Prediction for the acoustic performance of a floating floor: Novel probabilistic approach considering materials Gaussian uncertainties

Prediction for the acoustic performance of a floating floor: novel probabilistic approach considering materials Gaussian uncertainties.

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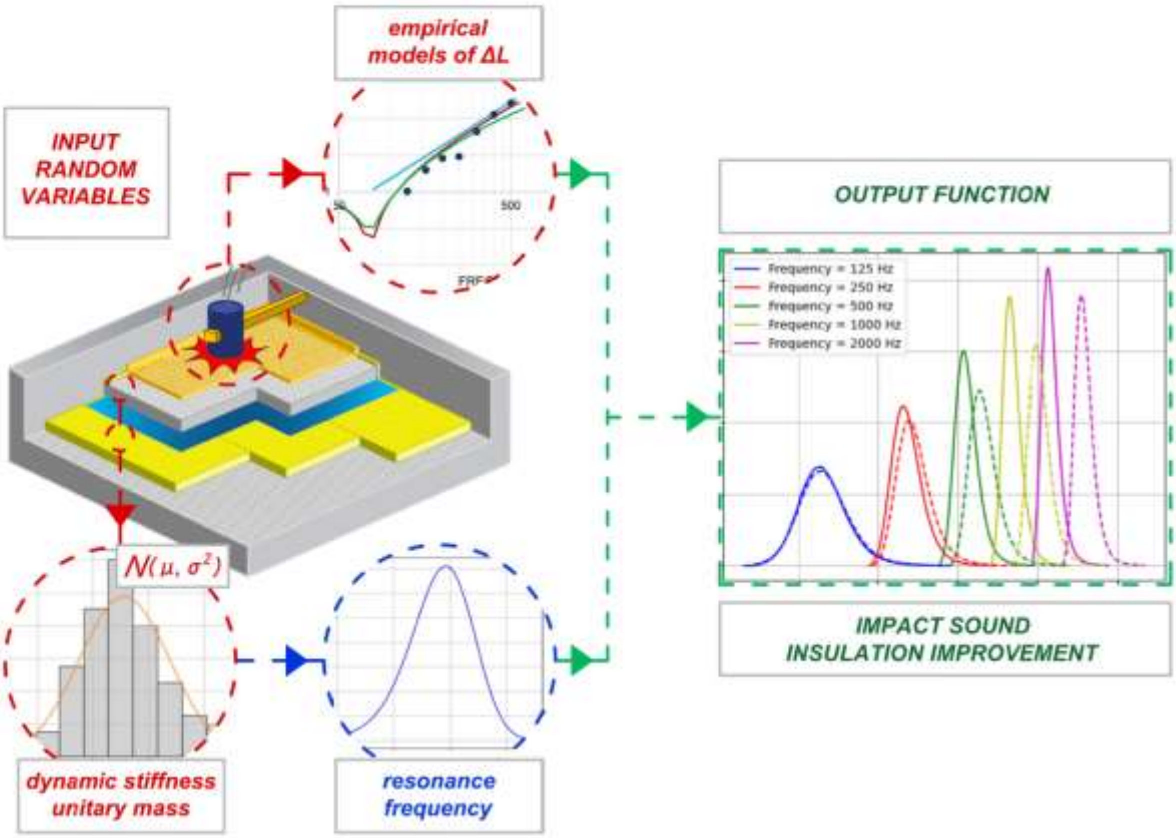
Keywords

impact sound insulation improvement; floating floor; uncertainty analysis; probability theory; Gaussian distribution; resonance frequency

Graphical abstract



Abstract

The primary aim of the presented research is to introduce a new approach based on the probabilistic analysis, which provides an accurate prediction for the improvement of the impact sound pressure level of a floating floor considering the uncertainty of material parameters. The theoretical background of the novel method was provided. It embraces two theorems describing the propagation of uncertainty of a random variables function. The introduced technique leads to the determination of the probability density function of a considered output parameter based on the joint distribution of an input random vector. Secondly, the algorithm was applied to investigate the insulation improvement of an exemplary floating floor. The considered construction was composed of extruded polystyrene as a resilient material and cement screed as a floating slab. The dynamic stiffness and the loss factor of the resilient layer and the density of cement screed served as input variable parameters with Gaussian distributions. The probabilistic analysis was based on two models providing the insulation improvement of a floating floor, e.g. Cremer-Vér formula and the recently proposed transmissibility model. Finally, the accuracy of the described methodology was verified by comparing it to the results obtained by Monte Carlo, performing 10^6 simulations. Additionally, we introduced a simplified model using the linearization of the Cremer-

Vér relation, which can be applied to determine the probability density of sound insulation improvement for floating floor composed of XPS and cement screed in a simple and efficient way.

1. Introduction

Every construction should be designed appropriately concerning acoustic properties to provide residents with sufficient comfort. Suitable acoustic conditions should be identified twofold, as the reduction of unwanted sounds, but also as the confidence that one's activities cannot be heard by other residents. In particular, this issue is decisive in the case of high residential buildings located in densely populated areas. In 2018 46% of the European Union population lived in apartments [1]. In such a dwelling type, the slabs separating two flats are subjected to airborne sounds (e.g. talking, dog barking, audio devices) and impact sounds mainly caused by footsteps or falling items. Due to such actions, the building structure is set into vibration, which can be gradually transmitted to consecutive elements of the construction skeleton, thereby affecting the comfort of living. In 1995 Swedish scientists carried out a survey on acoustic comfort among people living in multi-storey buildings [2]. The results evince that nearly 60% of the population are disposed to pay higher rent for improved acoustic insulation efficiency. According to numerous studies [3-8], the impact sounds are a primary source of residents' noise annoyance. Hence, sound insulation effectiveness is an issue frequently examined by scientists of many fields.

Sound insulation can be readily estimated experimentally. The standard ISO 10140-3 [9] specifies the laboratory procedure, which leads to the determination of normalized impact sound pressure level $L_n\,[dB]$. The standard method aims at generating impact noise by a tapping machine and measuring the sound pressure levels in the receiving room for particular frequencies. In the majority of European countries, the experimental test is conducted within 1/3-octave frequency bands ranging from 100 to 3150 Hz [10]. It is convenient to describe acoustic insulation properties using single-number quantities (SNQ) introduced by ISO 717-2 [11] such as the weighted normalized impact sound pressure level $L'_{n,w}\,[dB]$. The effectiveness of acoustic insulation can be clearly expressed by the reduction of impact sound pressure level $\Delta L_{n,w}[dB]$, which was introduced by Gösele [12]. The parameter corresponds to the decrease in sound pressure level in the receiving room due to applied insulation in comparison to floor containing solely a load-bearing slab.

Since a slab consisting only of structural material, such as concrete, does not provide sufficient comfort of living with regard to acoustic conditions, the proper insulating material has to be installed. An effective way of reducing impact noise is to attenuate vibrations before they reach the framework of a building, which can be readily achieved with floating floor systems. Such a construction consists of a layer of resilient material (e.g. expanded polystyrene, expanded polypropylene, glass fibers, rock wool, waste tyres, etc.) encasing the structural base floor, see Fig. 1. The insulating material is covered by the floating slab, usually made of screed, which is not supposed to be connected to lateral walls. Due to inherent elastic properties, the resilient layer dissipates the energy of impacts. Hence, the floating floor can be considered as a system of mass combined with spring and dampener [13]. The insulation effect can be further extended by the application of some flooring materials, whose insulating properties depend on texture [14].

In order to analyze the improvement of impact sound insulation, the deterministic approach is usually employed. It assumes that all material parameters and boundary conditions admit the scalar values. However, it is recognized that instead of the single value, the material properties should be taken as a random variable described by the probability density function. The randomness of input parameters propagates during the experimental and/or theoretical analysis implying the uncertainty of its result.

