1-D and 2-D Loss-Measuring Methods: Optimized Setup Design, Advanced Testing, and Results

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1D and 2D loss measuring methods: optimized setup design, advanced testing, results.

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Accurate measurements of magnetic losses in laminations are a prerequisite for their theoretical assessment, as well as for satisfying calculations of energy dissipation in engineering systems. The standardized and universally applied measurement method, used as a reference for the definition of the material quality in the specification standards, is based on the Epstein test frame magnetizer. Its success relies on the reproducibility of the performed measurements. Its limitations come, on the one hand, from cumbersome sample preparation and, on the other hand, from a certain divergence of the measured loss figures from the true loss figures. Similar systematic differences between measured and true loss values are also observed with the standard Single Sheet Tester method. In both cases, measurements under bi-dimensional induction are or cannot be envisaged. The design of new measurement setups and magnetizers overcoming the drawbacks of the Epstein and Single Sheet Tester methods and possibly becoming recognized Standards in the future is welcome, but challenging. This paper is devoted to a comprehensive discussion of the state of the art in the alternating and two-dimensional measurements of energy losses in soft magnetic materials for electrical applications. We will summarize, in particular, measuring solutions proposed in the current literature and we will discuss in detail recent developments achieved in the authors’ labs regarding 1D measurements with compensated permeameters and 2D measurements at high inductions and high frequencies.

\textbf{Index Terms}—Magnetic losses, Magnetic measurements, Permeameter, Two-dimensional magnetization.

I. INTRODUCTION

Electrical machine designers are nowadays facing difficult challenges. For example, the rapid growth of demand in embedded applications, such as hybrid/electric cars\textsuperscript{[1]}, require the design of very compact actuators\textsuperscript{[2]}. In this context, materials are often used at the limit of their thermal viability, so realistic designs impose accurate loss calculations. Should these be based on magnetic loss models or should they rely on empirical formulations, there is no alternative to precise loss measurements\textsuperscript{[3]}. The standard measurement methods, based on the use of the Epstein frame\textsuperscript{[4],[5]} or the Single Sheet Testing (SST) magnetizer with flux closing yokes\textsuperscript{[6]}, offer good reproducibility, as verified by a number of international comparisons\textsuperscript{[7],[8]}. They are therefore assumed as reference methods for the definition of the material quality in the specification standards. However, both Epstein and SST methods generate systematic contributions to the measurement uncertainty\textsuperscript{[9]} and the obtained loss figures can be appreciably different from the “true” loss values, those provided by accurate local measurements of the effective magnetic field strength and the flux density\textsuperscript{[8],[9],[10]}. An example concerning non-oriented and grain-oriented Fe-Si steel sheets is shown in Fig. 1. It is to remark that the Epstein strip samples require tedious preparation, including relief of cutting stresses. In addition, Epstein and SST methods are hardly compatible with two-dimensional induction loci\textsuperscript{[11]}, a typical working regime in electrical machine cores. It turns out that the losses due to the two components of the flux density locus are often computed separately and summed up, as if they were independent alternating magnetization processes\textsuperscript{[12]}. This procedure is clearly inaccurate, especially at high flux density values, where the hysteresis loss, always increasing with alternating flux, tends to vanish under circular flux density on approaching the saturation. No measuring standards exist for rotating flux density, though many research efforts have been devoted to the development of 2D induction setups. Different solutions have actually been envisaged for the 2D magnetizer, like the horizontal\textsuperscript{[13],[14],[15]} and vertical\textsuperscript{[16],[17]} cross-shaped yokes and the three-phase yoke with either hexagonal\textsuperscript{[18]} or circular\textsuperscript{[19]} samples. In the latter case, flux density values up to 1.85 T could be reached, with maximum frequency around a few hundred Hz, under both alternating and rotational flux\textsuperscript{[20]}. These limiting values of flux density and frequency should however be overcome, in order to meet the conditions of modern electrical machine cores, often attaining the saturated state and working at kHz frequencies.

In this paper, present-day developments in alternating and two-dimensional measurements of magnetic losses will be discussed, with emphasis on recent progress made in the authors labs (SATIE, INRIM, Politecnico di Torino), overcoming the previous limits. Two main points are addressed:

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A. Steel sheets.

II. The characterization of soft magnetic materials under
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- Alternating flux. The Single Sheet Tester will be specifically
considered, because it allows for a flexible approach to
material testing. SST measurements may show slightly
inferior reproducibility with respect to the Epstein
measurements [8], since the quality of the yoke, its losses,
and the reluctance of the sheet-yoke contact region can play
a role. A novel approach to the SST method [21], where
the potential drop occurring in different parts of the magnetic
circuit is automatically compensated by means of a
feedback system, will be here highlighted.

- Two-dimensional flux. The ambitious objective of reaching
very high flux density values and high frequencies under
controlled 2D fluxes and measuring the associated loss
figures has been addressed in recent times (see for example
[22] or [23]). To this end, two setups have been realized,
those employ circular samples and three-phase magnetizers.
One setup has been optimized to reach the kilohertz range,
the other for approaching the magnetic saturation. Both
fieldmetric and thermometric loss measurement methods
have been applied.
Significant results for rotating and alternating flux densities
will be discussed in the following.

II. LOSS MEASUREMENTS UNDER ALTERNATING MAGNETIC
FIELD STRENGTH

A. Standard methods

The characterization of soft magnetic materials under
alternating magnetic field strengths is well standardized, but
room exists for improved and wider-range measurements. We
summarize here the main features of the present-day measuring
approaches.
1) Toroidal sample.

Measurements in soft magnets can seldom be done on open
samples, because demagnetizing fields are large and generally
non-uniform. But a perfectly closed magnetic circuit, free of
macroscopic demagnetizing effects, can ideally be achieved
only using toroidal samples with uniformly distributed primary
winding [9]. This approach is widely used with bulk materials,
such as ferrites and Soft Magnetic Composites (SMC) [24],
described in the standards [25] [26]. It can be used with
materials in sheet or ribbon form, either tapewound or made by
stacking punched rings, possibly after release of bending or
residual stresses by annealing. Problems nevertheless arise,
because, with the length of the circumferential magnetic field
strength lines depending on the inverse of their diameter, a ratio
between outer and inner diameter \( D_o / D_i < 1.1 \) is prescribed, a
condition sometimes difficult to fulfill [27]. Limitations may
also appear regarding the maximum number of turns, that is, the
maximum applicable magnetic field strength value.

