependence on the datum length and on the distance of the toleranced feature to the datum. A high compatibility of the proposed method with experiment is visible.

4.3. Uncertainty of measurement of a circle diameter

A ring gauge with a diameter of 100.0119 mm was used for the tests. The ring was probed as a circle at 11 evenly spaced points. From the obtained coordinates of the points, the diameters of the circles defined by 3 points were calculated according to the following scheme (Fig. 6):

- 11 circles (arcs) spanning a central angle $\theta = 32.7^\circ$ from points: 1-2-3 (Figs. 6a), 2-3-4, …, 5-6-7 (Fig. 6a), …, 11-1-2,
- 11 circles spanning a central angle $\theta = 49.1^\circ$ from points 1-2-4, 2-3-5, …, 11-1-3, etc., up to
- 11 circles spanning the central angle $\theta = 163.6^\circ$ from points 1-6-11 (Figs. 6b), 2-7-1, …, 11-5-10.

For each case (the θ angle value) average diameter value was calculated. 9 diameter values are obtained for the following central angles θ: 32.7°, 49.1°, 65.5°, 81.8°, 98.2°, 114.5°, 130.9°, 147.3° and 163.6°.

Each experiment was repeated 20 times with measurements distributed in time in accordance with ISO 15530-3 recommendations.

Comparison of evaluated uncertainties for a diameter measurement using different measurement strategies (different centre angles) according to the proposed method (plain lines) and to experiment (dots) are presented on Fig. 7. The $E_{L,\ MPE}$ of the used CMM was $E_{L,\ MPE} = \pm(1.8 + 3L/1000)$ µm). The upper line corresponds to the evaluated uncertainty with the assumption of a uniform distribution ($b = 0.58$), the lower one takes into account the actual reverification test results ($b = 0.313$).

A clear overestimation occurs for small angles θ.

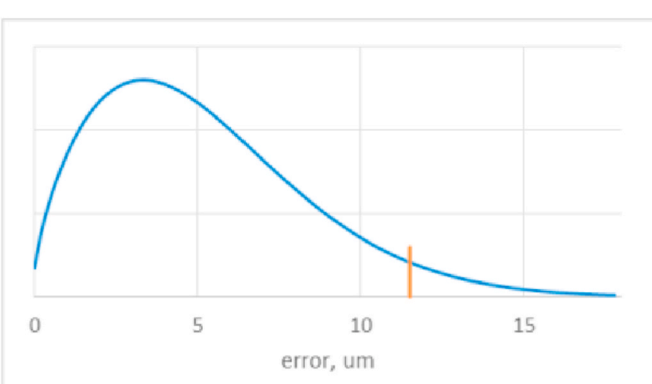

Fig. 4. Example of Weibull PDF with marked 0.95 quantile.

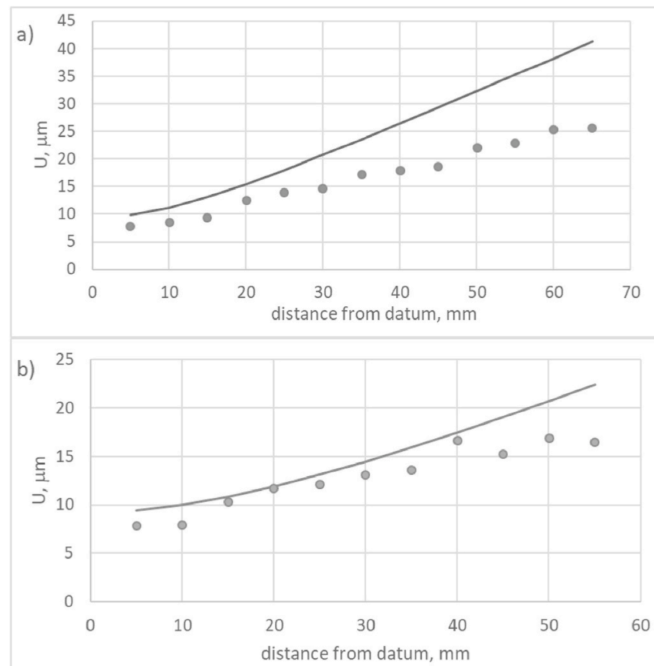

Fig. 5. Extended uncertainty for a coaxiality as a function of the distance of the toleranced feature from the datum according to the proposed method (lines) and to the experiment (dots), for example datum lengths: a) 15 mm, b) 25 mm.

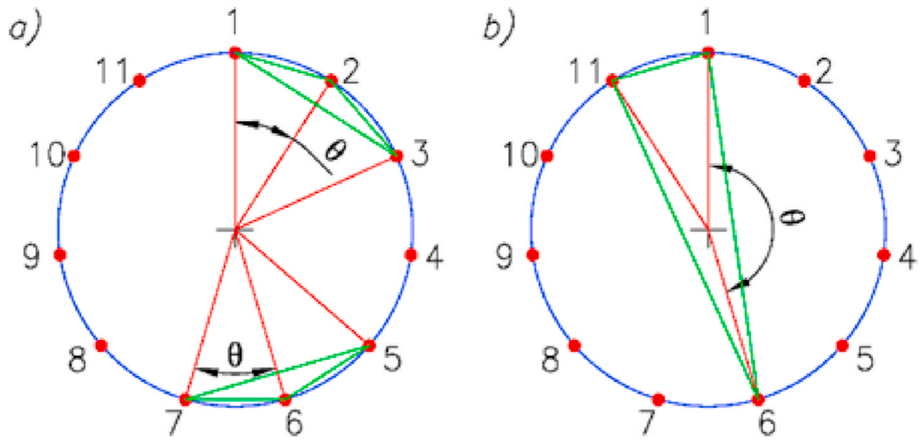

Fig. 6. Examples of triple of points and corresponding central angles: a) $\theta = 32.7°$, b) $\theta = 163.6°$.

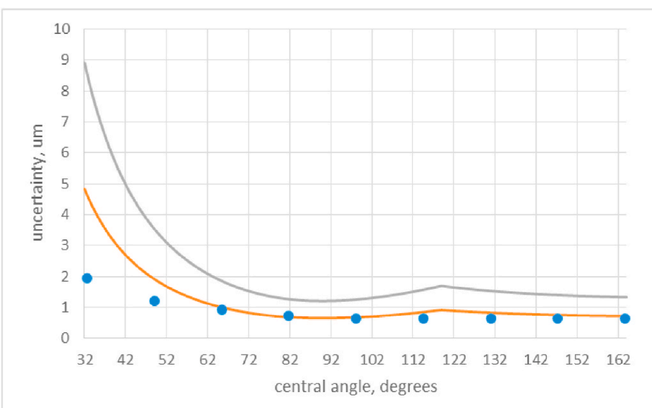

Fig. 7. Extended uncertainty of measurement of the diameter of the circle as a function of the central angle θ defining the arc on which the circle was sampled according to the proposed method (plain lines) and according to experiment (dots).

5. Conclusions

The results obtained confirmed that the cylinder square is ideal for UES verification, in particular for coaxiality measurements. The proposed UES method proved to be adequate in this case.

A ring gauge was used for validating the UES estimation of the diameter. When the central angle of the measured arc were full or large, then the measurement uncertainty was largely independent of the angle and the proposed method proved to be adequate. When short arcs were measured instead (central angle $\theta < 50°$), the method overestimated the uncertainty.

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