As it is known, both building structural materials (such as cement, mortar, bricks, blocks, etc.), and insulating materials (elastic, poroelastic, resilient, etc.), contain some uncontrolled inhomogeneities and inherent variabilities, which can severely affect the effectiveness of the acoustical performance of a final construction. Therefore, the main goal of the presented research is to propose a new approach based on stochastic analysis, which enables one to predict the distribution of the impact sound pressure level improvement of a floating floor considering the uncertainty of material parameters. Firstly, the theoretical framework of the novel approach is described. The procedure is based on two theorems adopted from probability theory, which allow to identify the probability distribution of a function, providing the probability distributions of its arguments are known. Secondly, the proposed approach is applied to a particular case of a floating floor. The considered resilient layer is made of 33-mm extruded polystyrene XPS, whereas the floating slab constitutes a cement screed of 30 mm. Probability distributions of input parameters, i.e. dynamic stiffness of the resilient layer, the cement screed density and loss factor are defined based on the conducted experimental research or taken from the literature. The stochastic analysis is conducted for two empirical models describing the improvement of impact sound insulation, i.e. the classical Cremer-Vér formula [15, 16] and the constitutive model for floating floors [13], based on the transmissibility theory. The first approach has been widely applied to determine the insulation effectiveness, for the frequencies above the resonance, for more than 40 years. The transmissibility model, presented in 2018, allows evaluating the amplification effects at the resonance, and the acoustical performance for frequencies below the resonance, based on the involved materials properties, and its suitability has been verified [17-19]. Additionally, a simplified model, which defines the probability density function of sound improvement considering the linearization of the Cremer-Vér formula was provided. Finally, the calculated results are compared to histograms obtained by the Monte-Carlo method, which is usually treated as the reference one but also the one of high computational cost. The comparison showed a good agreement.

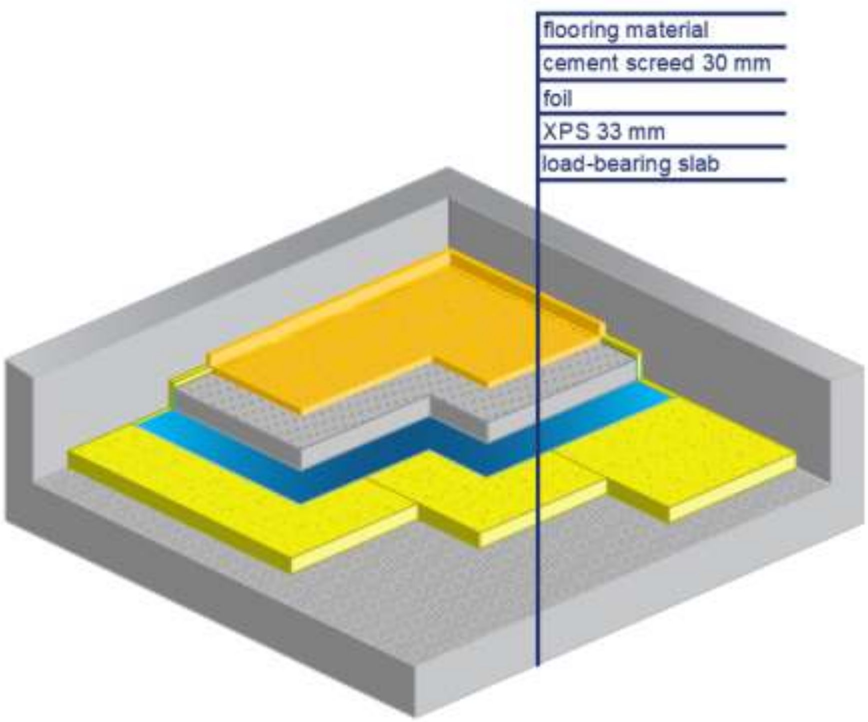


Figure 1. Schematic construction of an exemplary floating floor.

2. Theoretical background

2.1. Empirical models

There have been introduced several approaches leading to computation of the sound reduction index for floating floors. In 1952 Cremer introduced an empirical model [15], which has been improved over the years, mainly by Vér [16, 20]. The final version of the Cremer-Vér formula describes the relation between ΔL and frequency f as a straight line with the slope of 30 dB per decade (in 1/3 octave bands), as follows [21-23]:

$$\Delta L = 30\log\left(\frac{f}{f_0}\right), \tag{1}$$

where f_0 is the floating floor's mass-spring resonance frequency, determined according to the formula [13, 23]:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{s'}{m}}, \tag{2}$$

where $s'\left[N/m^3\right]$ is the dynamic stiffness of the resilient material and $m\left[kg/m^2\right]$ is the actual mass of the floating slab per unit area.

Cremer assumed that the insulation effectiveness of a floating floor is negligible below the resonance frequency, disregarding the amplification effects at the resonance. Hence, the Cremer-Vér model (1) does not allow predicting the improvement of sound insulation for such a range, which constitutes a certain limitation. Nevertheless, due to its simplicity, the formula gained widespread popularity and became adopted in the Standard ISO 12354-2 [24].

A constitutive model for floating floors, based on the force transmissibility theory [25], was recently proposed [13]. In this approach, a floating floor is considered as a vibrating system, which transfers motions towards the foundation, i.e. the load-bearing slab, through an elastic layer of insulating material. Thus, it can be considered as a mass-spring system, and the inherent insulation effectiveness can be determined in terms of transmissibility T_f [13, 22, 26]. Transmissibility also depends on the dissipative properties of the elastic layer of insulating material, which can be expressed in terms of total loss factor η [27, 28]. This dimensionless parameter usually ranges between 0.1 and 0.3, for common insulation materials [29]. For resonantly reacting floors, i.e. where the floating slab is rigid, lightly damped, and finite [16], the insulation effectiveness can be defined as follows [13]:

$$\Delta L = -15\log\left|T_f\right| = -15\log\sqrt{\frac{1+\eta^2\left(\frac{f}{f_0}\right)^2}{\eta^2\left(\frac{f}{f_0}\right)^2+\left(1-\frac{f^2}{f_0^2}\right)^2}}. \tag{3}$$

2.2. Stochastic approach

The main goal of the conducted probabilistic analysis is to determine the probability density function (*PDF*), g, of the impact sound insulation improvement. There are several approaches, which enable to determine distribution of a random variable function, knowing the *PDF* of its variable-argument. Each of them originates from the primary observation. Namely, suppose that a vector Y, in the general case, represents the vector function of a multivariate variable X, $Y=\varphi(X)=\left[\varphi_1,\varphi_2,..,\varphi_n\right]$, and $g_X(x)$ is the known joint probability distribution of the random vector X. The function vector Y occurs in the set

B if and only if the random variable X occurs in the set A_B. The set A_B is also called the inverse image of the set B corresponding to the function $\varphi(X)$ and is denoted by:

$$A_B = \varphi^{-1}(B) = \{x : \varphi(x) \in B\} \tag{4}$$

The probability that the random vector Y occurs in the set B is equal to the probability that the random vector X is in the set A_B. This observation can be rewritten in the following form:

$$P(Y \in B) = P(X \in A_B) = \int_{A_B} g_X(x)dx . \tag{5}$$

The first applied theorem employs the change of variables in order to transform the integral region of the eq.(5) into the set *B*. Nevertheless, to reach that goal some essential requirements have to be fulfilled [30, 31]. First of all, the function φ should be measurable, which means that A_B can be determined for each y. Secondly, each component of function vector *Y* needs to be continuously differentiable with regard to all components of *X*. Additionally, in the range of all probable *x* values, each $y = \varphi(x)$ needs to have a unique solution. Finally, the the Jacobian of the components of vector $\varphi(x)$ with respect to the components of random vector x is of the same sign in the region of possible argument values and vanishes only in the set of points being of measure zero. Then, the theorem can be expressed as follows [30]:

Theorem (transformed variables method): Suppose that the vectors Y and X are of the same dimensions, and the density function of the random variable X equals $g_X(x)$. *Assuming that in each case the function* $y = \varphi(x)$ *is injective and of* C^1 *class, the probability density of vector function Y can be expressed as:*

$$g_Y(y) = g_X\left(\varphi^{-1}(y)\right)\left|J(y)\right|, \tag{6a}$$

where $J(y)$ *describes the Jacobian of the components of* $x = \varphi^{-1}(y)$ *with respect to the components of* y:

$$J(y) = \frac{\partial\left(\varphi_1^{-1}, \varphi_2^{-1}, .., \varphi_n^{-1}\right)}{\partial\left(y_1, y_2, .., y_n\right)}. \tag{6b}$$

