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Quantum Correlations and Quantum Non-Locality: A Review and a Few New Ideas

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Abstract: In this paper we make an extensive description of quantum non-locality, one of the most intriguing and fascinating facets of quantum mechanics. After a general presentation of several studies on this subject dealing with different but connected facets of quantum non-locality, we consider if this, and the friction it carries with special relativity, can eventually find a “solution” by considering higher dimensional spaces.

Keywords: quantum correlations; quantum non-locality; entanglement, bell measurements

1. Introduction

Bell’s discovery, ref. [1] that local hidden variable theories (LHVT) cannot reproduce all the results of quantum mechanics when dealing with entangled states, demonstrated the peculiarity of correlations among quantum states introducing the concept of quantum non-locality [2–8].

Since then, the experimental studies of Bell inequalities [9–15] recently culminated in conclusive tests [16–18], together with the developing of quantum technologies [19,20] prompted a growing interest on this effect, which remains one of the most significant problems in understanding quantum mechanics.

The investigation on Bell non-locality properties has been developed and generalized in many directions, and in the present paper we aim to recall some of these achieved developments, in order to provide a reasoned and concise orientation between its multifaceted aspects. To pursue this purpose, the first crucial point to be considered is about the definition of non-locality and its meaning [9,10,21–30]. Looking at the Bell theorem, it is possible to understand how locality entails with the fact that if two different observers, henceforth called Alice and Bob, make two independent measures (for example, A and B) on a shared physical system and achieve respectively the results a, b , the resulting joint probability of their measurement can be expressed in the following factored form:

$$P(a, b|A, B, x) = P(a|A, x)P(b|B, x) \quad (1)$$

in which x plays the role of a so called “hidden variable”, endowed with its own probability density function $r(x)$, and acting an influence on the outcome in a deterministic or probabilistic way. This factorisation is dubbed Bell non-locality.

When this assumption is done, one can obtain different forms of Bell inequalities, as the Clauser–Horne [31] (or CHSH [32]) one:

$$CH = P(a, b) - P(a, b') + P(a', b) + P(a', b') - P(a') - P(b) \leq 0 \quad (2)$$

that are verified for every LHV, but violated in quantum mechanics. Therefore, this demonstrates that there exist quantum states whose outcome probabilities are inconsistent with locality. Thus, the non-locality property of entangled states leading to a violation of Bell inequality does not satisfy the former rule (1).

Very interestingly, after being a source of theoretical debate in the foundations of quantum mechanics and epistemology, more recently Bell non-locality has then emerged as a fundamental resource for quantum technologies [33], stimulating further studies on the topic and a consequent search for its deep understanding. Following this direction, a further form of quantum non-locality was then provided by Mermin [34], extending GHZ [35,36] analysis. In this case one directly shows that there exist quantum states whose outcome possibilities are inconsistent with locality. In little more detail, by considering the GHZ state

$$\cos(\alpha)|000\rangle + \sin(\alpha)|111\rangle \quad (3)$$

he demonstrated that no LHV can reproduce the quantum mechanics (QM) result for measurements $X_1 \otimes X_2 \otimes X_3$, $Y_1 \otimes Y_2 \otimes X_3$, $Y_1 \otimes X_2 \otimes Y_3$ and $X_1 \otimes Y_2 \otimes Y_3$. Interestingly, Mermin non-locality has also been recently included in categorical formulation of quantum mechanics [37]. Always in categorical formulation of quantum mechanics, it has been also demonstrated [21] that strong contextuality (exemplified by Mermin/GHZ non-locality) implies possibilistic non-locality (exemplified by Hardy non-locality [38]) that implies probabilistic non-locality (exemplified by Bell's inequalities).

It is interesting at this point to put in evidence that, aside important contributions on generalizations to non-deterministic hidden variable (HV) theories, Stapp coined also the inspired and elegant definition [39]: "the non-locality property of quantum mechanics is the logical need for information about a choice of experiment freely made in one region to be present in a second region that is space-like separated from the first".

