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Non-equilibrium thermodynamic approach to spin pumping and spin Hall torque effects

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Abstract. The paper introduces a non-equilibrium thermodynamic approach to describe vector magnetization in presence of a magnetic moment current. The Larmor term present in the theory describes in this case how the magnetization dynamics can be a source of magnetic moment currents and vice versa. The model here discussed is applied to describe the reciprocity between spin pumping and spin Hall torque effects found in bilayers composed of a metal with large spin Hall effect (e.g. platinum, Pt) and a ferromagnetic insulator (e.g. yttrium iron garnet, YIG). The result is that the two effects are related to the transport of the magnetic moment between layers of different materials. In particular for thin YIG the two effects are related each other by two parameters: the diffusion length of Pt, l_{Pt}, and the conductance of Pt, $v_{Pt} = l_{Pt}/\tau_{Pt}$, where τ_{Pt} is a time constant. Both parameters can be that can be determined by comparison with experiments.

Keywords: Non-equilibrium thermodynamics, magnetic moment current, spin pumping, spin Hall torque, Pt/YIG bilayers.

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1. Introduction

There have been recently several efforts to identify reciprocity relations present in spintronic effects [1, 2, 3]. The spin Seebeck and spin Peltier effects, found in bilayers made of an insulating ferrimagnet, like yttrium iron garnet (YIG), and a non magnetic metal with a large spin Hall effect, like platinum (Pt), are relevant examples of phenomena linked by the Onsager reciprocity relations [4, 5]. The reciprocity is related to the fact that the magnetic moment current (or equivalently, the spin current) in the ferromagnet carries also an heat current. Similarly to thermoelectrics, both the effects are described by a single parameter, the thermo-magnetic power coefficient ϵ_M of YIG, which can be experimentally measured as well as computed by statistical theories [6, 7]. Other two effects that are amenable to a detailed thermodynamic investigation are the spin Hall torque and the spin pumping. In the spin pumping, the ferromagnet, driven at the resonance by a radio frequency (RF) magnetic field, is able to generate a spin polarized current in the metallic layer and an enhancement of the damping in the ferromagnet [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. In the spin Hall torque, the magnetic moment current, generated by the spin Hall effect in in a metal like Pt, causes spontaneous self oscillations of the magnetization in the ferromagnet above a threshold current [8, 20, 21, 22]. Since the magnetic moment current has different carriers in the two layers, i.e. spin polarized electrons in the metal and spin waves in the insulator, the phenomenon is different from the well known spin transfer torque occurring in ferromagnetic metals because of the presence of spin polarized electric currents [23, 24]. Spin pumping is often interpreted by adapting the magneto-electronic theory of Tserkovnyak et al. [25] and the concept of spin mixing conductance to the interface between an insulating ferromagnet and a normal metal [26, 12]. Although there are attempts in the literature to address spin currents in bulk insulating ferromagnets [27, 28] from a statistical point of view, it is of interest to regard spin pumping and spin Hall torque from the viewpoint of non-equilibrium thermodynamics. The reciprocity of spin pumping and spin Hall torque is deeply rooted into the vector character of the magnetization of the ferromagnet and into the possibility to excite a dynamic mode (uniform or magnetetostatic) because of the presence of the Larmor term. The corresponding vector thermodynamic theory has been the subject of a few recent efforts by the groups of Ansermet [29, 3] and Saslow [1, 30, 31] and it is inspired by the thermodynamic theory of Johnson and Silsbee [32]. The main difficulty to build up such thermodynamic approaches is to properly address the typical length-scale of the theory.

On a small length-scale, micromagnetics is the relevant theory to be used [33]. The magnetization is defined as a local average over atomic magnetic moments and, at temperatures well below the Curie point, it results in a vector of constant modulus $M_s(T)$, depending on the temperature T only. In micromagnetics the spatially extended excitations, like spin waves, are not averaged out, but explicitly described. The thermal spin waves spectrum is therefore computed or numerically simulated by introducing random fluctuations in the magnetization mimicking the effect of temperature [34, 35, 36, 37, 38].

On a larger length-scale, the macroscopic thermodynamic approach of Johnson and Silsbee is the relevant theory[32]. In such a theory, all thermal excitations are averaged out and result in a scalar magnetization M that can be different from the equilibrium value. The distance from equilibrium is described by introducing the thermodynamic potential $H^* = H - H_{eq}(M)$, which is the difference between the magnetic field H and the equation of state at equilibrium $H_{eq}(M)$. The gradient of the potential H^* turns out to be the generalized driving force for magnetic moment currents and therefore it allows a careful description of thermodynamic effects like spin Seebeck and spin Peltier [6]. The aim of developing a vector thermodynamic theory should be to describe phenomena governed by characteristic length scales which are intermediate between those of micromagnetics and those of the Johnson and Silsbee thermodynamics.

