Comment on "Physics without determinism: Alternative interpretations of classical physics", Phys. Rev. A, 100:062107, Dec 2019

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Abstract

The paper "Physics without determinism: Alternative interpretations of classical physics" [Phys. Rev. A, 100:062107, Dec 2019] defines *finite information quantities* (FIQ). A FIQ expresses the available information about the value of a physical quantity. We show that a change in the measurement unit does not preserve the information carried by a FIQ, and therefore that the definition provided in the paper is not complete.

The expression of the state of knowledge about a measurand as a probabilitiy distribution (or some summary of it, such as its mean and standard deviation) is the conventional approach for expressing a measurement result [1–4]. However, it does not intuitively parallel the much more immediate concepts of "certain" and "uncertain digits" that every experimentalist feels when taking note of a measurement outcome in the lab notebook.

In [5], Del Santo and Gisin introduce the concept of *finite information quantities* (FIQ). A FIQ ranging in the interval $[0, 1]$ is expressed by the binary number $Q = 0.Q_1Q_2Q_3\ldots$, where the individual bits Q_k are Bernoulli random variables having propensities q_k for the realisation of the case $Q_k = 1$. A specific FIQ Q is thus defined by the vector of propensities $\boldsymbol{q} = [q_1, q_2, \ldots, q_k, \ldots, q_M, \frac{1}{2}, \frac{1}{2}, \ldots]$ of its bits Q_k; it is assumed that $q_k = \frac{1}{2}$ for $k > M$, *i.e.*, all bits beyond position M have a 50 % propensity of being either 0 or 1 and therefore carry no information. Only a finite number M of propensities are needed to specify Q.

The FIQ concept is very appealing and it is tempting to adopt it to express the value and uncertainty of a quantity as an alternative to probability distributions. However, for the concept of FIQ to become a practical alternative to the current way of representing the state of knowledge about a quantity, it is mandatory that calculations with them be possible and, hopefully, simple.

Consider for example the expression of the value of a quantity, traditionally written as $Q = \{Q\}[U]$, where $\{Q\}$ is the numerical value and $[U]$ is the unit. Changing the unit to $U' = U/L$, L being a constant, implies $Q = \{Q'\}[U']$, with $\{Q'\} = L\{Q\}$. So, even such an elementary tranformation as the change of measurement unit implies the multiplication of a FIQ by a constant.

Indeed, the FIQ definition suggests that it is possible to identify simple, practical calculation rules operating on the finite (and, intuitively, small) number of indeterminate bits and their propensities; rules suitable to be converted in efficient computation algorithms.

The arithmetic relevant to a unit change (Appendix A) shows that the transformation $Q' = LQ$ generates bits Q'_k of Q' which are not mutually independent even if the original Q_k bits are independent. Therefore, expressing Q' by providing only the propensities q'_k of its individual bits deletes some of the original information.

Random variables Q with independent binary digits Q_k have been considered in mathematical literature [6–8]. In general, Q has a 'reasonable' probability density function (pdf) only if the q_k satisfy strict conditions, and in that case the pdf is necessarily an exponential [6]; otherwise, it becomes a fractal [7], hence difficult to associate with a physical quantity.

In conclusion, it appears that a specification of the state of knowledge about a quantity Q by means of a FIQ should also include information on the dependencies among the Q_k, and therefore that, although the FIQ concept might be physically sound and useful, its definition as given in [5] is not complete, and deserves further development.

Appendix A: Minimal FIQ maths

A FIQ arithmetics can be established by generalizing operations on binary numbers. The sum $S = Q + R = 0.S_1S_2S_3\ldots$ of two FIQs, $Q = 0.Q_1Q_2Q_3\ldots$ and $R = 0.R_1R_2R_3\ldots$, is given by the full adder rule, Tab. I.

Q_k	R_k	C_{k+1}	S_k	C_k
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

TABLE I. Binary full adder truth table. C_k is the carry bit.

If $\boldsymbol{q}$ is the vector of propensities associated with Q, and $\boldsymbol{r}$ with R, then under the assumption of independence of q_k and r_k, the propensity s_k of each sum bit S_k can be written as the sum of the four propensities of the $S_k = 1$ cases in Tab. I:

$$\begin{aligned} s_k =& (1-q_k)(1-r_k)c_{k+1} + (1-q_k)r_k(1-c_{k+1}) \\ &+ q_k(1-r_k)(1-c_{k+1}) + q_k r_k c_{k+1} \\ =& q_k + r_k + c_{k+1} \\ &- 2(q_k r_k + q_k c_{k+1} + r_k c_{k+1}) + 4 q_k r_k c_{k+1} \end{aligned} \tag{A1}$$

and similarly the propensity c_k of the carry bit C_k is

$$c_k = q_k r_k + q_k c_{k+1} + r_k c_{k+1} - 2 q_k r_k c_{k+1} \tag{A2}$$

For example for the case $c_{k+1} = \frac{1}{2}$, we have $s_k = \frac{1}{2}$ and $c_k = \frac{1}{2}(q_k + r_k)$: the information provided by q_k and r_k is transferred, through the carry bit C_k, to bit S_{k-1}.

Multiplication by a deterministic constant L can be performed by repeated shifting and addition. Table II gives a simple example. If $P = LQ$, where $\boldsymbol{q} = [0, 0, q_3, \frac{1}{2} \ldots]$ and

0.	0	0	Q_3	...
$\times$		1	1	
0.	0	0	Q_3	...
+ 0.	0	Q_3	Q_4	...
= 0.	P_1	P_2	P_3	...

TABLE II. Multiplication table, $P = LQ$ where $Q = 0.0Q_2Q_3\ldots$ and $L = (11)_2 = (3)_{10}$.

$L = (11)_2 = (3)_{10}$, then

$$\begin{aligned} p_1 &= \frac{1}{2}q_3^2 + \frac{1}{4}q_3, \\ p_2 &= q_3 - q_3^2 + \frac{1}{4}, \\ p_3 &= \frac{1}{2}, \quad \ldots \end{aligned} \tag{A3}$$

The propensity of occurrence of specific digit couples can also be computed. For example, denoting as p_{12} the propensity of the event $\{P_1 = 1, P_2 = 1\}$ we have $p_{12} = 0$ (to have $P_1 = 1$, it should occur that $Q_3 = 1$ and $C_3 = 1$ at the same time, hence $C_2 = 1$. However, the case $\{Q_3 = 1, C_3 = 1\}$ always generates $P_2 = 0$, so $\{P_1 = 1, P_2 = 1\}$ is never possible). Since $p_{12} = 0 \neq p_1 p_2$, bits P_1 and P_2 are not independent.

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