Raman-Ramsey resonances in atomic vapor cells: Rabi pulling and optical-density effects

INRIM, Istituto Nazionale di Ricerca Metrologica,
Strada delle Cacce 91,
10135, Torino, Italy

(Dated: April 3, 2019)

Raman-Ramsey interference has proved as a very effective technique to implement compact and high performing vapor cell frequency standards. In this paper, we theoretically characterize Raman-Ramsey resonances in an optically thick atomic vapor. Specifically, some parameters of interest for frequency standards applications, like contrast and linewidth of the central Raman-Ramsey fringe, are evaluated at different temperatures for ^{133}Cs and ^{87}Rb vapor cells with buffer gas. Density narrowing and broadening effects are described and explained in terms of a three-level theory where laser fields propagation through the atomic medium is taken into account.

Also, we investigate light-shift both in low and high atomic density regimes. Light-shift, which potentially degrades the medium-long term stability of Raman-Ramsey clocks, is composed of two contributions. The first is a pulling effect exerted by the wide Rabi profile enclosing the interference pattern on the central Raman-Ramsey fringe. The second light-shift term is strictly related to the detection time.

Calculations derived from our model well describe already existing experimental results and new behaviors are predicted.

PACS numbers:

I. INTRODUCTION

τ_1

T

τ_2

$\nu_{1/2}$

$\nu_{1/2}$ / T

/ T

-15

/ T

133 87

II. THIN ATOMIC MEDIUM

A. CPT resonance in continuous operation

z 1

γ_1 γ_2

$*$

$_{bg}$

$_D$ $_{bg}$ $_D$

$* \quad \frac{1}{2} \quad _{bg} \quad _{bg}^2 \quad _D^2$

$\omega_{R1,2}$ * ω_R μ

0

δ_0 $_0/{}^*$

$$\omega_{LS} \quad \frac{\omega_{R2}^2 \quad \omega_{R1}^2}{*} \frac{\delta_0}{\delta_0^2} \quad \frac{\omega_{R1}^2 \quad \omega_{R2}^2}{\omega_{21}}$$

1

ω_{R1} ω_{R2} ω_R

$_0,$ * ω_{21} δ_0

γ_1 $_p$ $_p\delta_0\delta_{12}^i$

δ_{12} γ_2 $_p$ i $_\mu$ $\omega_R^2/$ ω_{21} δ_{12} $_p$ $i{}_p\delta_0$

ρ_{33} $\frac{\omega_R}{^*}$ ^{2} δ_{12}^r

$$_p \quad \frac{\omega_R^2}{*}$$

P_{fl} ρ_{33}

$$P_{fl} \propto \rho_{33} \quad \frac{\omega_R}{\square^*}^2 \quad \frac{p}{\square'} \frac{\square'^2 \quad {}_p^2\delta_0^2}{\square'^2 \quad {}_p^2\delta_0^2 \quad \mu \quad \omega_R^2 / \omega_{21}{}^2}$$

$\square'$ γ_2 p

δ_0

ω_{R1} ω_{R2} δ_0

B. CPT resonances in pulsed operation: Rabi pulling and detection light-shift

τ_1

T

τ_2

$$t \begin{pmatrix} \delta_{12}^r\ t \\ \delta_{12}^i\ t \end{pmatrix} t$$

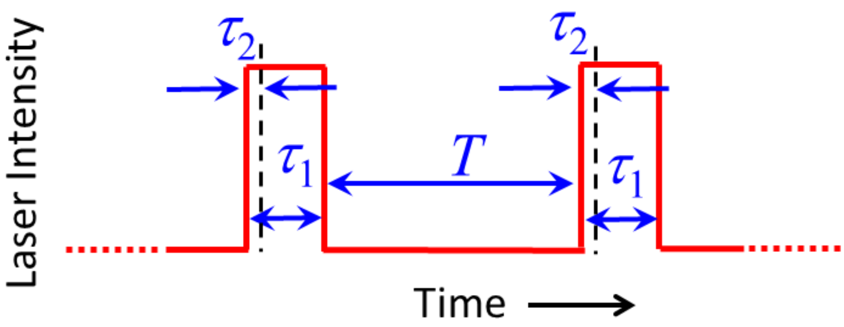


FIG. 2: (Color online) Time sequence of the Raman-Ramsey pulses: $_1$ is the duration of the Λ pulse, T the free evolution time and $_2$ the detection time.