2) Epstein frame

The Epstein frame is adopted in the IEC60404-2 standard
for measurements on magnetic sheets. These are cut as strip
samples (length 300 mm, width 30 mm), stacked and arranged
to form a square magnetic circuit with double lapped joints. The
related IEC standards cover the frequency ranges DC \( \leq f \leq 400 \text{ Hz} \) (700-turn primary and secondary windings) [4] and
400 Hz \( \leq f \leq 10 \text{ kHz} \) (200-turn windings) [5]. A fixed magnetic
path length \( l_m = 0.94 \text{ m} \) is assumed, independently of material,
flux density level, and frequency. Non-oriented (NO) alloys are
tested by stacking alternate layers of strips cut along the rolling
direction (RD) and the transverse direction (TD), in order to
cope with the non-negligible anisotropic response of these
materials. The properties of the grain-oriented (GO) alloys are
instead measured on strips cut along the rolling direction.
industry [8]. The statistical analysis is made on 212 loss figures (after excluding few outliers) provided by the participating laboratories. The data concern five different GO steel types, tested at the peak polarization values $J_0 = 1.3, 1.5, 1.7$ T. The global relative standard deviation of the measured $P_{\text{Epst}}$ values around the best estimate $<P>$ is $\sigma_{\text{Epst}} = 0.82\%$ ($\sigma_{\text{SST}} = 0.88\%$). Since the very same samples are tested and the stacking procedure is strictly defined, the dispersion among the laboratories best estimates is attributed to the performance and the calibration features of the measuring setups.

The imperfections (capacitive effects, leakage flux...) of the Epstein frame that can affect the loss measurements have been discussed by Brugel, et al. [28]. By assuming a defined magnetic path length $l_m = 0.94$ m one makes an obvious oversimplification, inevitably conflicting with the evolution of the flux patterns in the magnetic circuit with $f$, $J_0$, and the material type. We can nevertheless lump the objective complexity of such an evolution into an effective magnetic path length $l_{\text{eff}}(J_0)$, associated with the effective (true) power loss value $P_{\text{eff}}$, which relates then to $l_m$ and the standard Epstein loss figure $P_{\text{Epst}}$ according to

$$P_{\text{eff}}(J_0) = P_{\text{Epst}}(J_0) \cdot l_m / l_{\text{eff}}(J_0)$$

(1)

The results in Fig. 1 are therefore understood in terms of monotonical increase of $l_{\text{eff}}(J_0)$ with $J_0$, both in NO and GO sheets. Fig. 3. Two Epstein frames with different limb lengths have been employed to determine the effective magnetic path length $l_{\text{eff}}(J_0)$. Its derivation relies on the assumption that the power loss in the corner regions is independent of the frame size [29][30].

Fig. 4. Reduced value $J_0(x) / J_0(0)$ of the peak polarization in NO Fe-Si strip samples across the limb length $l_0 = 195$ mm (the length covered by the windings, with center at $x = 0$) of a standard Epstein frame. $J_0(x)$ is measured by means of a localized pickup coil, collinear with a narrow H-coil, allowing for simultaneous measurement of $P_{\text{Epst}}$ and the local power loss $P_{\text{loc}}(x)$ [10].

The problem of justifying from a physical viewpoint the behavior of $l_{\text{eff}}(J_0)$ has been considered both through Finite Element calculations [31] and by ad hoc experiments. Ahlers and Sievert compared Epstein frame and single strip measurements and justified the found behavior of $l_{\text{eff}}$ by expressing it as $l_{\text{eff}} = l_0 + (\mu_0/2\mu_1)l_0$, the sum of the legs length $l_0$ and a portion of the corners length $l_c$ depending on the ratio of the leg to corner permeabilities $\mu_0$ and $\mu_1$ [32]. A natural approach to the determination of $l_{\text{eff}}$ consists in assuming, as done in [29][30], that the total power loss $l_{\text{Epst}}$ measured with the Epstein frame can be decomposed in two terms, one ($l_{\text{Epst}}$) in the limbs, the portion of the frame of length $l_0$ covered by the windings, the other ($l_{\text{Epst}}$) in the corners. The somewhat crude assumption is made in [29][30] that $l_{\text{Epst}}$ is independent of the frame size and the polarization is uniform in the limbs, thereby making $l_{\text{Epst}}$ proportional to the limbs length. By comparing the loss figures obtained with two frames having different limb length, the two loss contributions are discriminated. A standard 25 cm frame and a reduced 17.5 cm frame have been considered in [29][30], as sketched in Fig. 3. If the total loss values $\Pi_{\text{Epst1}}$ and $\Pi_{\text{Epst2}}$ are measured in the larger and smaller frame, respectively, we can write

$$\Pi_{\text{Epst1}}(J_0) = \Pi_c(J_0) + \Pi_{\Pi1}(J_0)$$

$$\Pi_{\text{Epst2}}(J_0) = \Pi_c(J_0) + \Pi_{\Pi2}(J_0)$$

(2)

and we obtain $\Pi_{\text{Epst1}}(J_0) - \Pi_{\text{Epst2}}(J_0) = \Pi_{\Pi1}(J_0) - \Pi_{\Pi2}(J_0)$. In this way, the loss per unit volume $P_v(J_0) = \Pi_{\Pi1}(J_0) / A_{\Pi}$, where $A$ is the cross-sectional area of the sample, is obtained and the effective magnetic path length is retrieved from (1). A similar approach has been followed by the authors of [33], who used three Epstein frames. However, small deviations from uniformity of the magnetization inside the Epstein legs inevitably occur, impairing to some extent this conclusion. A detailed investigation on the evolution of effective magnetic field strength and polarization along the limbs, performed by
means of localized $H$- and $B$-coils, on non-oriented, conventional grain-oriented (CGO) and high-permeability grain-oriented (HGO) sheets, actually shows non-negligible decrease of the polarization value $J_0(x)$ across the limb length from center to corners in all materials [10]. Fig. 4 provides an example of decrease of $J_0(x)$ across the Epstein limbs in NO Fe-Si strips. If $J_{0p} = \frac{1}{l_0} \int_{-l_0/2}^{l_0/2} J_p(x) dx$ is the peak polarization value measured by the secondary Epstein winding, the conventional power loss figure is obtained as

$$P_{\text{epst}}(J_{0p}) = \frac{1}{T} \frac{N_H}{l_m} \int_0^T i_H(t) \cdot J_{0p} \cos(\omega t) \, dt$$