The hidden variable x is usually considered as a "property" that the two measured physical systems possess because of their generation from a certain common source (for example for an entangled photon pairs emerging from a spontaneous parametric down-conversion crystal). If the two measurements are space-like separated events their outcomes exhibit a local dependence on the experimental settings and on x itself. Given these conditions, following Ref. [30] it is possible to demonstrate that these models fail when an experiment is designed to implement two distinct and independent sources generating entangled pairs, therefore entailing two independent sets x_1, x_2 of HVs, and the visibility overcomes the threshold of 50% in entanglement swapping [40] (it is worth to notice that in the case of experiments verifying Bell inequalities the required visibility is 71%).

More in general, x can be explained as the state describing the two systems [22], or even the whole universe. In this case, referring to Equation (1) the requirements on A, B measurements imply that the dependence does not rely on x only, i.e., x can be a non-local beable with the only condition that the local inputs A, B together with the global state x determine the outcome probabilities (note that Ref. [22] uses the word "propensities" instead of probabilities for emphasizing that probability cannot be considered in Kolmogorov sense, since in this case it can always be interpreted as epistemic). Moreover, in Ref. [22] it is argued that whether a deterministic model is considered, or epistemic probabilities are taken into account, problems arise related to independence of outcomes from reference frame.

Bell non-locality (1) makes reference to hidden variables x ("ontological" version), and as stated in ref. in [41], operational definitions, i.e., signal locality (absence of signalling), $P(a|A, B) = P(a|A)$ for a certain preparation, and predicability (possibility to predict the outcomes of all possible measurements that can be performed on a system), $P(a, b|A; B) \in 0, 1$ for a certain preparation, were shown to be sufficient for demonstrating Bell inequalities.

Finally, it was demonstrated that it exists a complementarity for the resources needed for violating Bell inequalities. This can be written as $S + 2I > C$ both for ontic [42] and operational cases [43,44], where I and S are randomness and signalling resources respectively (note that these parameters were

defined in [42] as $S \equiv \max\{S^{2 \rightarrow 1}, S^{1 \rightarrow 2}\}$, $S^{1 \rightarrow 2} = \frac{\sup}{\{A, A', B, b, x\}} |P_{AB}^{(2)}(b|x) - P_{A', B}(b|x)|$ and $S^{2 \rightarrow 1}$ mutatis mutandis, $I \equiv \frac{\max}{\{j=1,2\}} [\frac{\sup}{\{A, B, x\}} \frac{\min}{z} (P_{AB}^{(j)}(z|x), 1 - P_{AB}^{(j)}(z|x))]$, C being the average communication in bits: thus, non-local correlations ($C > 0$) in presence of no signalling ($S = 0$) requires unpredictability ($I > 0$).

2. Various Questions on Quantum Non-Localities

Quantum non-locality and entanglement exhibit a strict and deep interrelation, being at the same time two well distinct concepts, and the understanding of the differences in a quantitative terms deserves attention in order to clarify the balance between these two inequivalent resources. As a starting point, an interesting aspect to be considered in this chapter concerns whether QM violates relativistic locality, and on the consequence if a superluminal influence between entangled entities can be a suitable explanation for quantum non-locality. It is important to notice that a preferred reference frame is not in contrast to special relativity and in particular that a universal preferred reference frame has been already observed, being the cosmic microwave background. Ref. [45] posed a lower limit of more than ten thousand time the speed of light in the vacuum. This limit has been recently enhanced [46] to $\beta > 5 \cdot 10^6$. In addition, the analysis reported in [47] shows that this hidden influence, although propagating to a subluminal speed in a preferred reference frame, does it allow always for superluminal communications: this peculiarity refers to four-partite systems, but it is not valid for bipartite states. On the other side, this is not true for all the other models [9] (including de Broglie-Bohm and GRW [48]) for which the propagation in space of the correlations is not continuous. Other interesting facets of the argument, from which it results evident that the exact relation between entanglement and nonlocality is still poorly understood, refers for example to the analytic demonstration [49] that quantum non-locality does not imply distillability of quantum correlations: this was deduced by considering a 3-qubit entangled state that is separable along any bipartition, but nevertheless violates a Bell inequality. Another case refers to the reasoning of [50], where it is demonstrated that despite a particular class of states can exhibit entanglement without violating Bell inequalities, furthermore it is possible to obtain more non-locality with less entanglement, not only at the qubit level [12] but also when there is no restriction on the size of the entanglement [51–53], a situation experimentally investigated in [54]. An argument related to states that exhibit on one side a maximal non-locality without possessing a maximum degree of entanglement has been investigated in [55]. This study put in evidence three possible scenarios plausible with this situation:

- (i) the case [12] in which states that are non-maximally entangled violate Bell inequalities when using single-particle detectors with non-ideal quantum efficiency below 82%;
- (ii) for non-maximally entangled states, the Kullback–Leibler distance with the closest local distribution [56] is larger;
- (iii) the simulation of entanglement with non-local resources is more favorable when maximally entangled states are considered rather than the non-maximally ones.

A peculiar property analyzed in [57] shows that the totality of non-fully-separable states with an arbitrary dimension in the Hilbert space and number of parties, violates a Bell inequality if mixed with another state that, if taken alone, could not act a violation of the same Bell inequality.

From another study it appeared evident that an emerging quantum non-locality [58,59] is always unveiled by mixing a σ_{AB} state not violating Bell inequalities with any entangled state ρ_{AB} , a peculiar trait still to be seized in detail for potential exploitation in quantum communication protocols. In the work reported in [60], an interesting comparison is analyzed between the resource theories of non-locality (local operations and shared randomness (LOSR) paradigm) and entanglement (local operations and classical communication (LOCC) paradigm), and it is shown how a class of non-local games can witness quantum entanglement, however weak, and reveal non-locality in any entangled quantum state.

This introduces another significant line of research, i.e., the quantification of non-locality, an important argument when this is considered as a resource for quantum technologies, but also for better understanding its fundamental meaning. A study related to the quantification of nonclassicality and focused on the global impact of local unitary evolutions [61] puts in evidence that only for those composite quantum systems exhibiting quantum correlations (evaluated in the context of quantum-versus-classical paradigm [62]) the global state can undergo a transformation operated by means of any nontrivial unitary evolution; this property holds also when states non violating the Bell inequality are considered. More recently it was demonstrated [63] that all entangled states allow implementing a quantum channel, which cannot be reproduced classically, in particular teleportation according to a specific “teleportation witness”. A relevant way to quantify non-locality [64,65] takes into account the minimal amount of information necessary to preserve quantum correlation in a communication exchange between two parties, which in the particular case of a 2-qubit maximally entangled system undergoing projective measurements this quantification is accounted to be exactly 1 bit (on the contrary the situation can be different when the system is non-maximal entangled, a matter still under debate). On a different perspective it is founded the argument detailed in [66], considering general hidden variable models that can be expressed in terms of both local and non-local parts, and demonstrating the existence of experimentally verifiable quantum correlations that are incompatible with any hidden variable model having a nontrivial local part. Following this reasoning, it is then conceivable that if the non-local HVs are not considered, measurement results rely exclusively on local HV parameters and experimental settings. In this sense, this is equivalent to saying that quantum mechanics cannot be mimicked by HV theories provided with a local component.

The following reported approach started by considering that in a typical Bell experiment, correlations are observed between the measurement results of Alice and Bob and averages are evaluated over measurements of many pairs of particles, but when non-locality appears from statistics, this does not directly means that all individual pairs behave non-locally. This inspired the reasoning [67] for which a set of the pairs behaves locally while another fraction in a non-local way. In other terms, it was argued that the relative quantum distribution probability $P_Q(a, b|A, B, \rho)$ for a given quantum state ρ can be expressed in terms of local (P_L) and non-local (P_{NL}) parts, where respectively the P_L satisfies Bell inequalities and the other P_{NL} does not depend on outcomes or experimental settings:

$$P_Q = p_L(\rho) \cdot P_L + [1 - p_L(\rho)] \cdot P_{NL}. \quad (4)$$

A tricky problem resides in determining the weight of the local part (that represents in some sense the measure of locality for P_Q) and in finding a suitable way to minimize it. A step in this direction was the demonstration that the local part is identically null for maximally entangled 2-qubit systems, returning a complete non-local behaviour, while an investigation analyzing non-maximally pure 2-qubit and 3-qubit states can be found in [68,69]. Instead, focusing on the non-local component of Equation (4) and making use of non-local resources uniquely, with no-signaling constraint, it is possible to simulate correlations for all pure 2-qubit entangled systems without communication [70], therefore exploiting no-signaling resources exclusively. A similar decomposition approach for contextuality has been validated in [71]. Taking advantage of the general way to express any q-qubit pure state in the form $|\Psi_\theta\rangle = \cos(\theta)|00\rangle + \sin(\theta)|11\rangle$, Ref. [72] shows that $P_L = \cos(\theta)$ represents the maximal value when non-maximally entangled states are considered.

The decomposition in Equation (4) has been then used for an analysis in terms of non-locality distillation [73]. In this case, when a number of copies of a non-local system is defined, two relevant parameters can be identified: (i) the non-local cost; and (ii) the distillable non-locality. This study put in evidence the existence of non-local boxes, showing what they define as bound non-locality, i.e., whose distillable non-locality is strictly smaller than their non-local cost, analyzing also whether the non-locality contained in such boxes can be activated.

In analogy with this, in [74] Bell inequalities are derived exploiting the known link between the Kochen–Specker and Bell theorems, and discovered other measurement bounds allowing a higher degree of non-local correlations than those reported for cases reported in.

Several further studies addressed the measurement of non-locality, for instance the study of interconversions among several copies of non-local resources was afforded in [75–77], the distillation of several copies of a non-local behaviour into a more non-local one in [73,78,79], the operations under which non-locality cannot increase used to find a proper definition of genuinely multipartite non-locality [80].

The study about quantification of non-locality naturally leads to the question of the amount of non-locality in quantum mechanics. In this sense, a different approach, that overturns in some sense the perspective on how non-locality can be conceived, is built on reversing the logical order of quantum mechanics [81], by making non-locality an axiom and indeterminism a theorem (QM relies on the opposite). In other words, it is argued that QM is not the most non-local theory (concerning with non-locality correlations) consistent with relativistic causality. This perspective gave the idea to account for non-local “superquantum” correlations (known also as PR-box, by Popescu and Rohrlic), that interrelate, without superluminal signalling, the outputs of two parties with their inputs A,B in this way:

$$\sum_b P(a, b|A, B) = P(a|A) \text{ and } \sum_a P(a, b|A, B) = P(b|B). \tag{5}$$

In Ref. [82] the properties of such kind of set of probabilities are detailed in terms of communication complexity.

For exemplifying PR-box one can consider the situation when one has the following relation between outputs and inputs:

$$a \oplus b = A \cdot B \tag{6}$$

where \oplus stands for addition modulus 2. Furthermore, Alice’s and Bob’s marginal distributions are completely random and relativity causality is satisfied (namely Alice and Bob cannot communicate directly by exploiting PR-boxes.)

It is worth to mention that these non-local superquantum correlations generate a stronger violation ($S = 4$) in CHSH inequality with respect to any other quantum correlation, remembering that Tsirelson’s bound [54] gives $S = 2\sqrt{2}$ (quantum mechanics) and $S = 2$ in classical theory, while eventually a stronger violation [54,83] can be achieved with a larger number of measurements (chained inequalities) [84].

Substantially, quantum correlations do not violate locality in a stronger way than as allowed by causality (at least for two dimensional systems, for higher dimensions the situation changes [85], using chained inequalities mentioned above). This raises the question if there is some physical principle limiting the correlations to QM ones. A partially negative answer was provided in [86], where it was shown that quantum theory is, at the level of correlations among systems, not as special as expected identifying a set of non-signalling correlations (defined in terms of efficiently solvable semi-definite programmes) strictly larger than the quantum correlations one, but satisfies all-but-one (information causality, not yet proved) of the device-independent principles proposed for quantum correlations (non-trivial communication complexity, no advantage for non-local computation, macroscopic locality, and local orthogonality). In a first approach, ref. [87], the problem was faced by a communication complexity point of view [82]. Taking into consideration the case of two parties dealing with a Boolean function $F(a; b)$, and for which each of the parts has knowledge of only one input (a or b), the interesting problem refers to the minimization in communication resources in order that a part can compute and solve the function. In general, the case in which a bit only is necessary results a trivial matter. For an analysis of nontrivial cases, it can be mentioned that [87] reported the comparison between the bounds for success probability, estimated to be: (i) 90.8% in the case of PR-box operating in probabilistic way; and (ii) 85.4% in the case Tsirelson case (considering that the classical level is fixed at 75%). The consequent partition of communication complexity problems in hard- and easy-solving classes