In practical application the dimension, *m*, of vector *Y* is usually smaller than the dimension, n, of vector X. For *m* components of vector X the equation $y = \varphi(x)$ has a unique solution. Let us denote x' the vector composed of those *m* components and x'' the vector composed of remaining *n-m* components of vector x. Then $y = \varphi(x', x'')$ and consequently its solution $x' = \varphi^{-1}(y, x'')$. Following the same reasoning as above the equations (6a) and (6b) might be rearranged for the case when $m < n$ in the following form:

$$g_Y(y) = \int_{-\infty}^{\infty} g_X\left(\varphi^{-1}(y,u),u\right)\left|J(y)\right|du \tag{7a}$$

and

$$J(y)=\frac{\partial\left(\varphi_1^{-1},\varphi_2^{-1},...,\varphi_m^{-1}\right)}{\partial\left(y_1,y_2,...,y_m\right)}. \quad (7b)$$

If one assumes that the equation $y=\varphi(x)$ has the set of solutions $x=\varphi_i^{-1}(y), i\in I(y)$, where i is the interval, the distribution of random vector Y might be determined according to the following formula [30, 31]:

$$g_Y(y)=\sum_{i\in I(y)} g_X\left(\varphi_i^{-1}(y)\right)\left|J_i(y)\right|. \quad (8)$$

The above-described theorem enables one to determine the explicit form of the *PDF* of the considered function. Having *PDF*, it is possible to calculate the cumulative distribution function (*CDF*), G, by the definition [31]:

$$G(x)=\int_{-\infty}^{x} g(t)\,dt \quad (9)$$

Since both, output and input vectors can be of any dimension, the approach is quite universal. Its restrictive assumptions can seem to be a serious obstacle, nevertheless they can be overcame by dividing the domain onto regions, in which all requirements providing an inversible function are met [30, 31]. The calculations are rather simple for injective transformation functions but become more complex for functions being injective only piecewise, such as eq. (3), and when the dimension of random vector is greater than one. To resolve this problem one might refer to the theorem, which regard the cumulative distribution function of a random variable. According to reasoning introduced at the beginning of this section, the assumption of $B=\{Y<y\}$ implies that the probability of the random vector $Y=\varphi(X)$ occurring in the set B is equal to the probability of X being in the corresponding set $A_B=A_y=\{x:\varphi(x)<y\}$ [30]. In case of the second applied theorem, it is sufficient for the $\varphi(x)$ functions to satisfy only one requirement. Namely, for each *y* the probability of *X* being contained in the set A_B has to be known, which means the functions have to be measurable. Then, the cumulative distribution function theorem can be expressed accordingly [30]:

Theorem (cumulative distribution function): Suppose the density function of the random variable vector X equals $g_X(x)$. *Assuming that each component of vector* $Y=\varphi(X)$ *is measurable, its cumulative density function* $G_Y(y)$ *can be determined by:*

$$G_Y(y)=\int_{\varphi(x)<y} g_X(x)\,dx \quad (10)$$

It is noticeable that the method is very general and does not depend on the dimensions of input and output random vectors X and Y. Such an approach is particularly effective when the input argument is a multicomponent vector and the function φ is not injective. Having a *CDF* it is possible to determine *PDF* by differentiation of eq. (10). Further, all discrete parameters of random variable, such as statistical moments and their functions, can be determined. Since the sound insulation improvement defined according to the transmissibility model, eq.(3) is the piece-wise injective function of frequency, in this case, the cumulative distribution function theorem was applied. Hence, the ΔL corresponds to the vector function Y, whereas the material parameters, i.e. the surface mass, the dynamic stiffness of the resilient layer and total loss factor represent the components of argument vector *X*.

3. Results and discussion

Let us analyze the improvement of impact sound insulation of the floor presented in Figure 1. The considered construction is composed of 33-mm layer of extruded polystyrene (XPS) as a resilient material, 30-mm of cement screed as a floating slab, and concrete base as a load-bearing element. As mentioned before, the resonance frequency, and as a consequence, the sound insulation effectiveness of such a construction depends only on the stiffness and the inertial properties of XPS and cement screed respectively. The performance of the examined floor was estimated by applying Cremer-Vér formula, eq. (1) as well as the transmissibility model, eq. (3). Fig. 2 presents the comparison between both models (in case of the transmissibility approach two different values of total loss factor were used) and experimental data [19]. It can be observed that the transmissibility model reflects the experimental results more accurately. Contrary to classical Cremer-Vér theory, the Schiavi approach enables one to predict the insulation effectiveness also below the resonance frequency. The model allows predicting the decrease of the floating floor acoustic insulation of a partition in the vicinity of the resonance frequency. The positive values of impact sound insulation improvement and thereby the benefits from application of sound insulation are observed at frequencies above approx. $1.5 f_0$. It also can be noticed that the lower total loss factor of a resilient layer is, the worse acoustic properties of floating floor around resonance frequency are observed.

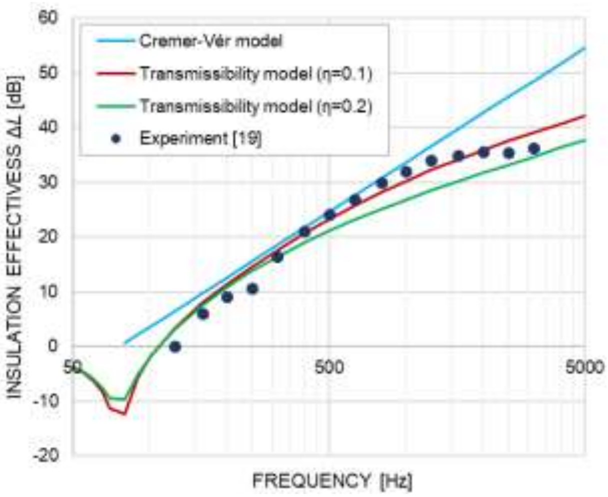


Figure 2. The comparison of impact sound insulation calculated using Cremer-Vér model and the transmissibility model assuming η=0.1 and η=0.2 with experimental data [19].

3.1 Materials uncertainties

To investigate the uncertainty of impact sound insulation improvement, the randomness of the cement screed density, dynamic stiffness and loss factor of the resilient layer were considered. Such a choice of variable arguments results from the fact, that they represent all of the input parameters of considered models, eq.(2), eq.(3). ~~Nevertheless, one has to be aware that~~ Actually, ~~also some~~ other material properties can affect the uncertainty of acoustic effectiveness, such as ~~Among them,~~ the resilient layer density ~~can be supposedly relevant~~. Indeed, creep effects (e.g., determined according EN 1606 […]), due to static deflection in time, increase density of resilient layers; this in turn, increases dynamic stiffness, reducing the sound insulation effectiveness, according to [PLEASE CITE].

~~As majority of material characteristics this parameter has a Gaussian distribution, however, it has not been clearly confirmed in the literature so far.~~ Nevertheless, the chosen models neglect the influence of other material characteristics on the acoustic performance of a floating floor. Although, the randomness of those parameters might increase the uncertainty of the improvement effectiveness. Hence, the attention was focused on the three mentioned parameters. The distribution of cement screed density was investigated on 90 cubic samples of dimensions equal to 100 mm. They were made of cement CEM I 42.5R and w/c ratio was equal to 0.5. The analysis of dynamic stiffness probability density was carried out for XPS, thickness 33 mm; 60 results were taken from the literature [32, 33]. The histograms of the density and dynamic stiffness are presented in Fig. 3. The normal distributions of dynamic stiffness and density were confirmed by the Shapiro-Wilk test [34]. Assuming $x_1 \leq x_2 \leq \ldots \leq x_n$ is an ordered sample tested for non-normality, the Shapiro-Wilk W statistic can be calculated according to the equation [34]:

$$W = \frac{\left[\sum_{i=1}^{n} a_i x_i\right]^2}{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}, \tag{11}$$

where n is the number of observations, $\bar{x}$ is the sample mean of the data and a_i are the coefficients. In this work, the approximations of the coefficients, W statistic and P-values, proposed by Royston [35, 36], have been adopted. For the dynamic stiffness of the resilient layer, the W was equal to 0.985 and the P-value to 0.387, while for the cement screed density the W was equal to 0.984 and the P-value to 0.594. The P-values were greater than 0.05, so it can be concluded that the hypothesis that normal distribution is appropriate is not rejected.