(3)

where $i_H(t)$ is the magnetizing current. The corresponding true power loss is given by the average value of the locally measured power loss $P_{\text{eff}}(x)$ across the length $l_0$

$$P_{\text{eff}}(J_{0p}) = \frac{l_0}{l_0} \int_{-l_0/2}^{l_0/2} P_{\text{eff}}(J_p(x)) \, dx$$

(4)

The power loss increases more than linearly with $J_0$, according to the power law $P_{\text{eff}}(J_0) \propto J_0^n$, with $n$, in turn, a function of $J_0$. $P_{\text{eff}}(J_0)$ becomes then dependent on the profile of $J_0(x)$ and the effective magnetic path length will consequently evolve with $J_0$ according to the equation

$$l_{\text{eff}}(J_{0p}) = l_{m} \cdot \frac{P_{\text{epst}}(J_{0p})}{P_{\text{eff}}(J_{0p})}$$

(5)

$l_{\text{eff}}$ is found to be an increasing function of $J_0$ in all the previous NO ($0.5 \, T \leq J_{0p} \leq 1.5 \, T$) and GO ($1.0 \, T \leq J_{0p} \leq 1.8 \, T$) alloys. It approximately ranges between 0.92 m and 0.98 m. Such an increase is understood and calculated in terms of $J_0(x)$ profile and $n(J_0)$ behavior [10].

3) Single Sheet Tester (SST)

Fig. 5. The loss figure measured with the SST method $P_{\text{SSST}}(J_0)$ in CGO and HGO alloys is higher (~1% ~ 5%) than the same quantity ($P_{\text{Epst}}$) measured using the Epstein frame (IEC60404-2) (adapted from [8], where the symbol $<>$ means the mean result between the different laboratories of the international comparison).

The SST standard IEC 60404-3 is based on the use of a single sheet of length 500 mm and width ranging between 300 mm and 500 mm, inserted between the pole faces of a double-C 500 mm x 500 mm flux-closing laminated yoke, made of GO Fe-Si sheets [6][34]. Primary and secondary windings are uniformly distributed on a former surrounding the sheet and the magnetic field strength is derived from the measured magnetizing current and Ampère’s law, assuming a fixed magnetic path length $l_m = 0.450 \, m$. Thanks to the large width of the test sample, edge effects are negligible and hardening due to sample cutting can be disregarded. It is a remarkable advantage of SST with respect to the Epstein method, where stress relief annealing of the cut strips is usually required [35][36]. SST testing of laser-scribed GO Fe-Si can, for example, be directly performed on the treated sheets. Epstein strips should instead be annealed before scribing, in order to keep the beneficial effect of the local deformation by the scratch lines on the domain structure. On the other hand, SST and Epstein methods show comparable reproducibility features, although SST may be more prone to outlying results, ensuing from imperfections, residual magnetism, and loss in the yokes [37][38]. Again, the fixed magnetic path length is conducive to a systematic deviation of $P_{\text{SSST}}(J_0)$ from $P_{\text{Epst}}(J_0)$, different from and difficult to reconcile with $P_{\text{Epst}}(J_0)$ through simple formulation. It is found, in general, that $P_{\text{SSST}}(J_0) > P_{\text{Epst}}(J_0)$, as illustrated for a number of CGO and HGO sheets in Fig. 5.

The typical solution adopted for overcoming the drawbacks associated with the IEC60404-3 standard consists in directly measuring the effective magnetic field strength at the sample surface by means of an $H$-coil. Since this is somewhat impractical with the 500 mm wide sheets, downsized fixtures with strip samples large enough (e.g. 60 mm) to avoid effects from work-hardened edges, have been proposed [39][40]. $H$-coil measurements are not easy, because a rigid multturn thin sensor must be realized and accurately calibrated [41][42] and
the induced signal at power frequencies can be very small and noisy, besides requiring integration. This makes this method appropriate for precise measurements in the laboratory, but unsuitable for the industrial practice. Use of Ampère’s law, where the magnetic field strength value is retrieved from the magnetizing current, without incurring in the systematic uncertainty associated with the definition of the magnetic path length in IEC 60404-3 or the difficult handling of $H$-coil low signals, would require some efficient compensation method. By this, we could neutralize the interference of the yokes on the determination of the loss figure, while maintaining zero magnetic potential drop.

**B. A novel approach to the compensated SST permeameter.**

![Diagram](image)

Fig. 6. a) Compensated permeameter with upper yoke working as zero MMF indicator and the related control loop. b) Equivalent reluctance scheme.