can be understood in terms of partial characterization of non-local correlations by measurements on entangled systems. It is of interest the experimental verification [88–91] reporting that post-selected measurement results can approach the PR bound, while it is well known that in quantum mechanics the limit is fixed by Tsirelson bound.

A study [92] focused on general properties relevant to the whole non-signaling theories (predicting Bell inequalities violation) discerns among the following phenomena, that are a consequence of the no-signaling principle and non-locality: intrinsic randomness, monogamy of correlations (in the sense that these correlations cannot be shared among a indefinite number of parties (e.g., when three parties Alice, Bob and Charles share an entangled state and a Bell inequality is maximally violated by Alice and Bob, than Alice and Charles result to be totally uncorrelated), and for this reason are different with respect to classical analogue), impossibility of perfect cloning, uncertainty due to the incompatibility of two observables, privacy of correlations (if two honest parties know to share correlations with some degree of monogamy, they can estimate and possibly bound their correlations with a third untrusted party), and bounds in the shareability of some states (shareability represents a natural property in the analysis of the monogamy of correlations: a bipartite probability distribution $P(a, b|A, B)$ is considered to be m shareable with respect to Bob, if there exists an $(m + 1)$ -partite distribution $P(a, b_1, \dots, b_m|A, B_1, \dots, B_m)$ that is symmetric with respect to $(b_1, B_1) \dots (b_m, B_m)$, with marginals $P(a, b_i|A, B_i)$ equal to the original distribution $P(a, b|A, B)$). Moreover, it was discovered that the properties of non-local, non-arbitrarily shareable, and positive secrecy contents result to be equivalent, and this turns out to be valid for any distribution.

In general, Bell inequality violation implies that: (i) the considered quantum state must be entangled, (ii) the sets of local quantum measurements performed by one or the other party must be incompatible. A logical question to be asked is whether this link is reversed; Werner demonstrated that entanglement does not imply violation of Bell inequalities.

One of the most recent and interesting lines of investigation related with quantum non-locality is the connection with the absence of uncertainty free joint measurements of observables eventually used in Bell inequalities tests. In [93] it was shown that, for dichotomic POVM, every pair of incompatible quantum observables enables the violation of a Bell inequality and therefore it must remain incompatible within any other no-signaling theory; moreover, from [94,95], further elaborated in [96–99], evidence emerged on the fact that a set of measurements is not jointly measurable if and only if it can be used for demonstrating EPR steering (note that it occupies an intermediate position between Bell non-locality and entanglement); on the contrary, the work [100] showed that the result of [93] does not hold in general, as the joint measurability problem cannot be reduced to a pair of POVMs only (result extended in [101]). In [102] it was established that the degree of non-locality of any theory is determined by two factors: the strength of the uncertainty principle and the strength of steering, i.e., the possibility of explaining the Tsirelsons bound by exploiting the uncertainty limit of conditional quantum states deriving from the measurement of one system of a bipartite entangled state. In particular, it was examined the game where Alice and Bob win when their answer satisfies relation (6) between outputs and inputs. Then they considered the concept of steerability, i.e., what states Alice can remotely prepare on Bob’s system: when they share a state ρ_{AB} , the reduced density matrix pertaining Bob’s system is

$$\rho_B = \sum_a p(a|s) \rho_{s,a}. \tag{7}$$

Schrödinger noticed that for all s there exist a measurement on Alice’s system that allows steering the state $\rho_{s,a}$ with probability $P(a|s)$ on Bob’s site. This steering procedure does not allow to violate no-signalling because for each of Alice’s settings, Bob’s state is the same when averaging over Alice’s measurements outcomes. The tight bound for the game success is:

$$P^{game} = \sum_s p(s) \sum_a P(a|s) \zeta_{s,a} \tag{8}$$

where $\zeta_{s,a}$ is a proper set of uncertainty relation in entropic form [102] (one cannot obtain a measurement outcome with certainty for all the measurements simultaneously whenever $\zeta_{s,a} < 1$).