δ_{12}

ρ_{11} ρ_{22}

τ_1

$$\tau_1 \quad \begin{pmatrix} \frac{\square_p \square'}{\square'^2 \quad {}'^2_\mu} & \frac{e^{-\Gamma'\tau_1}\square_p \quad ' \quad {}'_\mu\tau_1 \quad {}'_\mu \quad {}'_\mu\tau_1}{\square'^2 \quad {}'^2_\mu} \\ \frac{\square_p \quad {}'_\mu}{\square'^2 \quad {}'^2_\mu} & \frac{e^{-\Gamma'\tau_1}\square_p \quad {}'_\mu \quad {}'_\mu\tau_1 \quad \square' \quad {}'_\mu\tau_1}{\square'^2 \quad {}'^2_\mu} \\ \frac{{}^2_p\delta_0 \quad {}'_\mu}{\square' \quad {}'^2 \quad {}'^2_\mu} & \frac{e^{-\Gamma'\tau_1}\square^2_p\delta_0}{\square' \quad {}'_\mu} \quad \frac{{}^2_p\delta_0 e^{-\Gamma'\tau_1}}{{}'_\mu \quad {}'^2 \quad {}'^2_\mu} \quad ' \quad {}'_\mu\tau_1 \quad {}'_\mu \quad {}'_\mu\tau_1 \end{pmatrix}$$

${}'_\mu$ μ $\frac{\omega_R^2}{2\omega_{21}}$

$e^{-\Gamma'\tau_1}$ $'\tau_1$ $\square$

$$\tau_1 \quad /\square' \quad \begin{pmatrix} \frac{\square_p \square'}{\square'^2 \quad {}'^2_\mu} \\ \frac{\square_p \quad {}'_\mu}{\square'^2 \quad {}'^2_\mu} \\ \frac{{}^2_p\delta_0 \quad {}'_\mu}{\square' \quad {}'^2 \quad {}'^2_\mu} \end{pmatrix}$$

T

$M_D\ T$

$$M_D\ T \quad \begin{pmatrix} e^{-\gamma_2 T} & {}_\mu T & e^{-\gamma_2 T} & {}_\mu T \\ e^{-\gamma_2 T} & {}_\mu T & e^{-\gamma_2 T} & {}_\mu T \end{pmatrix} e^{-\gamma_1 T}$$

$T \quad \tau_1 \quad M_D\ T \quad \tau_1$

μ

$\omega_R^2 / \ \omega_{21}$

$$T \quad \tau_1 \quad \begin{pmatrix} \frac{\ _p}{\ '^2 \quad '^2_\mu} \ ' \quad {}_\mu T \quad '_\mu \quad {}_\mu T \ e^{-\gamma_2 T} \\ \frac{\ _p}{\ '^2 \quad '^2_\mu} \ ' \quad {}_\mu T \quad '_\mu \quad {}_\mu T \ e^{-\gamma_2 T} \\ \frac{\ ^2_p}{\ '} \frac{\delta_0 \quad '_\mu}{\ '^2 \quad '^2_\mu} e^{-\gamma_1 T} \end{pmatrix}$$

τ_2

$\delta_{12}^r\ T \quad \tau_1 \quad \delta_{12}^i\ T \quad \tau_1 \quad T \quad \tau_1$