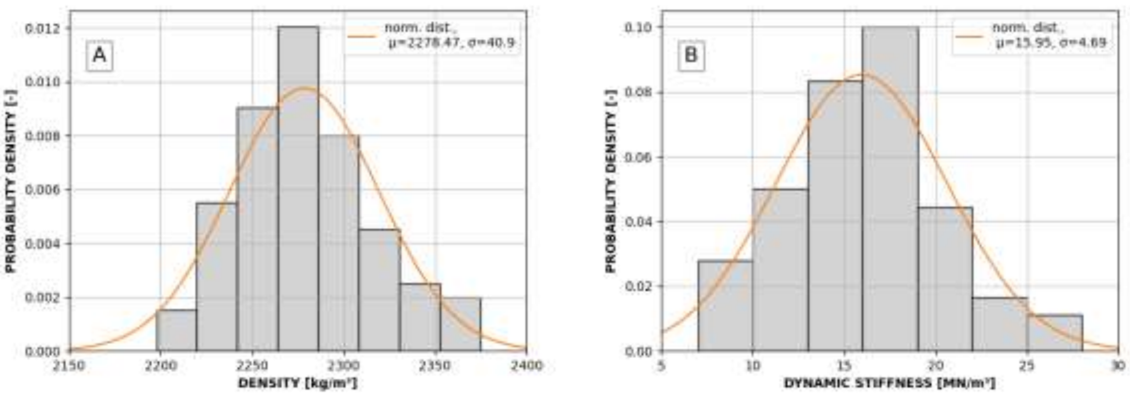


Figure 3. Histograms of the dynamic stiffness of resilient layer (A) and cement screed density (B) and parameters of normal distributions fitted to the experimental data.

Parameters of the normal distributions have been determined using regression analysis. As a result, the distributions of $N(15.95, 4.69^2)$ and $N(2278.47, 40.9^2)$ have been assumed for dynamic stiffness of the resilient layer and the cement screed density, respectively. It was assumed that the thickness of the cement screed layer in the floating slab is certain and equal to d=3cm along the area of the floor. One can apply directly the properties of expectation and variance to calculate the distribution parameters of the mass per unit area:

$$\mu_m = \mu_\rho d, \tag{12a}$$

$$\mathrm{Var}(m) = \mathrm{Var}(\rho) d^2. \tag{12b}$$

Since the thickness of the cement screed is assumed to be constant, the mass per unit area of the cement screed layer, m, is treated as the random input parameter in further acoustic analysis. Consequently, one can assume that the mass per unit area of the floating slab is normally distributed with the following parameters: $\mu_m = 68.35 kg/m^2$ and $\sigma_m = \sqrt{Var(m)} = 1.23 kg/m^2$. Concerning the loss factor, there is lack of any information regarding the distribution of this parameter. Some national regulations indicate that the loss factor should lie in the interval [0.1,0.3] [37]. Most of the random variables describing the materials properties have the Gaussian distribution. Therefore, one assumed that loss factor is normally distributed with the expected value and standard deviation equal to 0.2 and 0.03, respectively. Such assumption ensures that the loss factor admits the value from the interval [0.1,0.3] with the probability larger than 99.9%. This made, according to the transmissibility theory, the improvement of impact sound insulation the function of three random variable: surface density, dynamic stiffness and loss factor.

3.2 Resonance frequency

The randomness of material parameters is transferred into the insulation effectiveness uncertainty of the floating floor. Let us first analyze the probability density of resonance frequency, which is given by the function of two random variables, namely, dynamic stiffness and surface mass, see eq. (2). Applying the method described in the section 2.2 one is able to determine directly the density of resonance frequency. One assumes the vector argument $x = [s', m]$, the output $y = f_0$ and the function $\varphi(s', m) = \frac{1}{2\pi}\sqrt{\frac{s'}{m}}$. Since the dimension of the argument vector is equal to 2 while the dimension of output vector is equal to 1, one has to apply the equation (7a) and 7(b). The Jacobian takes the form of scalar. It can be noticed that one can obtain two equivalent formulations describing the distribution of resonance frequency. To complete this task, one needs to calculate the inverse functions of resonance frequency with regard to dynamic stiffness as well as unitary mass along with their derivatives:

$$\varphi_{s'}^{-1}(f_0, m) = s' = 4\pi^2 m f_0^2 \tag{13a}$$

$$\frac{\partial \varphi_{s'}^{-1}(f_0, m)}{\partial f_0} = \frac{\partial s'}{\partial f_0} = 8\pi^2 m f_0 \tag{13b}$$

and consequently, the probability density function of resonance frequency:

$$g_{f_0}(f_0) = \int_{-\infty}^{\infty} g_{s',m}\left(\varphi_{s'}^{-1}(f_0, m), m\right) \left| \frac{\partial \varphi_{s'}^{-1}(f_0, m)}{\partial f_0} \right| dm \tag{13c}$$

where $g_{s',m}$ is the joint probability distribution of (s', m). Following the same reasoning for surface mass one can obtain the following expressions:

$$\varphi_m^{-1}(f_0, s') = m = \frac{s'}{4\pi^2 f_0^2} \tag{13d}$$

$$\frac{\partial \varphi_m^{-1}(f_0, s')}{\partial f_0} = \frac{\partial m}{\partial f_0} = -\frac{s'}{2\pi^2 f_0^3} \tag{13e}$$

and consequently:

$$g_{f_0}(f_0) = \int_{-\infty}^{\infty} g_{s',m}\left(\varphi_m^{-1}(f_0,s'),s'\right)\left|\frac{\partial \varphi_m^{-1}(f_0,s')}{\partial f_0}\right| ds' \tag{13f}$$

For the readers' convenience, there are given both partial derivatives, although only one of them is needed to determine the Jacobian, eq. (7). The *PDF* of the resonance frequency is directly determined – see Fig. 4a. Having the *PDF,* it is possible to calculate the *CDF* (Fig. 4b). One can notice that although both arguments of the resonance frequency have symmetric, normal probability densities, the resulting variable has the nonsymmetrical *PDF*. The expected value of resonance frequency is equal to 76.1 Hz while its standard deviation is 11.7 Hz. The probability that resonance frequency is higher than 100 Hz is around 1%.

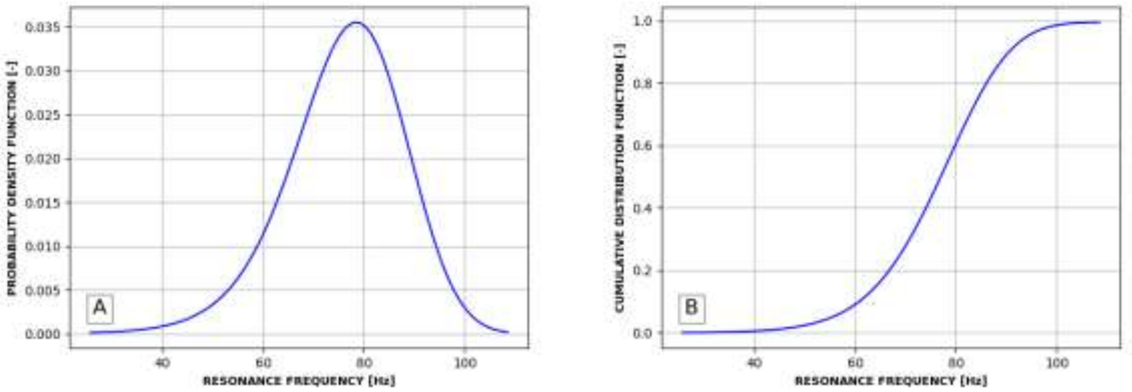


Figure 4. a) Probability density (a) and cumulative distribution function (b) of resonance frequency being function of two random variables: dynamic stiffness of XPS and mass per unit area of cement screed.