On the other hand, in the study Ref. [103] (see also [104]) a link was proved between Tsirelson's bound and measurement incompatibility in two qubits Hilbert space (i.e., with the optimal degree of unsharpness). By analysing covariance matrix, it was demonstrated Ref. [105] the inequality:

$$|B| \leq 22 + \text{Min}[S(a), S(b)], \quad (9)$$

where B is the Bell–CHSH parameter and $S(a), S(b)$ are Tsallis entropies concerning local measurements of Alice and Bob. The use of weak values [106–108] was suggested for experimentally testing this inequality. Ref. [109] characterised quantum correlations in terms of generalized uncertainty relations as well as specific notion of locality (local uncertainty relations cannot be affected by space-like separated events). Finally, a bound on non-local correlations (stronger than Tsirelson's one) in terms of local measurements of two independent joint measurements on two quantum correlated systems was demonstrated in Ref. [103], in particular it was shown how the requirement of positivity for experimental probabilities measured in a combination of two local joint measurements realized on two separate quantum systems leads to a bound on non-local correlation between these. The fact that steering occurs for mixed entangled states that are Bell-local has been experimentally demonstrated [110] by observing that a violation occurs when specific inequalities are considered (see also [111]), which extends previous works [112–115] relying on inferred variances of complementary observables [116,117] (also when transition-edge detectors, that are intrinsically single-particle number resolving, are experimentally implemented to avoid detection loophole [118]). However, the inequalities considered in [116,117] are not violated by some particular entangled states: Ref. [119] describes and experimentally tests a new criterion founded on entropy functions, violated by more states.

In [120] it was investigated whether the phenomenon of EPR steering can also be generalized beyond quantum theory demonstrating that, whilst post-quantum steering does not exist in the bipartite case, it is present in the case of three observers and it is fundamentally different from postquantum non-locality.

An interesting discussion on the connections between non-locality and complementarity can be found in [121], in which the key point focuses on the possibility to attribute different operational meanings to the concept of joint measurability of two observables (A,B): (i) A and B values can be obtained by the measurement of a third observable C (in the sense that the probability distribution of A and B outcomes are included as marginals in the distribution of C results); (ii) when the sequence ABA is measured, the two measurements of the same observable A return identical outcomes; (iii) if the measurement sequence AB is considered, the same probability distribution over B outcomes is obtained as a direct B measurement. In the context of quantum mechanics there is a total equivalence among all these statements, nevertheless Ref. [121] describes that for any no-signalling theory not showing complementarity (as specified at item (iii) it is allowed a local realistic description, but this is not valid with respect to properties i and ii (the demonstration took advantage of non-local box correlations models).

In addition, it is worth to mention the research [75] that analyzes the comparison among the structures of entanglement correlations with respect to the non-local ones, putting in evidence significant differences. The concept of "unit of non-locality" is speculated as a possible explanation for a PR-box, and proven that this is not applicable to describe correlations when cluster states are considered. In [122–125] an investigation on other aspects of PR-box is extended. Moreover, the paper [126] analyzes the possibility to build a covariant deterministic non-local HV extension of quantum theory, and demonstrates how it is possible to reduce any covariant non-local variable to Bell local variable whose existence is known to be incompatible with well-tested quantum predictions.

Another fundamental problem considered in Ref. [127] is related to whether the non-locality of a quantum state can be superactivated, in other words if a state $\rho \otimes \rho$ can be considered non-local if

ρ is local. This aspect, that refers to a sort of additivity property of non-locality, has been analyzed by considering unbounded [128] Bell inequalities, returning the conclusion that if a number of local entangled states are combined by direct product, it is possible to get a non-local global state, falsifying in this way the forementioned additivity. In conclusion, as an extension of this study it was considered the possibility for any kind of local entangled state to generate non-locality superactivation, and demonstrated [129] that all quantum states useful for teleportation represent non-local resources.