δ_0 6

δ_0

δ_0

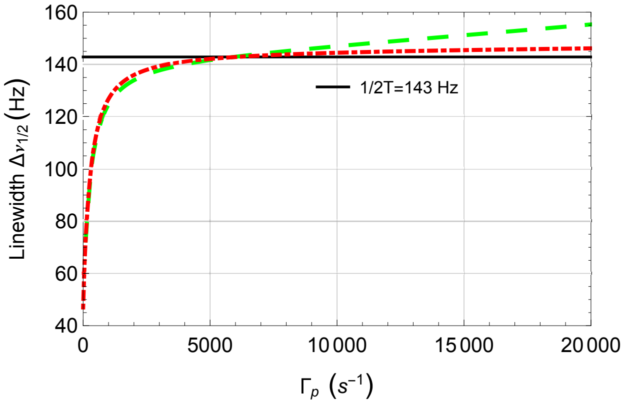


FIG. 3: (Color online) Calculated linewidth of the central Ramsey fringe versus Γ_p; $_2 = 300$ s^{-1}; T $= 3{:}5$ ms; dashdot line: $_2 = 10$ s; dash line: $_2 = 100$ s.

δ^r_{12} τ_1 T τ_2 $\frac{\square_p \square'}{\square'^2 \quad '^2_\mu}$ $\frac{\square_p e^{-\Gamma'\tau_2}}{\square'^2 \quad '^2_\mu}$ h $'$ $_\mu T$ $'_\mu$ $_\mu T$ $e^{-\gamma_2 T}$ $\square'$ i $'_\mu \tau_2$

$\frac{\square_p e^{-\Gamma'\tau_2}}{\square'^2 \quad '^2_\mu}$ h $'$ $_\mu T$ $'_\mu$ $_\mu T$ $e^{-\gamma_2 T}$ $'_\mu$ i $'_\mu \tau_2$

$/$ T

$'_\mu$ $\square'$ τ_2 T

$$\delta_{12}^{r}\ \tau_1 \quad T \quad \tau_2 \qquad \frac{□_p}{□'} e^{-\gamma_2 T - \Gamma' \tau_2} \qquad {}_{\mu}T \quad \frac{□^* □_p}{\omega_{21}} \tau_2 \quad \frac{□_p}{□'} \ e^{-\Gamma' \tau_2}$$

ω_{DT} τ_2

$$\frac{\omega_{DT}}{\omega_{21}} \qquad \frac{□^* □_p}{\omega_{21}^2} \frac{\tau_2}{T}$$

τ_2

μ $^{-12}/$

$'_{\mu}$ $_{\mu}$

τ_2 !

$$\delta_{12}^{r}\ \tau_1 \quad T \quad \tau_2 \qquad \frac{□_p}{□'^2 \qquad '^2_{\mu}} \ \text{h} \ ' \quad {}_{\mu}T \quad '_{\mu} \quad {}_{\mu}T \ \text{i} \ e^{-\gamma_2 T}$$

$_{\mu}$

$_{\mu}$

$$\frac{\omega_{RP}}{\omega_{21}} \qquad \frac{□^*}{\omega_{21}^2 T} \frac{□_p}{\gamma_2 \qquad _p}$$

$$\frac{\partial \quad \omega_{RP}/\omega_{21}}{\partial □_p / □_p} \qquad \frac{□^*}{\omega_{21}^2 T} \frac{\gamma_2 □_p}{\gamma_2 \qquad _p \ ^2}$$

$^{-13}/$

$'$ $/\tau_1$

$_p$ □

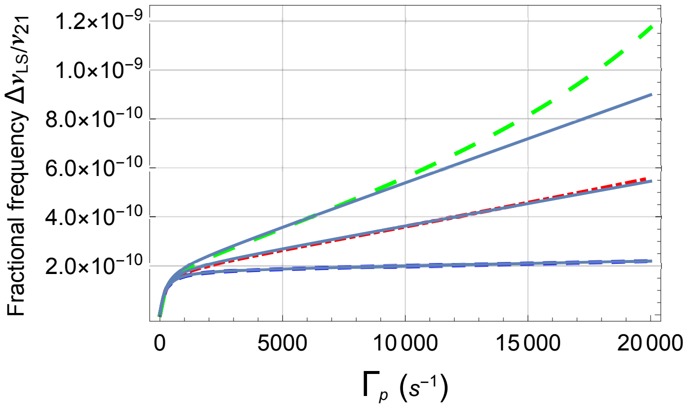


FIG. 4: (Color online) Calculated light-shift of the central Ramsey fringe versus Γ_p; $_2 = 300$ s^{-1}; T $= 3{:}5$ ms; $\Gamma^* = 2$ 684 MHz; small dash line: $_2 = 5$ s; dashdot line: $_2 = 50$ s; large dash line: $_2 = 100$ s. For each curve, the approximated expression of the shift given by Eq. (19) has been superimposed to the numerically calculated behavior.