3.3 Improvement of impact sound insulation

Since the transmissibility model covers the frequencies higher and lower than the resonance, the insulation effectiveness is only the piecewise injective function. Assuming three input random variables (loss factor, dynamic stiffness and surface mass), the application of equations (7a) and (7b) might cause some difficulties. Hence, the distribution of the improvement of impact sound insulation was obtained applying eq. (10) – see section 2.2. Then the probability density was calculated by differentiation of a distribution function. It must be pointed out that the distribution is calculated by numerical integration, hence it might be not smooth. The differentiation of such a function might lead to oscillations, which disturb the *PDF*. Alternatively, the inverse functions with respect to the arbitrary variables and their derivatives needed to apply the change of variable method are given in the appendix. They are expressed by very complex equations. However, using them one can formulate the exact form of probability density functions of improvement of impact sound insulation by transmissibility model. Cremer-Vér function is monotonic with frequency and with respect to both investigated random variables. Consequently, it possesses the single inverse function concerning both arguments in the entire domain. Therefore, for Cremer-Vér model the direct determination of insulation improvement probability density is possible and rather easy to obtain. One assumes the vector argument $x = [s',m]$, the output $y = \Delta L$ and the function $\varphi(s',m) = 30\log\left(2\pi\sqrt{\frac{m}{s'}}\right)$. As in the previous case the dimension of the argument vector equals 2 and the dimension of output vector equals 1, therefore one has to apply the equation (7a) and 7(b). To reach the goal the inverse functions with regard to dynamic stiffness as well as unitary mass have to be calculated. Then, their partial derivatives with respect to ΔL are needed to determine the Jacobian. Two equivalent expressions describing probability density function might be obtained:

$$\varphi_{s'}^{-1}(\Delta L, m) = s' = 4\pi^2 f^2 \cdot m \cdot 10^{-\frac{\Delta L}{15}}, \tag{14a}$$

$$\frac{\partial \varphi_{s'}^{-1}(\Delta L, m)}{\partial \Delta L} = \frac{\partial s'}{\partial \Delta L} = 4\pi^2 \cdot \left[-\frac{\ln(10)}{15}\right] \cdot f^2 \cdot m \cdot 10^{-\frac{\Delta L}{15}}, \tag{14b}$$

and consequently, the probability density function of the improvement of impact sound insulation:

$$g_{\Delta L}(\Delta L) = \int_{-\infty}^{\infty} g_{s',m}\left(\varphi_{s'}^{-1}(\Delta L, m), m\right) \left|\frac{\partial \varphi_{s'}^{-1}(\Delta L, m)}{\partial \Delta L}\right| dm. \tag{14c}$$

For the surface mass one can obtain the following expressions:

$$\varphi_{m}^{-1}(\Delta L, s') = m = 0.25\pi^{-2} f^{-2} \cdot s' \cdot 10^{\frac{\Delta L}{15}}, \tag{14d}$$

$$\frac{\partial \varphi_{m}^{-1}(\Delta L, s')}{\partial \Delta L} = \frac{\partial m}{\partial \Delta L} = 0.25\pi^{-2} \cdot \left[\frac{\ln(10)}{15}\right] \cdot f^{-2} \cdot s' \cdot 10^{\frac{\Delta L}{15}}, \tag{14e}$$

and the probability density of the improvement of impact sound insulation:

$$g_{\Delta L}(\Delta L) = \int_{-\infty}^{\infty} g_{s',m}\left(\varphi_{m}^{-1}(\Delta L, s'), s'\right) \left|\frac{\partial \varphi_{m}^{-1}(\Delta L, s')}{\partial \Delta L}\right| ds'. \tag{14f}$$

Distribution is calculated by integration of the *PDF*, see eq. (9). To present the results legibly, they were divided into two groups: 1) frequency lower or equal to 100 Hz (f = 50 Hz, 60 Hz, 70 Hz, 80 Hz, 90 Hz, and 100 Hz), 2) frequency higher than 100 Hz, the following octave band frequencies are taken: 125 Hz, 250 Hz, 500 Hz, 1000 Hz, 2000 Hz. For lower frequencies (the first group) the improvement of impact sound insulation was calculated only using the transmissibility model, while for the higher frequencies (the second group) both Cramer-Vér and Schiavi relations were employed. The *PDF* and *CDF* determined for lower frequencies are presented in Fig. 5a, b. It is noticeable that although the random input parameters are normally distributed, the output function has strongly nonsymmetrical distribution – see Fig. 5a. The *CDF* presented in Fig. 5b might be used to determine the quantile of any desirable order. According to the presented results, the probability that floating floor will impair the insulation effect of a partition equals 70% for sound frequency of 100 Hz and increases up to 100% along with the decreasing frequency.

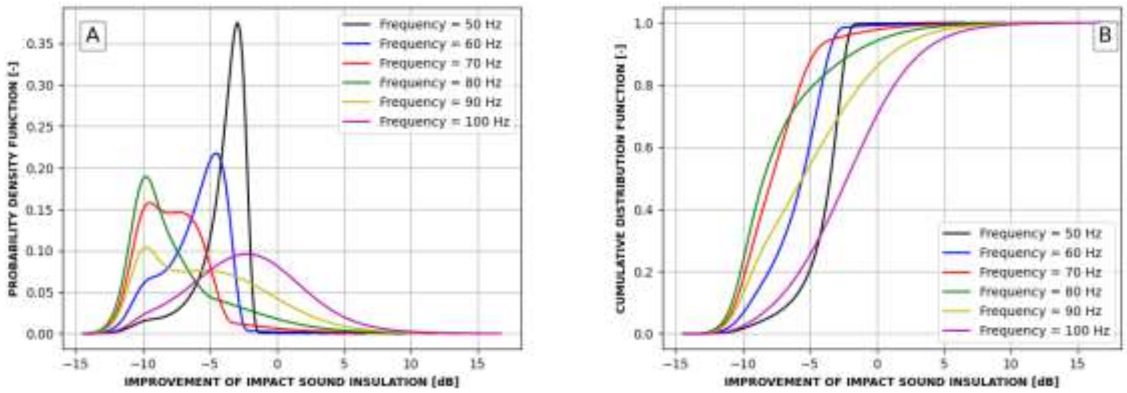


Figure 5. The a) *PDF* and b) *CDF* of the impact sound insulation improvement determined using the transmissibility model for frequencies equal to 50 Hz, 60 Hz, 70 Hz, 80 Hz, 90 Hz, and 100 Hz.

The discrete parameters of insulation improvement density are gathered in Table 1. Based on the *PDF*, determined by the proposed approach, it is possible to calculate the expectation and standard deviation. The non-symmetrical shape of the probability density of the improvement of impact sound insulation determined for frequencies lower than 100Hz results from the features of relation between insulation effectiveness and frequency of the noise, which is highly nonlinear – see eq. (3) and Fig. 2. Consequently, one is not allowed to linearize the improvement of impact sound insulation in this region. Since the expected value of the resonance frequency for analyzed case equals 76.1 Hz, the worst improvement (deterioration) can be observed for adjacent frequencies, namely 70Hz and 80Hz – see Table 1. The standard deviation monotonously grows along with the frequency.

Table 1. The expected value and standard deviation of insulation improvement calculated using the transmissibility model for frequencies less or equal than 100 Hz.

Frequency [Hz]	Expectation [dB]	Standard deviation [dB]
50	-4.04	1.86
60	-6.04	2.34
70	-7.59	2.76
80	-7.35	3.11
90	-5.09	4.46
100	-2.18	4.22

For frequencies higher than 100 Hz both Cremer-Vér and Schiavi relations were applied. The comparison of these approaches are presented in Figure 7a, b. The nonsymmetrical behavior of *PDF*s might be still recognized by applying both approaches (Cremer-Vér and Schiavi) – see Fig. 6a and Fig. 7a. but become much softer than for frequencies below 100 Hz. The *PDF* peak determined using the transmissibility model grows and becomes more soaring along with the band frequency while the shape of *PDF* calculated applying Cremer-Vér relation remains unchanged. They are only shifted toward larger values of sound insulation improvement along with the band frequency. Based on the *PDF,* the expected values and standard deviations of insulation improvement using both models might be determined. Their comparison is presented in Table 2. The standard deviation calculated using Cremer-Vér equation remains unchanged for all band frequencies. Assuming the transmissibility model the standard deviation decreases along with the growing frequency and admits slightly lower values for larger total loss factor. Generally, the transmissibility model implies that for higher sounds the efficiency of insulation increases, and simultaneously particular results are closer to the expected values.