The classical compensated DC permeameters, like the Burrows [43] and the Iliovici [44] permeameters are based on the idea of compensating the drop of magnetomotive force (MMF) occurring in the flux-closing yoke by supplying and suitably adjusting the current flowing in auxiliary windings adjacent to the yoke pole faces. A modern version of AC SST compensated permeameter, discussed in [45], is based on the idea of using a Chattock coil, placed upon a defined central portion of length $l_m$ of the sheet sample as a zero signal indicator. The sample is inserted between the pole faces of a double-C laminated yoke and the auxiliary windings are supplied via a high-gain amplifier by the signal generated in the Chattock coil. With magnetizing solenoid and sample suitably longer than $l_m$, near-zero Chattock signal can be maintained, which implies uniform tangential magnetic field strength over the magnetic path length $l_m$. A weak point of the Chattock coil method consists in the difficulty of handling the very low signal generated by the coil. We have recently demonstrated that the MMF can actually be controlled with high sensitivity on a defined length of the sample, without using a sensor [21]. Such a control is exerted by means of auxiliary windings located on the yoke branches, as sketched in Fig. 6. It works in such a way that the magnetic path length becomes exactly equal to the distance $L_i$ between the pole faces of the yoke. To this end, the upper yoke is endowed with sharp wedge-shaped pole faces, whose tip lines are in contact with the sample sheet at the precise distance $L_i$. By imposing that the MMF drop along the upper yoke is zero, one obtains a magnetic path length coincident with $L_i$, according to the reluctance scheme shown in Fig. 6b. We denote here by $R_s$, $R_Y$, and $R_Y$ the reluctance of the sample sheet, the lower and upper yokes, and the wedge-shaped poles, respectively. The auxiliary winding on the lower yoke generates the MMF $N_iL_i$, which is controlled in such a way as to cancel the flux circulating in the upper yoke. The PID controller on the feedback loop keeps in fact the voltage $V_i = \frac{d\phi_i}{dt}$ detected on the upper yoke vanishingly small by supplying via a high-gain amplifier a magnetizing current to the winding on the lower yoke, where all the flux is eventually deviated. Since the flux in the upper yoke is zero, wedge-shaped bulk poles are perfectly appropriate. The adopted magnetic field strength value $H = \frac{N_i}{l_i}L_i$, for a current $i$ circulating in the exciting $N$-turn solenoid, is compared with the effective magnetic field strength value, obtained by integrating the voltage simultaneously detected by a many-turn flat $H$-coil placed on the sample surface. By using the yoke itself as a zero MMF indicator, we obtain much larger signal than achievable with the Chattock coil, with ensuing easier and more precise control around the zero value.
Only slight modification of standard SST setups is required; 5) This method remains effective at very low frequencies, for which the $H$-coil method becomes too noisy to get reliable results.

III. TWO-DIMENSIONAL MAGNETIZATION MEASUREMENTS: SETUPS AND RESULTS

A. Measurement setups

The characterization of soft magnetic sheets under two-dimensional flux has a long story, going back to the end of the 19th century [46]. The interest in this type of measurements has chiefly to do with the study of the energy losses in rotating machines [47] and three-phase power transformers [48][49][50]. No standards exist for 2D magnetic measurements and a variety of measuring methods are in use [9]. This fact, in conjunction with a certain experimental complexity, is the main reason for the substantial discrepancies found in the results from different laboratories [51][52].

![Fig. 8. Example of Vertical Rotational Single Sheet Tester [53].](image)

In early attempts to measure the 2D magnetic losses, either cross-shaped samples with orthogonal pickup coils [54][55] or disk samples placed at the center of two perpendicular rectangular Helmholtz pairs coils supplying a maximum magnetic field strength around 20 kA/m [56], were tested. Nowadays, the Rotational Single Sheet Testers (RSST), of either vertical or horizontal type, are typically employed. The vertical RSST derives from the standard SST magnetizer for alternating flux density, to which an orthogonal second double-C yoke is added [53][57], as sketched in Fig. 8. The magnetizing coils are wound around the yokes and a large sheet sample is generally used, in order to achieve good magnetic field strength homogeneity in the measuring region, typically located at the center of the sample.

The horizontal RSST is realized as a cross-shaped laminated magnetizer with a gap, inside which a square sheet sample is placed [58][59][60][13]. It is verified that a square sample ensures better magnetic field strength homogeneity than a circular one [61], much more so if the yoke laminations are stacked perpendicular to the sample plane, thereby hindering flux leakage between the adjacent salient poles [62][63]. Acceptably good flux homogeneity is actually obtained upon a relatively small region across the sample center [16] and modifications of the salient poles, supported by Finite Element calculations, have been realized for the sake of widening such a region [64][15].

Improved homogeneity of the magnetic flux is obtained by adopting a circular sample inserted in the stator core of a
rotating machine [32][19][22], either two-phase or three-phase. A variant of this approach consists in using an hexagonal sample inside a three-phase magnetizer [48][18]. The three-phase choice ensures a power advantage in supplying the 2D magnetic field strength (three power amplifiers can be used instead of two) and the flexibility of an additional degree of freedom in the control of the flux loci [19]. The price to pay is a certain complexity of the feedback algorithm, requiring a matrix transformation before the conventional fixed point iteration scheme [65]. Relatively sophisticated control methods, based on adaptive correction algorithms [66], can be found in literature. Other authors prefer simpler methods, even if a non-negligible number of iterations might be required [67]. Another interesting variant is the system presented in [68]. The rotating flux density is generated by a system of electromagnets working as an array of Halbach magnets. Good homogeneity of the flux density can be reached by such a system.

Recently, 3D magnetizers have been developed, for the specific objective of testing bulk Soft Magnetic Composites (SMC). They are obtained by arranging C-shaped yokes along three orthogonal planes, by which cubic samples can be tested with flux loci lying on a generic plane [69].

Fig. 9. Arrangement of $H$- and $B$- windings for the 2D characterization of magnetic sheets. a) Crossed $H$-coils for the measurement of $(dH/dt, dH/dt)$; b) Crossed $B$-windings, made of few turns threaded through small holes, providing $(dB/dt, dB/dt)$; c) Final arrangement of the $H$- and $B$- coils for the fieldmetric measurement of 2D hysteresis loops and losses.

B. Measurement of 2D magnetic losses

The measurement of the magnetic losses under a rotating magnetic field strength is typically accomplished on open samples. Consequently, magnetizing current and effective magnetic field strength are in a complex relationship and an $H$-coil is preferably used to determine the magnetic field strength on the measuring area. With the additional knowledge of the local flux density and the usual integration of their product, the loss figure is obtained. This is the rule for 2D loss measurement, but it is in some cases associated with or substituted by the thermometric method [56], where the increase of the sample temperature upon energy dissipation is measured.