$$\frac{\omega_{LS}}{\omega_{21}} \quad \frac{\omega_{DT}}{\omega_{21}} \quad \frac{\omega_{RP}}{\omega_{21}} \quad \frac{\square^*\square_p}{\omega_{21}^2 T} \quad \tau_2 \quad \frac{}{\gamma_2} \quad _p$$

$_p$ $\square$

$_p$ $\square$

$_p$ $\square$ τ_2 μ

III. THICK ATOMIC MEDIUM

A. Theory

ω_{R1}

ω_{R2}

$$\gamma_1 \quad \frac{\omega_{R1}^2 \quad \omega_{R2}^2}{*} \qquad \frac{\omega_{R1}^2 \quad \omega_{R2}^2}{*}$$

$$\delta_{12} \quad \gamma_2 \quad \frac{\omega_{R1}^2 \quad \omega_{R2}^2}{*} \quad i \quad \mu \quad \frac{\omega_{R1}^2 \quad \omega_{R2}^2}{\omega_{21}} \quad \delta_{12} \quad \frac{\omega_{R1}\omega_{R2}}{*}$$

$$\frac{\partial \omega_{R1}}{\partial z} \quad \alpha \left[\frac{\omega_{R1}}{*} \qquad \frac{\omega_{R2}}{\square^*}\delta_{12}^r \right]$$

$$\frac{\partial \omega_{R2}}{\partial z} \quad \alpha \left[\frac{\omega_{R2}}{*} \qquad \frac{\omega_{R1}}{\square^*}\delta_{12}^r \right]$$

α

$$\alpha \quad \frac{\omega_L d_e^2}{\epsilon_0 \hbar c} n$$

ω_L d_e D_1

n ϵ_0 $\hbar$

c

δ_0

δ_0

[1]

ω_{R1} z ω_{R2} z

ω_{R1} z ω_{R2} z ω_R z z

α' $\frac{\alpha}{2I+1}$ I

α'

$_p$

$$\delta_{12} \quad \gamma_2 \quad _p \quad i \quad _\mu \quad \frac{^*_p}{\omega_{21}} \quad \delta_{12} \quad _p$$

$$\frac{\partial_p}{\partial z} \quad \frac{_p}{^*} \; \alpha \quad \alpha' \delta^r_{12}$$

^{133} I / ^{87} I /

δ_{12}

z $_p$

$'$ z

$'\tau_1$

z

τ_1

$$\delta^r_{12} \; z, \tau_1 \qquad \frac{_p '}{'^2 \quad '^2_\mu} \quad \frac{e^{-\Gamma' \tau_1} \, _p \quad ' \quad '_\mu \tau_1 \quad '_\mu \quad '_\mu \tau_1}{'^2 \quad '^2_\mu}$$

$$\delta^i_{12} \; z, \tau_1 \qquad \frac{_p \quad '_\mu}{'^2 \quad '^2_\mu} \quad \frac{e^{-\Gamma' \tau_1} \, _p \quad '_\mu \quad '_\mu \tau_1 \quad ' \quad '_\mu \tau_1}{'^2 \quad '^2_\mu}$$

$$\frac{\partial_p \; z}{\partial z} \qquad \alpha \frac{_p \; z}{^*} \qquad \frac{}{I} \delta^r_{12} \; z, \tau_1$$

$$_p \; z \qquad _{p0}$$

T

$\tau_1 \quad T \quad z$

$$\delta^r_{12} \; z, \tau_1 \quad T \qquad e^{-\gamma_2 T} \qquad _\mu T \; \delta^r_{12} \; z, \; \tau_1 \qquad _\mu T \; \delta^i_{12} \; z, \; \tau_1$$

$$\delta^i_{12} \; z, \tau_1 \quad T \qquad e^{-\gamma_2 T} \qquad _\mu T \; \delta^r_{12} \; z, \; \tau_1 \qquad _\mu T \; \delta^i_{12} \; z, \; \tau_1$$