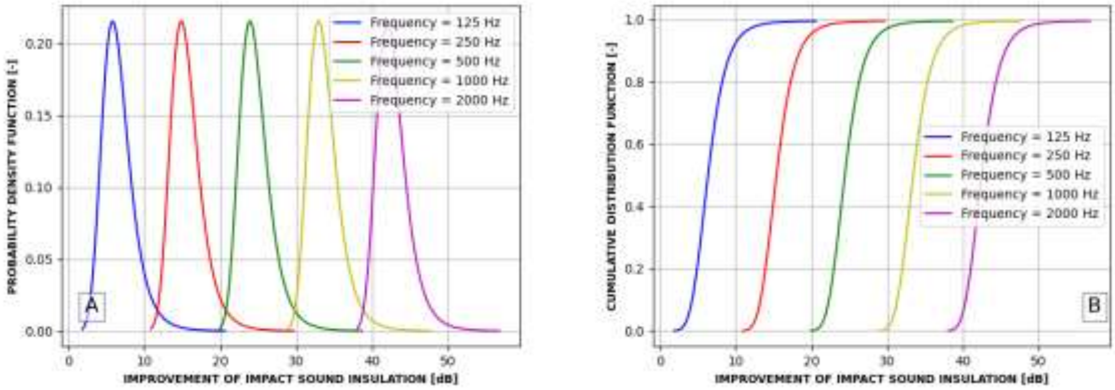


Figure 6. The a) *PDF* and b) *CDF* of the impact sound insulation improvement determined using the Cremer-Vér model for frequencies equal to 125 Hz, 250 Hz, 500 Hz, 1000 Hz, and 2000 Hz.

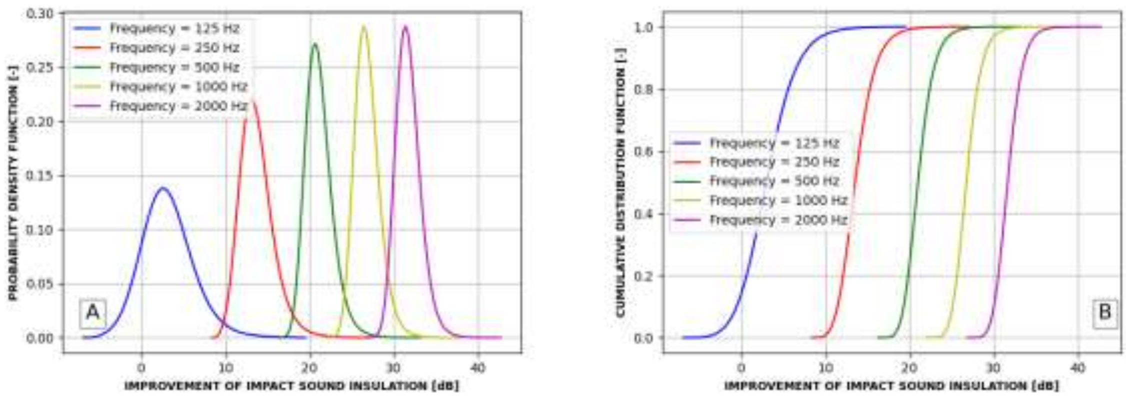


Figure 7. The a) *PDF* and b) *CDF* of the impact sound insulation improvement determined by using the transmissibility model for frequencies equal to 125 Hz, 250 Hz, 500 Hz, 1000 Hz, and 2000 Hz.

Table 2. The comparison of expected values and standard deviations of insulation improvement calculated using Cremer-Vér model and the transmissibility model assuming η=0.1 and η=0.2 for frequencies higher than 100 Hz.

	Expectation [dB]		Standard deviation [dB]	
f [Hz]	Cremer-Vér	Schiavi	Cremer-Vér	Schiavi
125	6.65	3.15	2.17	3.10
250	15.68	13.74		2.00
500	24.71	21.15		1.62
1000	33.74	26.87		1.49
2000	42.77	31.76		1.47

To validate the results calculated using eq. (7), the comparison with the Monte-Carlo data [38] was made. In order to decrease the number of simulations, the total loss factor was assumed to be the constant value equal to $\eta = 0.1$ and $\eta = 0.2$, appropriately. For two random variables (surface density and dynamic stiffness) 10^6 calculations were performed. The number of bins (k) was taken as the maximum integer (46) satisfying the inequality $k \le 5\ln(n)$, where n is the number of sampling points. The comparison is presented in Fig. 8 and concerns data obtained for frequency equal to 90 Hz. Since this value is relatively close to the expectation of the resonance frequency, the transmissibility model is used for this purpose. One can notice that the probability density of the improvement effectiveness obtained using eq. (7) resembles the Monte Carlo results. The slight difference between the results obtained using those two techniques is visible only in the sharp peak. This peak stems from the non-monotonic relation between improvement of the impact sound insulation and frequency. Since the number of beans in MC method is limited by the empirical inequality, therefore the accuracy of MC method is worse than the accuracy of method based on transformed variables. By increasing the number of sampling point the result calculated using MC would converge to the one obtained by the transformed variable method, but it would require much longer computational time.

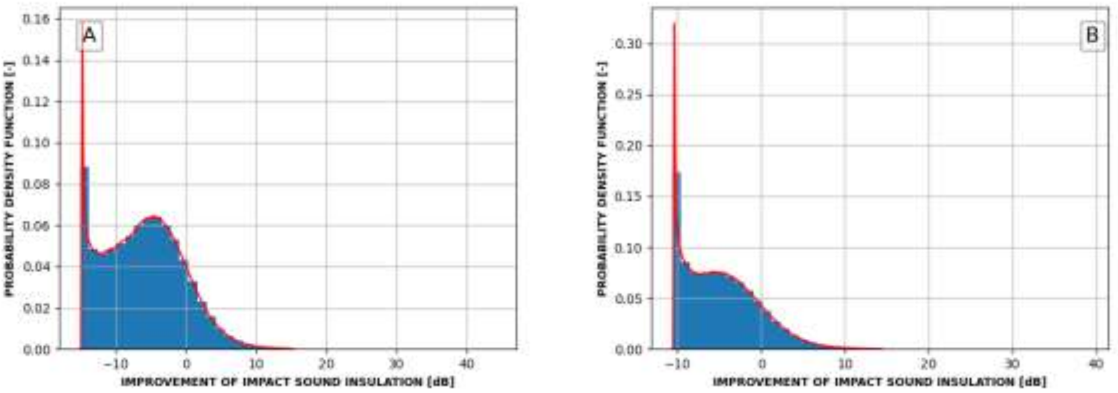


Figure 8. The comparison of improvement of impact sound insulation probability density calculated using transformed random variable method with Monte Carlo simulation, with 90 Hz of frequency for a) $\eta = 0.1$ and b) $\eta = 0.2$.

3.4 Linearization of Cremer-Vér formula

According to the aforementioned results, the dynamic stiffness s' of the insulating layer, as well as mass per unit area m of cement screed, are characterized by normal distributions, which can be denoted as $N\left(\mu_{s'}, \sigma_{s'}^2\right)$ and $N\left(\mu_m, \sigma_m^2\right)$ respectively. Let us presume that standard deviations of both input parameters admit relatively low values. Such an assumption implies that the output variable, i.e. the improvement of impact sound insulation might be approximated using linear relation in the vicinity of its expected value. Consequently, the improvement of impact sound insulation is normally distributed, with PDF, $g_{\Delta L}$, given by the formula:

$$g_{\Delta L}(\Delta L) = \frac{1}{\sigma_{\Delta L}\sqrt{2\pi}} \exp\left[-\frac{\left(\Delta L - \mu_{\Delta L}\right)^2}{2\left(\sigma_{\Delta L}\right)^2}\right], \tag{15a}$$

Applying the Cremer-Vér model expected value and standard deviation are given accordingly:

$$\mu_{\Delta L} = 30\log\left[\frac{2\pi f}{\sqrt{\mu_{s'}/\mu_m}}\right] \tag{15b}$$

$$\mathrm{Var}(\Delta L) = \sigma_{\Delta L}^2 = \left(\frac{15}{\ln(10)} \cdot \frac{1}{\mu_m}\right)^2 \sigma_m{}^2 + \left(-\frac{15}{\ln(10)} \cdot \frac{1}{\mu_{s'}}\right)^2 \sigma_{s'}{}^2 \tag{15c}$$

Eq. (15a) allows predicting the probability density of the floating floor construction, whose component parameters' distributions, i.e. dynamic stiffness and unitary mass are known. The equation (14c) was derived from the first order Taylor expansion of the variance [30]. Assuming a linear relation between the improvement of sound impact insulation and surface mass and dynamic stiffness truncation of terms of order higher than one does not introduce any error. However, for strongly nonlinear function such a simplification might cause substantial error. The accuracy of the determined formulas (eq. 15b – 15c) was verified based on the statistical parameters of materials data considered in the manuscript. The expected values and standard deviations describing the insulation improvement of the floating floor composed of XPS and cement screed were calculated assuming their statistical parameters as presented in section 3.1. Then they were compared to the expectation and standard deviation calculated by integration of PDF determined using the transformed variables theorem, see Table 3. The comparison evinces remarkable accuracy with regard to standard deviation, which implies that the proposed formula can be successfully applied to predict the insulation effectiveness uncertainty concerning floating floor composed of XPS and cement screed.