If we define by $x$ and $y$ the reference axes on the sample plane, the associated ($H_x$, $H_y$) magnetic field strength components are thus determined by means of a couple of crossed multturn flat $H$-coils or by crossed Chattock coils [70][71]. The two orthogonal $H$-windings are wound on a rigid thin ($d \approx 1$ mm) epoxy plate, as shown in Fig. 9a. The induced voltages $V_{Hx}$ and $V_{Hy}$ are linked to the magnetic field strength through the equations

$$H_x(t) = \frac{1}{\mu_0 (NA)_x} \cdot \int V_{Hx}(t) \, dt$$

$$H_y(t) = \frac{1}{\mu_0 (NA)_y} \cdot \int V_{Hx}(t) \, dt$$

where $(NA)_x$ and $(NA)_y$ are the turn-area products of the coils. With uniform flux density in the measuring area, tangential ($H_x(t)$, $H_y(t)$) and effective magnetic field strength components coincide and associate with the components ($B_x$, $B_y$). These are typically obtained by threading a couple of few-turn windings through small (0.5-0.75 mm) holes drilled symmetrically on the measuring region, as illustrated for a disk sample in Fig. 9b. The flux density components are then calculated from the detected e.m.f.s ($V_{Bx}(t)$, $V_{By}(t)$), according to
where \( N_{Bx} \) and \( N_{By} \) are the number of turns of the \( B \)-windings, and \( (A_x, A_y) \) are the linked cross-sectional areas of the material. To avoid drilling the sample and the possible perturbations induced by this process (which can in any case be removed by annealing), the needle method, where point-like contacts are made in place of holes and the related voltage drop is detected, has been implemented [72],[73]. The detected signal, however, is generally very low and noisy, and this method is not in general use.

The precise measurement of \( (B_x(t), B_y(t)) \) requires the accurate determination of the cross-sectional areas \( (A_x, A_y) \) linked with the \( B \)-windings. Given the rounded profile of the holes, the geometrical determination of \( (A_x, A_y) \) in the sheet samples can be inaccurate. In addition, on approaching very high polarization values, precise correction for the air-flux is required, but the actual turn-area of the \( B \)-coils is not accurately known. For its precise determination, the saturation polarization of the material is first measured by a standard method (e.g., using a permeameter) on a conventional strip sample with well-known cross-sectional area. The disk sample is then inserted between the pole faces of a Type-B permeameter ([9], pp. 311) and the flux \( \Phi_x = N_{Bx}(J A_x + \mu_0 H_0 A_x) \), where \( J \) is the polarization and \( A_x \) is the total area (air plus sample) linked with the \( B_x \) winding, is measured. The effective magnetic field strength \( H_x \), measured at the coil position by means of a Hall plate, is increased beyond about 150 kA/m, thereby ensuring full magnetic saturation. \( J = J_x \). The linear increase of \( \Phi_x \) versus \( H_x \) is exclusively due to the term \( \mu_0 H_0 A_x \) in high-magnetic field strength region. We obtain in this way the area \( A_x \) and, for any sufficiently high magnetic field strength \( H_0 \), the measuring cross-sectional area of the sheet sample:

\[
A_x = \frac{\Phi_x(H_x) - N_{Bx} \mu_0 H_0 A_x}{N_{Bx} V_x}
\]  
(8)

By repeating the same procedure along the \( y \)-direction, \( A_y \) is obtained and the loss can finally be calculated from the measured magnetic field strength and flux density components and their time dependence according to the Poynting theorem:

\[
W = \int_0^\tau \left( H_x \frac{dB_x}{dt} + H_y \frac{dB_y}{dt} \right) dt \quad [J/m^2]
\]  
(9)

To remark here the inevitable approximation involved with the measurement of the effective magnetic field strength in the open sample, with its large and relatively inhomogeneous demagnetizing field. The \( H \)-coil provides, because of its finite thickness, an overestimate of the true magnetic field strength value in the sample. This fact, however, does not interfere with the loss measurement, because true and measured magnetic field strengths differ by a quantity proportional to the magnetization.

A further drawback of the fieldmetric 2D loss measurement can derive from possible slight misalignment and imperfect orthogonality of the \( H \)- and \( B \)-windings [74],[75],[76]. This leads to different loss values when measured with the magnetic field strength rotating clockwise (CW) or counterclockwise (CCW). While the difference is reduced, though not negligibly [77], at low flux density values, dramatic divergence between CW and CCW rotational losses can be observed at high flux densities (e.g., beyond 1.5 T in non-oriented alloys) [77],[78]. This discrepancy can be largely, though not fully, compensated by averaging the results of the CW and CCW measurements [79],[80],[78], as the geometrical phase lag is reversed relatively to the electromagnetic phase lag.

At high flux densities, the phase shift between flux density and magnetic field strength (actually, its fundamental harmonic) may become too small to be measured with sufficient accuracy. Consequently, the thermometric method, where the magnetic power loss is associated with the rate of heating of the sample, would rather be used instead of the usual fieldmetric method. In the ideal case of adiabatic measurements on a material of specific heat \( c_p \), the power loss per unit mass is related to the time derivative of the temperature according to

\[
P = c_p \cdot \frac{dT}{dt} \quad [W/kg]
\]  
(10)

The temperature is measured either with a thermocouple stuck on the sheet surface [9],[56] or a thermistor [81]. Good agreement between the fieldmetric method and the thermocouple-based thermometric method is verified in NO steel sheets [20]. The thermistor sensors actually appear better suited to localized loss investigations, as those concerning high-permeability wide-domains GO steels [81].

C. 2D magnetizers for high frequencies and high flux densities: design and analysis

The relative complexity of the 2D measurements, due to the open sample configuration, the related need for bulky magnetizers, and the relatively awkward control of the flux loci, have traditionally limited the range of frequency and flux density \( B_B \). This becomes apparent when the reported capabilities of the main laboratories dealing with the 2D characterization of nonoriented steel sheets are considered and...
compared, as shown in the peak flux density versus test frequency map shown in Fig. 10. The maximum experimental frequency is typically around a few hundred Hz and reliable results are hardly obtained beyond 1.7 T – 1.8 T [78]. The reported 2D measuring limits regard INRIM (before year 2010) [19][20], Wolfson Centre WCM [82], G2ELAB Grenoble [83][84], PTB Braunschweig [85], Gifu Univ. [86]. Actually, present-day high-speed rotating machines involve frequencies in the kHz range ([87]), and mass reduction requirements force machine designers to deal with flux density levels around 2 T, as highlighted in Fig. 10. The dashed line in this figure, covering such (\(B_0, f\)) limiting profile, reflects the enhanced 2D measuring capabilities recently attained by the joint efforts of the SATIE/INRIM/Politecnico di Torino labs. Two magnetizers have been designed, built, and employed for this purpose. We summarize in the following the properties of these devices, the related measuring methods, and a few significant results.