τ_2

$$\delta^r_{12} \; z, \tau_1 \quad T \quad \tau_2 \qquad \frac{\square_p \square'}{\square'^2 \qquad {}'^2_\mu} \quad e^{-\Gamma' \tau_2} \; \delta^r_{12} \; z, \tau_1 \quad T \quad \frac{\square' \square_p}{\square'^2 \qquad {}'^2_\mu} \qquad {}'_\mu \tau_2$$

$$e^{-\Gamma' \tau_2} \; \delta^i_{12} \; z, \tau_1 \quad T \quad \frac{{}'_\mu \square_p}{\square'^2 \qquad {}'^2_\mu} \qquad {}'_\mu \tau_2$$

$$\delta^i_{12} \; z, \tau_1 \quad T \quad \tau_2 \qquad \frac{\square_p \quad {}'_\mu}{\square'^2 \qquad {}'^2_\mu} \quad e^{-\Gamma' \tau_2} \; \delta^i_{12} \; z, \tau_1 \quad T \quad \frac{{}'_\mu \square_p}{\square'^2 \qquad {}'^2_\mu} \qquad {}'_\mu \tau_2$$

$$e^{-\Gamma' \tau_2} \; \delta^r_{12} \; z, \tau_1 \quad T \quad \frac{\square' \square_p}{\square'^2 \qquad {}'^2_\mu} \qquad {}'_\mu \tau_2$$

$$\frac{\partial \square^{det}_p \; z}{\partial z} \qquad \alpha \frac{\square^{det}_p \; z}{\square^*} \qquad \frac{}{I} \delta^r_{12} \; z, \tau_1 \quad T \quad \tau_2$$

$\square^{det}_p$

$$^{det}_{p} \qquad \square \qquad z \quad L \;\; L$$

$$_p \; z \square \qquad \square^{det}_p \; z \qquad \square_{p0}$$

133 $\circ$ 87 $\circ$

$_{p0}$ $\square$ -1 133

$\mu \quad / \quad 2$ -1 $\mu \quad / \quad 2$ 87

1

133

2

$\square_{lg} \quad \pi$ 1 133

$\circ$ $_D \; \square \quad \pi$ $_D \quad \square$

2

$\square_{lg} \qquad \square_D$ 87

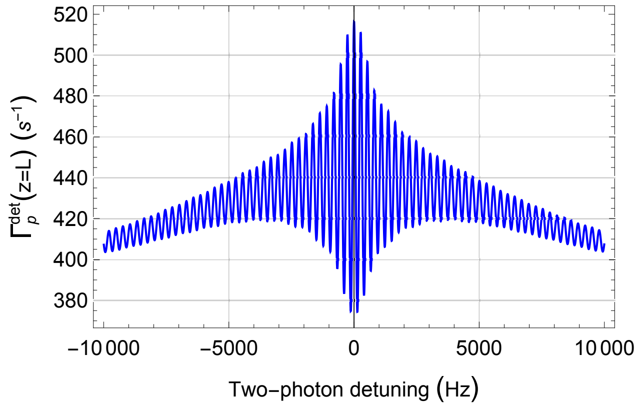


FIG. 5: (Color online) Calculated Ramsey fringes for a cell containing ^{133}Cs and 25 Torr of N_2; $\Gamma_{p0} = 8000$ s^{-1}, cell temperature 35 °C; $_1 = 1$ ms; T $= 3$:5 ms; $_2 = 50$ s. Cell length: L $= 2$ cm; radius: R$=$ 1 cm.