The results of linearization might be compared with the procedures enclosed in ISO/IEC GUIDE 98-3:2008 [39]. According to ISO/IEC 98-3:2008, point 4.1.5 the combined standard uncertainty equals the estimated standard deviation, which is equal to the variance (point 4.2.2, ISO/IEC 98-3:2008). Consequently, combined standard uncertainty of the improvement of impact sound insulation presented by eq. (10), p. 5.1.2 of ISO/IEC 98-3:2008, equals its variance eq. (15c) of the manuscript:

$$\left[u_c(\Delta L)\right]^2 = \left(\frac{\partial \Delta L}{\partial m}\right)^2 \sigma_m{}^2 + \left(\frac{\partial \Delta L}{\partial s'}\right)^2 \sigma_{s'}{}^2 = \left(\frac{15}{\ln(10)} \cdot \frac{1}{\mu_m}\right)^2 \sigma_m{}^2 + \left(-\frac{15}{\ln(10)} \cdot \frac{1}{\mu_{s'}}\right)^2 \sigma_{s'}{}^2 = \mathrm{Var}(\Delta L).$$

However, it shall be mentioned that the guide ISO/IEC 98-3:2008 considers only variables with symmetrical distribution, while the presented approach does not involve such limitations.

Table 3. The comparison of expected values and standard deviations of insulation improvement calculated by means of transformed variables theorem and linearization of the Cremer-Vér model

	Expectation [dB]		Standard deviation [dB]	
f [Hz]	Transformed variables theorem	Linearization of Cremer-Vér formula	Transformed variables theorem	Linearization of Cremer-Vér formula
125	6.65	6.33	2.17	1.92
250	15.68	15.36		
500	24.71	24.39		
1000	33.74	33.42		
2000	42.77	42.45		

4. Conclusions

The main object of the present paper was to propose an innovative method, which enables one to predict the uncertainty of the improvement of impact sound insulation of a floating floor. So far the acoustic analyses have been usually performed assuming a deterministic approach. Since the mechanical properties of resilient materials are often related to certain randomness and uncertainties, such an approach is inevitably related to some inaccuracy. Based on the stochastic analysis one proposed the algorithm, in which the randomness of input material properties is taken into consideration. The procedure was applied to empirical estimations of the acoustical performance of actual floating floor construction, consisting of XPS (33 mm) as a resilient material, cement screed (30 mm) as a floating slab, and concrete base, with regards to the experimental result. According to the obtained results, the following conclusions can be drawn:

- The insulation effectiveness of a floating floor depends solely on elastic and inner properties of the resilient and floating layer respectively. However, since material parameters are related to some randomness, the acoustic service of a final partition is also burdened with some uncertainty. The statistical test applied with regard to the experimental data concerning dynamic stiffness of XPS and density of cement screed indicates that the distribution of both random variables might be described by Gaussian probability density functions. Despite that, the resonance frequency, which is the output function, has a non-gaussian probability distribution. This is caused by the non-linear relation between input and output variables.
- The probability density function of insulation effectiveness determined by the transmissibility model is bimodal for frequencies close to the resonance one especially in the range 60 Hz - 90 Hz. In the direct vicinity of the resonance frequency, the presence of floating layers even aggravates the acoustic properties of a slab in comparison to the bare concrete slab, whereby negative values of ΔL are greater for resilient material with a lower total loss factor η. According to the given distribution of total loss factor, dynamic stiffness of resilient layer and cement screed density, the probability that floating floor will impair acoustic comfort equals 70% for noise with 100 Hz of frequency and increases up to 100% along with the decreasing frequency. This effect is practically independent of the total loss factor.
- Above the resonance frequency, the insulation effectiveness can be equivalently estimated by using either Cremer-Vér and transmissibility models. However, Cremer-Vér model does not take into account energy dissipation within the resilient layer, whereas the transmissibility model depends on the total loss factor. Hence, Cremer-Vér formula overestimates the benefits accruing from the installation of a floating floor. In both models, the acoustic improvement is described by a non-symmetrical probability distribution function, but for Cremer-Vér formula, standard deviation is constant along with rising frequency. On the other hand, the expected values provided by transmissibility models are closer to the experimental data along with rising frequency. The simplified formula derived based on linearization of the Cremer-Vér model provides accurate results concerning the probability density function of insulation effectiveness and can be practically applied to assess the floating floor performance uncertainty. The results obtained by the linearization were confirmed with the data provided by ISO/IEC GUIDE 98-3:2008. However, apart from the discrete features the linearization technique provides additionally the distribution function of output variable.
- The accuracy of the proposed stochastic algorithm was verified using Monte Carlo methods. Since the calculated probability density functions closely reflect the MC histograms (for 10^6 simulations) it is evident that the introduced technique evinces high efficiency, simultaneously demanding less calculation cost than sampling methods.
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Appendix A. Formulas providing the explicit forms of probability density functions of the impact sound insulation improvement according to the transmissibility model.

Application of transformed variables theorem (section 2.2.) enables to formulate explicitly the *PDF* of output variable. To that end, the inverse functions of a considered parameter with regard to its variable-arguments needs to be established along with their derivatives. For readers' convenience the general expression for the *PDF* determined by means of variables transformation is inserted below [31,32]:

$g_Y(y) = g_X\left(\varphi^{-1}(y)\right)\left|J(y)\right|$, where the Jacobian is determined accordingly: (A1)

$$J(y) = \frac{\partial\left(\varphi_1^{-1}, \varphi_2^{-1}, .., \varphi_n^{-1}\right)}{\partial\left(y_1, y_2, .., y_n\right)} \tag{A2}$$

Due to the complexity of established formulas, the appropriate components necessary for inserting to eq. (A1) are provided below. Both, the inverse functions and derivatives needed to *PDFs* of resonance frequency (Fig. 4A), and the impact of sound insulation according to Cremer-Vér formula (Fig.6A) are contained in the main text of the manuscript. Below the expressions concerning the transmissibility model, eq. (3) are provided. Since the function is not injective, the formulas concerning dynamic stiffness and unitary mass differ for frequencies below and above the resonance frequency.

Inverse functions with regard to the impact sound insulation improvement

1. Loss factor η

$$\eta=\sqrt{\frac{2\cdot 10^{\frac{-2\Delta L}{15}}\cdot\frac{f^2}{f_0^2}+1-10^{\frac{-2\Delta L}{15}}\cdot\frac{f^4}{f_0^4}-10^{\frac{-2\Delta L}{15}}}{10^{\frac{-2\Delta L}{15}}\cdot\frac{f^2}{f_0^2}-\cdot\frac{f^2}{f_0^2}}}$$

2. Dynamic stiffness *s'*

2.1. For $f>f_0$:

$$s'=\frac{8\pi^2 m\cdot f^4\cdot 10^{\frac{-2\Delta L}{15}}}{-\left[\eta^2 f^2\left(10^{\frac{-2\Delta L}{15}}-1\right)-2f^2\cdot 10^{\frac{-2\Delta L}{15}}\right]+\sqrt{f^4\left[\left(\eta^2\cdot 10^{\frac{-2\Delta L}{15}}-2\cdot 10^{\frac{-2\Delta L}{15}}-\eta^2\right)^2-4\cdot 10^{\frac{-4\Delta L}{15}}+4\cdot 10^{\frac{-2\Delta L}{15}}\right]}}$$