3. Non-Locality in Higher Dimensional Spaces

The list of studies connected to quantum non-locality of the previous sections, albeit incomplete and a little schematic, demonstrates as this is not only a fundamental point for understanding the very foundations of quantum mechanics, but it is also becoming an important resource for emerging quantum technologies. This prompts the need of a clear understanding of quantum non-locality and, in particular, of its compatibility with special relativity. Even if one can rigorously demonstrate that it can not lead to any superluminal transmission (signalling) [130], peaceful coexistence between special relativity and QM would require more, i.e., it would be necessary understanding a coherent description among different observers [131]. For instance, how to reconcile two observers that observe a different temporal order in the collapse of two entangled particles? The answers to this question span from being an unsolved problem [132–139], requiring a preferred foliation to relativistic space–time, to being only apparent, since accounts of entangled systems undergoing collapse yielded by different reference frames can be considered as no more than differing accounts of the same process and events [140]; there is a form of holism associated to QM description of composite systems, the factorizable state after collapse on a certain hypersurface is merely one way of slicing together local parts, entangled states are superposition of such splittings.

Be that as it may, *rebus sic stantibus*, an undoubted difficulty in understanding the collapse in composite systems persists. Our suggestion is that one should explore the possibility that this problem can be solved in higher dimensional spaces. Let us consider a $n > 4$ space-time with $n - 4$ more spatial dimensions and let us suppose that, while the usual fields only “live” in $3 + 1$ dimensions, the collapse is mediated by “some field” propagating also in the other dimensions. Since every n -dimensional space can be “contracted” around a single point of a higher dimensional space (for example a plane can be wrapped as tied as one wishes around a single point in three dimensional space), two far points in the “ordinary” space can be as close as wanted considering extra-dimensions. Incidentally, this “contraction” is an isometry and leaves invariant Gaussian curvature.

Thus, if this higher dimensions are extremely small, every event of the $3+1$ dimensional space could be connected with any another event in an “extremely short” effective distance through the compactified dimensions. Therefore, the collapse of one of two remote entangled particles could cause the collapse of the other with the propagation of a subluminal signal through the compactified dimensions. Quantum non-locality would reduce to the fact that only wave function collapse would be affected by these extra dimensions.

This kind of phenomenon could either be included in a more general theory going beyond quantum mechanics (for example, some Planck scale theory already predicts compactified dimensions) or find a use in building relativistic collapse models, for example a simple generalization of Tumulka’s one [141] should be able to incorporate it.

For instance, one could think to some general non-unitary evolution operator

$$U(t) = \text{Exp}\left\{-i \int d^4x d^n y O_I(x, y, t)\right\} \quad (10)$$

where $O_I(x, y, t) = H_I(x, t) + C_I(x, y, t)$, $H_I(x, t)$ being the usual Hermitian–Hamiltonian density of quantum fields leaving in a $3+1$ dimensional space (x), while $C_I(x, y, t)$ is not Hermitian and eventually induces the collapse. When one traces over the degrees of freedom corresponding to the not Hermitian component, $C_I(x, y, t)$, the usual unitary evolution is eventually restored, as in ‘t Hooft finite degrees of

freedom model [142]. Lastly, it would be worth investigating if this extra component could contribute to solve other issues of modern physics, as dark matter and dark energy.

Finally, one can notice that this idea could present some vague analogy with the one proposed in [143], where a connection between quantum non-locality and Einstein–Podolsky bridges was conjectured; nonetheless the nature of the phenomenon is rather dissimilar and also potential consequences could be rather different: for example in this second case, as discussed in [143], the two regions remain causally disconnected due to non traversability of Lorentzian wormholes (Of course, following [143], we do not consider here eventual traversable wormholes [144,145]), while in principle the connection through a compactified dimension does not prevent causal connection (if no other effect intervenes).

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