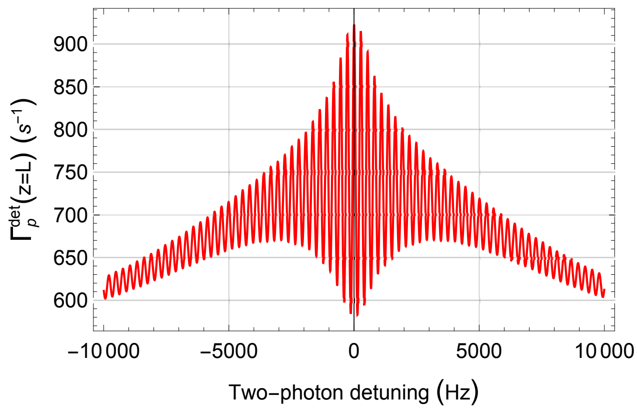


FIG. 6: (Color online) Calculated Ramsey fringes for a cell containing ^{87}Rb and 25 Torr of Ar+N_2 in the pressure ratio 1.6; $\Gamma_{p0} = 10000$ s^{-1}, cell temperature 50 °C; $_1 = 1$ ms; T $= 3$:5 ms; $_2 = 50$ s. Cell length: L $= 2$ cm; radius: R$=$ 1 cm.

B. Contrast and linewidth of the central Raman-Ramsey fringe

γ_2

^{133} ^{87}

T .

γ_2

$_{p0}$ □

τ_2

$/\ T$

τ_2

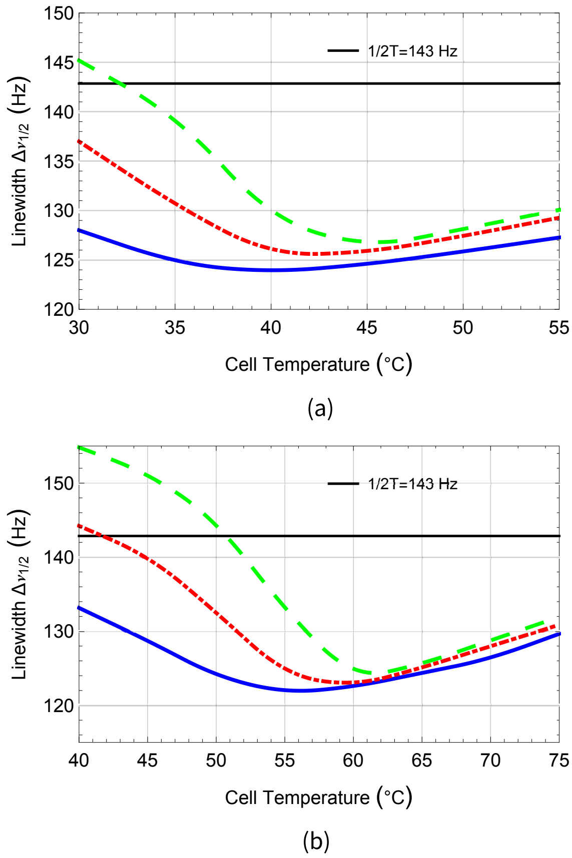


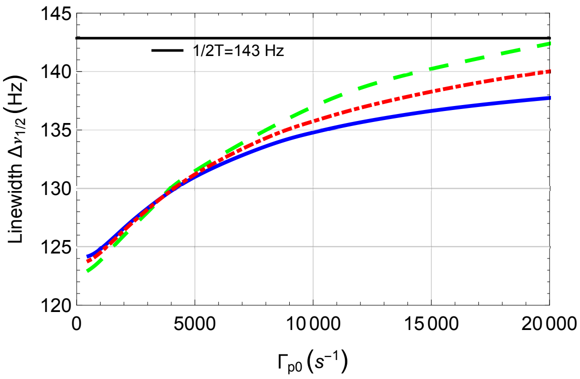
FIG. 7: (Color online) Linewidth of the central Ramsey fringe for the same Cs (a) and Rb (b) cells considered in Figs. 5 and 6. The straight line is the $1{=}2T$ value; $t_p = 1$ ms; $T = 3{:}5$ ms; $_2 = 50$ s. Continuous line: $\Gamma_{p0} = 3000$ s^{-1}; dashdot line: $\Gamma_{p0} = 10000$ s^{-1}; dash line: $\Gamma_{p0} = 30000$ s^{-1}.