2.2. For $f<f_0$:

$$s'=\frac{8\pi^2 m\cdot f^4\cdot 10^{\frac{-2\Delta L}{15}}}{-\left[\eta^2 f^2\left(10^{\frac{-2\Delta L}{15}}-1\right)-2f^2\cdot 10^{\frac{-2\Delta L}{15}}\right]-\sqrt{f^4\left[\left(\eta^2\cdot 10^{\frac{-2\Delta L}{15}}-2\cdot 10^{\frac{-2\Delta L}{15}}-\eta^2\right)^2-4\cdot 10^{\frac{-4\Delta L}{15}}+4\cdot 10^{\frac{-2\Delta L}{15}}\right]}}$$

3. Surface mass *m*

3.1. For $f>f_0$:

$$m=\frac{s}{4\pi^2}\cdot\frac{-\left[\eta^2 f^2\left(10^{\frac{-2\Delta L}{15}}-1\right)-2f^2\cdot 10^{\frac{-2\Delta L}{15}}\right]+\sqrt{f^4\left[\left(\eta^2\cdot 10^{\frac{-2\Delta L}{15}}-2\cdot 10^{\frac{-2\Delta L}{15}}-\eta^2\right)^2-4\cdot 10^{\frac{-4\Delta L}{15}}+4\cdot 10^{\frac{-2\Delta L}{15}}\right]}}{2f^4\cdot 10^{\frac{-2\Delta L}{15}}}$$

3.2. For $f<f_0$:

$$m=\frac{s}{4\pi^2}\cdot\frac{-\left[\eta^2 f^2\left(10^{\frac{-2\Delta L}{15}}-1\right)-2f^2\cdot 10^{\frac{-2\Delta L}{15}}\right]-\sqrt{f^4\left[\left(\eta^2\cdot 10^{\frac{-2\Delta L}{15}}-2\cdot 10^{\frac{-2\Delta L}{15}}-\eta^2\right)^2-4\cdot 10^{\frac{-4\Delta L}{15}}+4\cdot 10^{\frac{-2\Delta L}{15}}\right]}}{2f^4\cdot 10^{\frac{-2\Delta L}{15}}}$$

Derivatives with regard to the impact sound insulation improvement

1. Loss factor η

$$\frac{\partial \eta}{\partial \Delta L} = \frac{1}{2\cdot\sqrt{\dfrac{2\cdot 10^{\frac{-2\Delta L}{15}}\cdot\dfrac{f^2}{f_0^2}+1-10^{\frac{-2\Delta L}{15}}\cdot\dfrac{f^4}{f_0^4}-10^{\frac{-2\Delta L}{15}}}{10^{\frac{-2\Delta L}{15}}\cdot\dfrac{f^2}{f_0^2}-\cdot\dfrac{f^2}{f_0^2}}}}\times$$

$$\times\frac{\left[-\frac{2}{15}\ln(10)\cdot 10^{\frac{-2\Delta L}{15}}\cdot\left(2\frac{f^2}{f_0^2}-\frac{f^4}{f_0^4}-1\right)\right]\cdot\left(10^{\frac{-2\Delta L}{15}}\cdot\frac{f^2}{f_0^2}-\cdot\frac{f^2}{f_0^2}\right)-\left(2\cdot 10^{\frac{-2\Delta L}{15}}\cdot\frac{f^2}{f_0^2}+1-10^{\frac{-2\Delta L}{15}}\cdot\frac{f^4}{f_0^4}-10^{\frac{-2\Delta L}{15}}\right)\cdot\left(-\frac{2}{15}\ln(10)\frac{f^2}{f_0^2}\cdot 10^{\frac{-2\Delta L}{15}}\right)}{\left(10^{\frac{-2\Delta L}{15}}\cdot\frac{f^2}{f_0^2}-\cdot\frac{f^2}{f_0^2}\right)^2}$$

2. Dynamic stiffness *s'*

$$\frac{\partial s'}{\partial \Delta L} = \frac{A'B - AB'}{B^2}$$

2.1. For $f > f_0$:

$$B = B_1 + B_2$$

$$B' = B_1' + B_2'$$

2.2. For $f < f_0$:

$$B = B_1 - B_2$$

$B' = B_1' - B_2'$, where:

$$A = 8\pi^2 m\cdot f^4\cdot 10^{\frac{-2\Delta L}{15}}$$

$$B_1 = -\left[\eta^2 f^2\left(10^{\frac{-2\Delta L}{15}}-1\right)-2f^2\cdot 10^{\frac{-2\Delta L}{15}}\right]$$

$$B_2 = \sqrt{f^4\left[\left(\eta^2\cdot 10^{\frac{-2\Delta L}{15}}-2\cdot 10^{\frac{-2\Delta L}{15}}-\eta^2\right)^2-4\cdot 10^{\frac{-4\Delta L}{15}}+4\cdot 10^{\frac{-2\Delta L}{15}}\right]}$$

$$A' = -\frac{16}{15}\pi^2\ln 10\cdot f^4\cdot 10^{\frac{-2\Delta L}{15}}$$

$$B_1' = \frac{2}{15}\ln 10 \cdot 10^{\frac{-2\Delta L}{15}} f^2\eta^2 - \frac{4}{15}\ln 10 \cdot 10^{\frac{-2\Delta L}{15}} f^2$$

$$B_2' = \frac{1}{2\sqrt{f^4\left[\left(\eta^2 \cdot 10^{\frac{-2\Delta L}{15}} - 2\cdot 10^{\frac{-2\Delta L}{15}} - \eta^2\right)^2 - 4\cdot 10^{\frac{-4\Delta L}{15}} + 4\cdot 10^{\frac{-2\Delta L}{15}}\right]}} \times$$

$$\times\left[-\frac{4}{15}\ln 10 \cdot f^4 \cdot 10^{\frac{-2\Delta L}{15}}\left(\eta^2 - 2\right) - \frac{8}{15}\ln 10 \cdot f^4 \cdot 10^{\frac{-2\Delta L}{15}}\left(1 - \frac{2}{15}\cdot 10^{\frac{-2\Delta L}{15}}\right)\right]$$

3. Surface mass *m*

$$\frac{\partial m}{\partial \Delta L} = \frac{s'}{4\pi^2}\frac{C'D - CD'}{D^2}$$

3.1. For $f > f_0$:

$$C = C_1 + C_2$$

$$C' = C_1' + C_2'$$

3.2. For $f < f_0$:

$$C = C_1 - C_2$$

$C' = C_1' - C_2'$, where:

$$C_1 = -\left[\eta^2 f^2\left(10^{\frac{-2\Delta L}{15}} - 1\right) - 2 f^2 \cdot 10^{\frac{-2\Delta L}{15}}\right]$$

$$C_2 = \sqrt{f^4\left[\left(\eta^2 \cdot 10^{\frac{-2\Delta L}{15}} - 2\cdot 10^{\frac{-2\Delta L}{15}} - \eta^2\right)^2 - 4\cdot 10^{\frac{-4\Delta L}{15}} + 4\cdot 10^{\frac{-2\Delta L}{15}}\right]}$$

$$D = 2 f^4 \cdot 10^{\frac{-2\Delta L}{15}}$$

$$C_1' = \frac{2}{15}\ln 10 \cdot 10^{\frac{-2\Delta L}{15}} f^2\eta^2 - \frac{4}{15}\ln 10 \cdot 10^{\frac{-2\Delta L}{15}} f^2$$

$$C_2' = \frac{1}{2\sqrt{f^4\left[\left(\eta^2 \cdot 10^{\frac{-2\Delta L}{15}} - 2\cdot 10^{\frac{-2\Delta L}{15}} - \eta^2\right)^2 - 4\cdot 10^{\frac{-4\Delta L}{15}} + 4\cdot 10^{\frac{-2\Delta L}{15}}\right]}} \times$$

$$\times\left[-\frac{4}{15}\ln 10\cdot f^4\cdot 10^{\frac{-2\Delta L}{15}}\left(\eta^2-2\right)-\frac{8}{15}\ln 10\cdot f^4\cdot 10^{\frac{-2\Delta L}{15}}\left(1-\frac{2}{15}\cdot 10^{\frac{-2\Delta L}{15}}\right)\right]$$

$$D' = -\frac{4}{15}\ln 10\cdot f^4\cdot 10^{\frac{-2\Delta L}{15}}$$