1) The high-frequency 2D magnetizer

A two-pole three-phase machine has been designed for 2D magnetic testing up to the kilohertz range of disk-shaped samples. A 3D FEM calculation has been conducted and optimized machine geometry has been designed for an 80 mm diameter sample. The calculations take into account the specifications of the supply system, consisting of three CROWN AUDIO 5000VZ power amplifiers (peak-to-peak output voltage 300 V, maximum output current 40 A), driven by three synchronized Agilent 33220a function generators. The feedback system employed for the control of flux density loci is based on the contraction mapping principle [65]. The scheme of the magnetizer and the adopted 3D FEM meshing is shown in Fig. 11, together with the actual device. This is equipped with a toroidal winding, which avoids long overhangs. Three slots per pole are used, totaling 18 slots. The laminated core is made of 0.35 mm thick non-oriented Fe-Si sheets, whose experimental anhysteretic curve is taken as magnetic constitutive equation of the material in the FEM calculations. Details on the modeling criteria and procedure are discussed in [88]. Two main points are stressed here:

1. The disk sample is separated from the magnetizing core by a 1 mm thick gap. This narrow air gap allows to minimize the apparent power required by the magnetizer.
2. The axial height \(T\) of the magnetizing core is optimized by 3D FEM calculations to maximize the flux density in the sample under a given magnetizing power. By taking the anhysteretic constitutive equation for a standard type of non-oriented sample of thickness \(d\), the calculations show, in particular, that, for a ratio \(T/d\) ~ 75, minimum apparent power is required in order to achieve a defined peak flux density value (e.g. \(B_0 = 1.5\) T at 1 kHz). For \(d = 0.20\) mm, a 15 mm thick core is therefore predicted.

For measurements performed with the fieldmetric method, a 1 mm thick 20 mm \(\times\) 20 mm crossed \(H\)-coil, placed in the central region of the disk sample, is employed. The capacitive effects are minimized by adequately spacing the turns, for kHz range operation.
and conclusions for the high-sized good linearity of the strength of size centered dependence of surface.

The same 3D FEM analysis was used to evaluate the uniformity of the effective magnetic field strength at the sample surface. Fig. 14 provides the behavior of $H_\nu(x,y)$, parallelizing the dependence of $B_s(x, y = 0)$ shown in Fig. 13, on the square centered region of size 40 mm (a) and the inner square region of size 20 mm (b). The inhomogeneity of the magnetic field strength (about 31% decrease from $y = 0$ to $y = \pm 20$ mm) with respect to the flux density is inevitably due to the strong non-linearity of the $B(H)$ dependence at $J_p = 1.5$ T. In order to have good congruence between the $B$ and $H$ signals, a square 20 mm sized $H$-coil is adopted. This analysis leads to similar results and conclusions for the high-flux density 2D magnetizer, to be discussed in the following.

![Fig. 12. An axis system $x$-$y$ is defined on the sample surface. The three-phase currents are set in such a way that an alternating flux density of peak value $B_p = 1.5$ T is established along the $y$ axis. The shadowed 20 mm square region centered on the sample surface is the $H$-coil zone.](image)

![Fig. 13. Behavior of the magnetic flux density $B_s(x, y = 0)$ across the 80 mm x-diameter of the non-oriented Fe-Si disk sample calculated for alternating magnetic field strength directed along the y-diameter. The $B$-measuring region is restricted to the $-20 \text{ mm} \leq x \leq 20 \text{ mm}$ region. The relative difference between $B_s(0,0)$ and $B_s(\pm 20 \text{ mm},0)$ is equal to 0.2%.](image)

![Fig. 14: Condition $B_p=0$, $B_s=1.5$ T, as in Fig. 12. a) Magnetic field strength homogeneity $H_\nu(x,y)$ calculated by FEM analysis on the sample surface over a 40 mm wide centered square region. b) Enlarged view of the inner 20 mm wide area. The relative 31 % difference found between $H_\nu(0, x)$ and $H_\nu(\pm 20 \text{ mm}, x)$ is reduced to 7 % for $H_\nu(\pm 10 \text{ mm}, x)$ .](image)

### 2) The high-flux density 2D magnetizer

In order to fully cover the aimed at $(B, f)$ domain envisaged in Fig. 10, a novel magnetizer, specifically designed for 2D measurements at high flux densities, was developed. In such a device, the basic structure of the previous high-frequency magnetizer was retained, together with the 80 mm diameter sample. However, with the same two-pole/three-phase 18 slot configuration, re-sizing of the magnetizer was required. The thermometric measurements at high flux densities, in fact, are preferably performed under near-adiabatic conditions, which are in the present case emulated by keeping the sample inside a vacuum bell ($p \approx 10^{-4}$ Pa), which occupies the stator bore, as sketched in Fig. 15. The 6 mm thick walls of the vacuum chamber, made of PVC, introduce a relevant airgap and the optimized $T/d$ ratio will differ from the value found for the previous setup. An additional constraint is posed by the maximum current of 40 A per phase delivered by the CROWN AUDIO 5000VZ power amplifiers (little constraint on the voltage is posed at power frequencies). By keeping the previous winding scheme, the magnetomotive force (MMF) per slot necessary to reach $B_p = 2$ T at the sample center has been computed by FEM analysis as a function of $T/d$ [88]. From the results shown in Fig. 16, obtained with $d = 0.20$ mm, the optimized value $T/d = 150$, that is $T = 30$ mm, is estimated, with
the MMF per slot around 1000 A. This condition is satisfied, taking into account the 40 A upper limit for the magnetizing current, by inserting 25 turns per slot. Assuming a maximum current density of 5 A/mm² (forced air convection is used to cool the windings), the geometrical parameters of teeth and back core of the magnetizer are then calculated, as summarized in Table I. To remark that the thermal transfer due to Joule heating of the windings and their forced cooling does not influence the temperature of the sample, which is protected in the vacuum chamber.