Here we present a possible approach to such a vector thermodynamic theory aiming to understand the reciprocity of spin Hall torque and spin pumping effects as found in Pt/YIG bilayers. The theory is appropriate to understand the interaction between thermal spin currents and the uniform or the magnetostatic modes excited in a ferromagnet [21]. In Section 2 we outline the vector generalization of the thermodynamic theory of Ref.[6]. In Section 3 we introduce the appropriate generalized magnetization dynamics. Spin pumping and spin Hall torque effects arising in YIG/Pt bilayers are discussed in Section 4. The outcome of the theory is discussed in Section 5.

2. Non-equilibrium thermodynamics of magnetic systems

The starting point to build up a non-equilibrium thermodynamics theory is the description of situations in which the extensive variables of the system, the magnetization $\boldsymbol{M}$, the entropy density s and the internal energy density u, are space dependent. Therefore, an appropriate description of the system at coarse-grained level is needed. Indeed, in non-equilibrium thermodynamics, the densities of the extensive quantities must be the result of thermodynamic averages over a proper volume Δv^* centered around the considered point $\boldsymbol{r}$. Δv^* must be small enough to allow description of the system as a continuum and large enough for taking meaningful statistical averages of the thermal effects, including thermal spin waves, to be averaged out. Furthermore, it is necessary to introduce current densities, i.e. the magnetic moment current $\boldsymbol{j_M}$ (a second rank tensor), the entropy current $\boldsymbol{j}_s$ and the energy current $\boldsymbol{j}_u$, to account for the flow of the extensive quantities from one point of the medium to another one. By means of the non-equilibrium thermodynamics of fluxes and forces it is possible to associate to each current density a generalized thermodynamic force [39]. Finally, the kinetic constitutive equations of the medium, i.e. the relations between current densities and generalized forces, are introduced by assuming linear relations. This linear case valid as soon as the distance from the equilibrium state caused by the presence of flows is not too large. For example, the charge current is associated to the gradient of the electric potential and their product is the amount of heat which is dissipated as entropy production. The application of non-equilibrium thermodynamics to magnetic systems is not as straightforward as for the electric ones because the magnetic moment is not a conserved quantity. Then, in a continuity equation, it is always necessary to introduce an additional term describing sources and sinks for the magnetic moment. In this section we outline the main features concerning the non-equilibrium thermodynamic theory for vector magnetization by making a straightforward generalization of the scalar theory presented in Ref.[6]. The formal expression for the continuity equation is

$$\frac{\partial \boldsymbol{M}}{\partial t} + \nabla \cdot \boldsymbol{j_M} = \frac{\boldsymbol{H}^*}{\tau_M} \tag{1}$$

where at the right hand side of Eq(1) we find the source/sink term for the magnetic moment expressed as a function of the Johnson and Silsbee potential

$$\boldsymbol{H}^* = \boldsymbol{H} - \boldsymbol{H}_{eq}(\boldsymbol{M}) \tag{2}$$

The vector $\boldsymbol{H}^*$ represents the distance of the actual magnetization state, at the magnetic field $\boldsymbol{H}$, from the equilibrium state characterized by the equation of state $\boldsymbol{H}_{eq}(\boldsymbol{M}) = (1/\mu_0)\partial u/\partial \boldsymbol{M}$. In the latter equation μ_0 is the magnetic permeability of vacuum. In absence of currents ($\boldsymbol{j_M} = 0$), the continuity equation (1) expresses the low of relaxation of the magnetization towards equilibrium. The law states the proportionality between the generalized velocity, $\partial \boldsymbol{M}/\partial t$, and the generalized force, $\boldsymbol{H}^*$. The proportionality is characterized by the material dependent parameter τ_M, that has the units of a time constant. To derive the generalized force associated to the magnetic moment current one follows the classical method and expresses the entropy production rate σ_s in terms of a sum of products of each current density times its conjugated thermodynamic force[39]. By generalizing the equations of Ref.[6] to account for vector quantities, we obtain

$$\sigma_s = \nabla\left(\frac{1}{T}\right)\cdot \boldsymbol{j}_q + \frac{1}{T}\mu_0 \nabla \boldsymbol{H}^* \cdot \boldsymbol{j_M} + \frac{1}{T}\frac{\mu_0\,(H^*)^2}{\tau_M} \tag{3}$$

where the heat current is defined as $\boldsymbol{j}_q = T\boldsymbol{j}_s$. Eq.(3) shows that the generalized force conjugated with the magnetic moment current is the gradient $\nabla\boldsymbol{H}^*$ (a second rank tensor) and it also contains an additional term which describes the entropy production generated by local relaxation processes depending only on the modulus H^*.