$_{p0}$ ▯

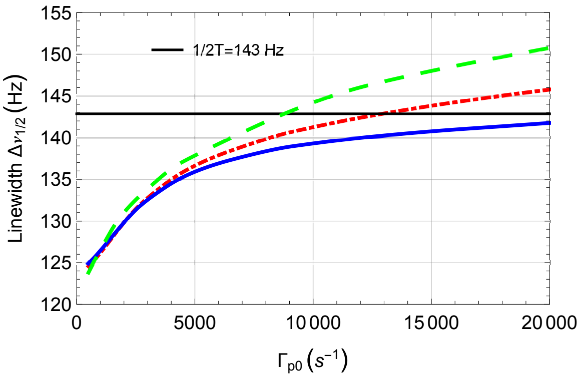
$/ \ T$

T $/ \ T$

t_1



(a)



(b)

FIG. 8: (Color online) Linewidth of the central Ramsey fringe as a function of the laser pumping rate for different values of the detection time $_2$. Cs cell with the same buffer gas content and pressure of Fig. 5 at 30 °C (a); Rb cell with the same buffer gas content and pressure of Fig. 6 at 40 °C (b). The straight line is the 1=2T value; $t_p = 1$ ms; T $= 3{:}5$ ms. Continuous line: $_2 = 5$ s; dashdot line: $_2 = 20$ s; dash line: $_2 = 50$ s.

$$\frac{1}{2(T+4t_1/\pi)}$$

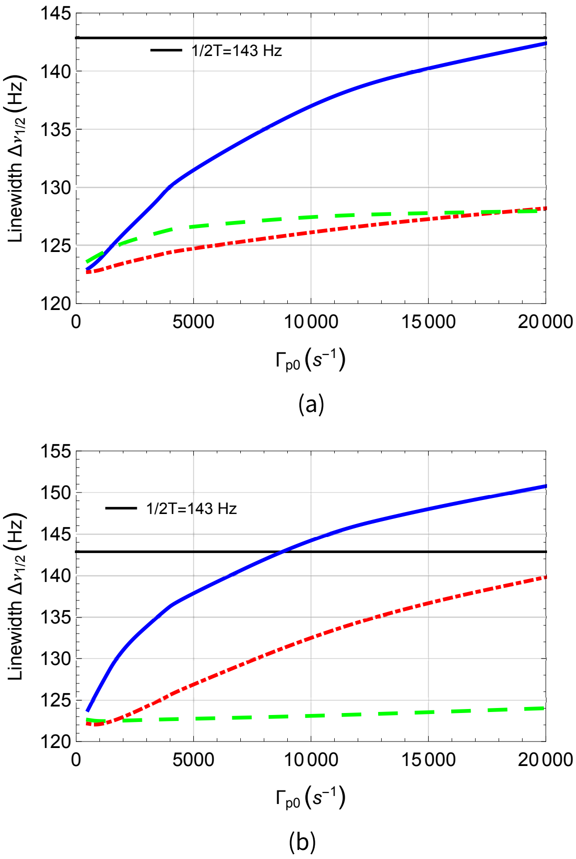


FIG. 9: (Color online) Linewidth of the central Ramsey fringe of Raman-Ramsey resonances versus the laser pumping rate. The straight line is the 1=2T value; $_1 = 1$ ms; T $= 3{:}5$ ms; $_2 = 50$ s. (a) Cs cell: Continuous line: 30 °C; dashdot line: 40 °C; dash line: 50 °C; (b) Rb cell: Continuous line: 40 °C; dashdot line: 50 °C; dash line: 60 °C.