![Three-phase toroidal winding](image)

Three-phase toroidal winding

Vacuum chamber and sample

Stator core

Disk sample

Plexiglas top of the vacuum chamber

Plastic vacuum chamber

Vacuum pump

Fig. 15. Front and cross-sectional views of the high-flux density magnetizer. It holds the vacuum chamber employed for emulating quasi-adiabatic conditions during the measurement of the magnetic power loss with the thermometric method.

![Magnetomotive force per slot](image)

B_p = 2 T

Fig. 16. Magnetomotive force per slot required to reach B_p = 2 T at the center of the 0.20 mm thick disk sample as a function of the ratio T/d between axial height of the magnetizer and sample thickness.

As previously, stressed, the large uncertainty of the loss value measured by the conventional fieldmetric method at high flux densities can be overcome by measuring the dissipated power through the rise of the sample temperature. This can be easily detected, in particular, by a copper-constantan extended junction, carefully glued by silver paint on the sample surface, along and close to the B-windings. The reading of the junction signal is made by a calibrated nanovoltmeter, which output signal upon switch-on and switch-off of the magnetic field strength displays the typical behavior shown in Fig. 17. The magnetic field strength is applied at time t_on, after stabilizing the whole device temperature, and switched off at time t_off. The temperature difference between the temperature at a certain instant t and the one at time t_on is called . Since the system is not fully adiabatic, we observe a typical first order system increase of vs. t till switch-off at time t = t_off, followed by an exponential decrease. Consequently, (10) does not apply, but we can nevertheless precisely retrieve the power loss figure by modeling the heat exchange process according to the balance equation

\[ P = \frac{dQ_{in}}{dt} + \frac{dQ_{out}}{dt} = c_p \frac{d\Delta T}{dt} + K_{ext} \Delta T, \quad (t > t_{on}) \]  

where we denote by \( \frac{dQ_{in}}{dt} \) and \( \frac{dQ_{out}}{dt} \) the rates at which heat is stored in the sample and lost to the environment, respectively, and by \( K_{ext} \) the heat transmission coefficient. The increase of the specimen temperature \( T \) with time is then obtained from (11) as

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample diameter</td>
<td>( D )</td>
<td>80 mm</td>
</tr>
<tr>
<td>Airgap thickness</td>
<td>( a )</td>
<td>6 mm</td>
</tr>
<tr>
<td>Magnetizer axial thickness</td>
<td>( T )</td>
<td>30 mm</td>
</tr>
<tr>
<td>Teeth depth</td>
<td>( t_s )</td>
<td>45 mm</td>
</tr>
<tr>
<td>Teeth width</td>
<td>( w_s )</td>
<td>15 mm</td>
</tr>
<tr>
<td>Yoke thickness</td>
<td>( t_y )</td>
<td>20 mm</td>
</tr>
<tr>
<td>Number of turns per coil</td>
<td>( N )</td>
<td>25</td>
</tr>
</tbody>
</table>

**TABLE I**

**MEASUREMENTS AT HIGH FLUX DENSITIES**

<table>
<thead>
<tr>
<th>MMF (A)</th>
<th>4000</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 17.** Temperature versus time in a non-oriented Fe Si alloy subjected to rotational magnetic field strength at 20 Hz from time t_on to time t_off. The temperature difference \( (t) \) follows an exponential dependence on time, with time constant \( \tau = c_p \), where \( c_p \) is the specific heat of the alloy and \( K_{ext} \) is the heat transmission coefficient, by which we lump the imperfect adiabatic response of the sample. The continuous fitting lines are calculated from (12) and (13).
\[
\Delta \theta = \frac{P}{K_1} \left[ 1 - \exp \left( -\frac{K_{\text{ext}}}{c_p} (t - t_{\text{on}}) \right) \right], (t_{\text{on}} \leq t \leq t_{\text{off}}) \quad (12)
\]

In the limit \( K_{\text{ext}} \to 0 \), this equation reduces to the linear relationship (10). The coefficient \( K_{\text{ext}} \) is unknown, but it can be found by fitting the exponential decay of temperature observed after switch-off. If at the instant of time \( t_{\text{off}} \) the temperature difference is \( \Delta \theta \), the time dependence of the temperature difference is obtained, by posing \( P = 0 \) in (11), as

\[
\Delta \theta = \Delta \theta_0 \cdot \exp \left( -\frac{K_{\text{ext}}}{c_p} (t - t_{\text{off}}) \right), (t > t_{\text{off}}) \quad (13)
\]

We find that the temperature decay is accurately described by (13), with the time constant \( \tau = c_p / K_{\text{ext}} \). For the specific case of the Fe-(3.5 wt%)Si sheet sample of Fig. 17, having specific heat \( c_p = 470 \text{ J kg}^{-1} \text{ K}^{-1} \), we obtain from (13) the time constant \( \tau = 48.7 \text{ s} \) and \( K_{\text{ext}} = 9.65 \text{ J kg}^{-1} \text{ K}^{-1} \text{ s}^{-1} \). The value of \( P \) is then determined by fitting through (12) the experimental vs. \( t \) increase in the time interval \((t_{\text{off}} - t_{\text{on}})\). It is noted that, by limiting the measurement to very short times \((t_{\text{off}} - t_{\text{on}}) \ll \tau \) (\( \tau = 49 \text{ s} \) in Fig. 17) a straight line is a good approximation to (12) and we can retrieve \( P \) by (10). In the example reported in Fig. 17, we find by (12) \( P = 1.16 \text{ W/kg} \).

3) A few significant results

![Graph](a)

![Graph](b)

Fig. 18. Alternating \( W_{\text{rot}} \) and rotational \( W_{\text{alt}} \) energy losses measured with the use of the high-frequency RSST magnetizer in Fe-Si (a) and Fe-Co (b) non-oriented steels sheets up to the kHz range.