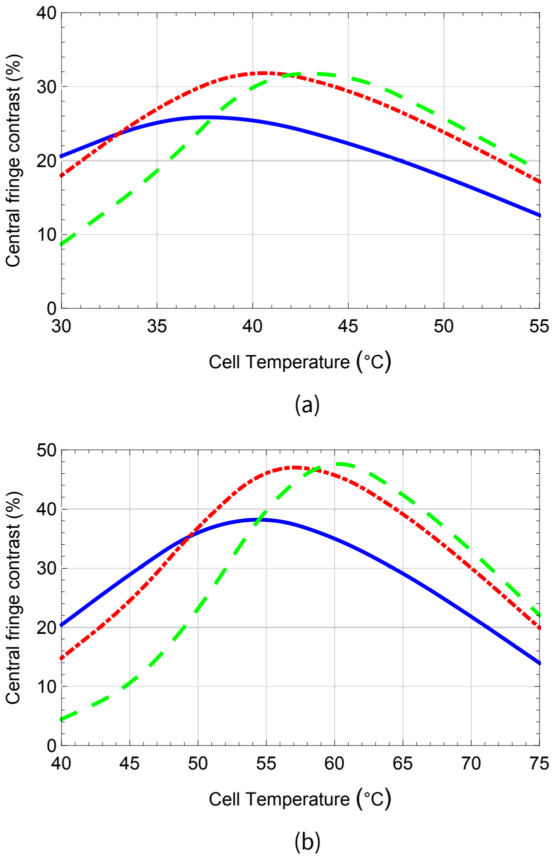


FIG. 10: (Color online) Contrast of the central fringe of Raman-Ramsey resonances for the same Cs (a) and Rb (b) cells considered in Figs. 5 and 6; $_1 = 1$ ms; T $= 3{:}5$ ms; $_2 = 50$ s. Continuous line: $\Gamma_{p0} = 3000$ s^{-1}; dashdot line: $\Gamma_{p0} = 10000$ s^{-1}; dash line: $\Gamma_{p0} = 30000$ s^{-1}.

C. Light-shift in an optically thick vapor

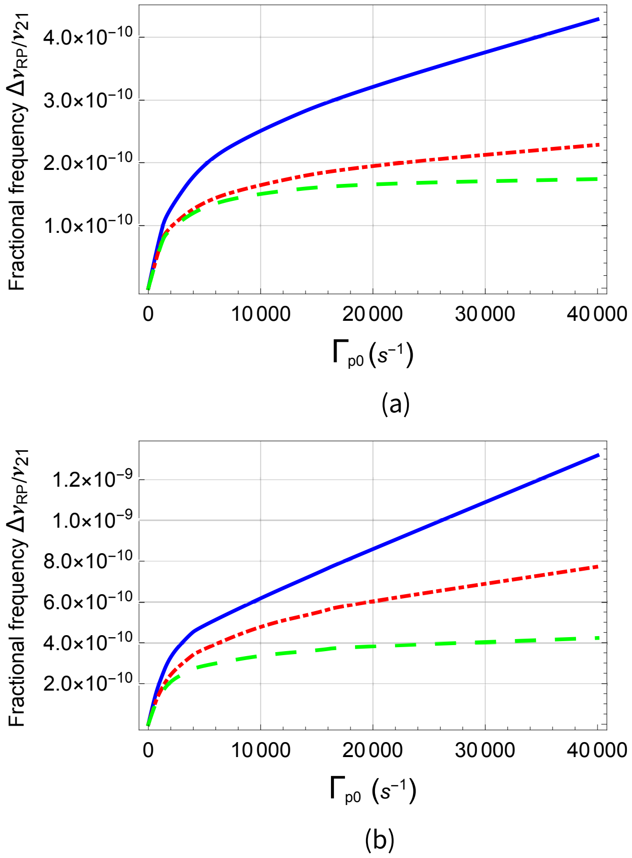


FIG. 11: (Color online) Fractional light-shift of the central fringe of Raman-Ramsey resonances versus the laser pumping rate; $_1 = 1$ ms; T $= 3{:}5$ ms; $_2 = 50$ s. (a) Cs cell: Continuous line: 30 °C; dashdot line: 40 °C; dash line: 50 °C; (b) Rb cell: Continuous line: 40 °C; dashdot line: 50 °C; dash line: 60 °C.

$_{p0}$

◦

$_{p0}$ $^{-1}$. $^{-12}/$ ◦ . $^{-14}/$

◦

. $^{-12}/$. $^{-13}/$

◦ ◦

IV. DISCUSSION AND CONCLUSIONS

μ ω_R^2/ω_{21} μ

τ_2

$/ \ T$

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