![Graph](a)

![Graph](b)

Fig. 19. Broadband energy loss under alternating and circular magnetic polarization measured with the high-frequency RSST in a Soft Magnetic Composite. The curve shown in the inset roughly delimits the accessible \((f, J_p)\) measuring domain.
The high-frequency RSST magnetizer has been used for unidirectional and 2D loss measurements up to the kHz range in steel sheets and Soft Magnetic Composites (SMC). Fig. 18 provides an example of measured alternating $W_{alt}$ and rotational (circular flux loci) $W_{rot}$ losses in 0.20 mm thick non-oriented Fe-Si (up to 1.5 T / 2 kHz) and Fe-Co (up to 2.1 T / 5 kHz) sheets. The skin effect under alternating and circular polarization is put in evidence at the highest frequencies [89] [90]. One can notice in Fig. 18 the progressive disappearance of the maximum of $W_{rot}$ vs. $J_p$ on increasing the magnetizing frequency. This behavior is understood in terms of growing share of the classical loss component in the total loss, a quantity monotonically increasing with $J_p$ [23]. Measurements of $W_{alt}$ and $W_{rot}$ at high frequencies in SMC materials, which, because of their granular structure, are expected to bear some advantage in terms of loss behavior, are reported in Fig. 19. The 80 mm diameter tested SMC disks have a thickness of 3 mm. Because of their low permeability ($\mu_r \sim 100$ - 500) and bulk shape, these materials pose a challenge to their 2D characterization at medium-to-high flux densities and frequency values in the kHz range. The domain ($f, J_p$) accessible to measurements can be roughly identified with the area subtended by the experimental limiting $f(J_p)$ line shown in the inset of Fig. 19. This measuring capability is superior to the state of the art [91]. Typical behaviors at 50 Hz (i.e., close to quasi-static excitation) of $W_{alt}$ and $W_{rot}$ versus $J_p$ in different types of SMC are shown in Fig 20. These samples have been obtained following different processes, leading to different average particle size and density $\delta$. It is observed how the loss increases with decreasing material density (from $\delta = 7450$ kg/m$^3$ to $\delta = 7110$ kg/m$^3$ on passing from sample 1 to sample 3), that is, increasing the thickness of the intergrain boundaries. At the same time, $W_{rot}$ moves its maximum value to lower $J_p$ values. Grain decoupling, ensuing from thicker non-magnetic boundaries, yields higher coercivity (i.e. higher quasi-static losses). On the other hand, the coherent moment rotations also start at lower $J_p$ values, because the required applied magnetic field strength, largely increased to compensate higher internal demagnetizing fields, becomes high enough to induce coherent rotations in the favored grains.

The high-flux density magnetizer shown in Fig. 15 permits to approach the magnetic saturation under alternating and two-dimensional magnetic field strengths in steel sheets. To this end, both fieldmetric and thermometric methods are applied upon the low- and high-flux density range, respectively. An example is provided in Fig. 21, concerning the behavior of $W_{alt}$ and $W_{rot}$ in a non-oriented Fe-(3 wt %)Si sample, brought up to $B_p = 2.0$ T ($J_p = 1.95$ T). The fieldmetric method is applied up to about 1.6 T, the thermometric method beyond this limit, with a short overlap region. The capability of this system to measure the value of $W_{rot}$ very close to the saturated state allows reaching the limiting condition, where domain walls collapse, and only coherent reversible rotations are deemed to provide the macroscopic rotation of the magnetization. In this case, the measured loss should reduce to the classical loss component, according to the equation

$$W_{rot} = W_{rot, class} = \frac{\pi^2}{3} \frac{\sigma d B_p^2}{\delta} f$$

[J/kg] (14)

where $\sigma$ is the electrical conductivity, and $d$ and $\delta$ are the previously introduced sheet thickness and density, respectively. Using (14) and the experimental $W_{rot}(f)$ behavior at different $J_p$ values [23], we can separate $W_{rot}(J_p)$ in the NO Fe-Si sheet of Fig. 21 into its hysteresis $W_{rot}(J_p)$, excess $W_{exc}(J_p)$, and classical $W_{class}(J_p)$ components up to saturation, as shown in Fig. 22. Here we prove that by (14) we identify the limiting value of the rotational loss for $J_p = J_s$, a previously predicted result.
IV. CONCLUSIONS

We have discussed the problems connected with the measurement of the magnetic properties of soft magnetic materials under alternating and two-dimensional magnetic fields and related recent advances, with a focus on recent investigations and advances obtained by the cooperative activity of the SATIE, INRIM, and Politecnico di Torino labs.

The Epstein and SST standard measuring methods and the systematic deviations of their power loss figures from the true values, follow from the specifically involved magnetic circuits and the practical need to define a magnetic path length. This, however, inevitably depends on the sample type, the flux density level, and the magnetizing frequency. Detailed analysis of the Epstein measurements made on different NO and GO Fe-Si sheets shows, for example, that in all materials the effective magnetic path length increases with the peak polarization, differing by about 5% with respect to the standard 0.94 m path length [10]. A Single Sheet Tester with double-C flux-closing yoke has therefore been developed, by which the magnetic path length is independent of the material type and flux density level by means of an original compensation method, based on the use of the yoke itself as a zero-MMF indicator. Systematic deviations from the true loss values are in this way reduced to less than 1%, that is within the typical uncertainty of this kind of measurements.

The present-day capabilities of the two-dimensional measuring systems lack in general the access to frequencies in the kHz range at technical flux densities and to high flux densities at power frequencies. But testing at high frequencies and high flux densities meets the demand by today machine designers. To overcome current limitations, the authors have developed novel setups, based on 3D FEM designed three-phase magnetizers, employed with disk-shaped samples, minimum airgap, high-performance power amplifiers, and dedicated feedback algorithms. Loss measurements up to 2 kHz at 1.5 T could be made, for example, in 0.20 mm thick non-oriented Fe-Si sheets, a range broadened to 2.1 T and 5 kHz in the high-saturation Fe-Co alloys. Soft Magnetic Composites could also be tested up to 1 kHz at 1.25 T. These figures largely exceed previous literature reports. On the other hand, the saturation magnetization could be closely approached in non-oriented Fe-Si sheets at power frequencies. It is here verified, in particular, that \( W_{\text{tot}} \) is reduced to the classical component, as expected on theoretical grounds, but never experimentally proved.

V. REFERENCES