Once the generalized forces have been identified it is possible to directly state the constitutive equations for a ferromagnetic insulator by writing down the most general relation relating the magnetic moment current tensor $\boldsymbol{j_M}$ and the heat current vector $\boldsymbol{j}_q$ with the generalized forces, i.e the gradients $\nabla\boldsymbol{H}^*$ (a tensor) and ∇T (a vector). By restricting to the case of linear relations, we have

$$\boldsymbol{j_M} = \sigma_M\left(\mu_0\nabla\boldsymbol{H}^* - \epsilon_M \boldsymbol{m}\nabla T\right) \tag{4}$$

$$\boldsymbol{j}_q = \epsilon_M T(\boldsymbol{m}\cdot\boldsymbol{j_M}) - \kappa\nabla T \tag{5}$$

where σ_M is the magnetic moment conductivity, a tensor relating the components of the magnetic moment current $\boldsymbol{j_M}$ to the components of the gradient $\nabla\boldsymbol{H}^*$ and κ is the thermal conductivity under zero magnetic moment current. As a consequence of the Onsager reciprocity relations, the cross effects are described by a single parameter, ϵ_M, the so called thermo-magnetic power coefficient, and by the direction of the magnetization in the ferromagnet identified by the unit vector $\boldsymbol{m}$ [39].

The importance of the non-equilibrium thermodynamic approach to the description of transport phenomena involving vector magnetization and magnetic moment currents is appreciated by looking at the possibility to solve conduction problems. In fact, the solution of the transport of the magnetic moment is obtained by joining the continuity equation (Eq.(1)) with the constitutive equation (Eq.(4)). Stationary states solutions are described by imposing the condition $\partial\boldsymbol{M}/\partial t = 0$ and, in the isothermal case ($\nabla T = 0$), the two equations combine in a diffusion equation for the potential: $l_M^2\nabla^2\boldsymbol{H}^* = \boldsymbol{H}^*$, where $l_M = (\mu_0\sigma_M\tau_M)^{1/2}$ is the diffusion length. As the solution of the diffusion equation is straightforward, the problem reduces to a boundary value problem in which appropriate boundary conditions for the magnetic moment current and for the potential shall be imposed. A relevant case is obtained by joining different media and studying the the conditions for the passage of the magnetic moment current from one medium to the other one. To address the problem in such a case, the diffusion equation is solved independently for each layer, each one characterized by its own diffusion length (see Ref. [6] for the diffusion equation in Pt and Appendix B for the solutions), and then proper boundary conditions are imposed at the interfaces. The amount of current traversing the interface is determined by the balance between the action of the active source layer and the counteraction of the passive layer. This counteraction is due to the appearance of a non zero value of H^* in the passive layer and is equivalent to the effect of a backflow spin current flowing from the passive to the active layer, a concept often used in the literature employing the micromagnetic viewpoint with fluctuations [34, 36]. Although the theory developed up to now is appropriate to describe the spin Seebeck and spin Peltier effects [6], it is not sufficient yet to deal with the spin pumping and spin Hall torque effects, because they both depend on the precession of the magnetization arising from the Larmor term present in the equation of magnetization dynamics. Therefore the continuity equation (1) needs to be generalized by adding the conservative Larmor term.

3. Generalized magnetization dynamics in the thermodynamic framework

The precession of the magnetization is the result of the torque exerted on the magnetization vector by an effective field non collinear to it [40, 41, 33]. From the point of view of the conservation of the physical quantities, one has to better specify the physical system into consideration. In magnetic solids one has to consider the joint presence of a magnetic subsystem (M) and a reservoir subsystem (R). The magnetic subsystem (M) is characterized by the magnetization vector $\boldsymbol{M}$ and by the angular momentum $\boldsymbol{J}_M$ associated to it. The reservoir subsystem (R) includes all the non magnetic degrees of freedom of body and the externally applied magnetic fields and is characterized by the angular momentum $\boldsymbol{J}_R$. Between the two subsystems the torque $\boldsymbol{T}$ exerted by R on M will produce a change in the angular momentum $\boldsymbol{J}_M$ as $\partial \boldsymbol{J}_M/\partial t = \boldsymbol{T}$. Since the angular momentum $\boldsymbol{J}_M$ is also the source of the magnetization $\boldsymbol{M}$, we can write $\partial \boldsymbol{M}/\partial t = \gamma \boldsymbol{M} \times \delta u_{e,L}/\delta \boldsymbol{M}$, where γ is the gyromagnetic ratio ($\gamma = |e/m_e| = 1.76 \cdot 10^{11}$ s^{-1}T^{-1} for electrons) and $\delta u_{e,L}/\delta \boldsymbol{M}$ is a term having the meaning of an effective field (see Appendix A). The previous equation expresses the precession of the vector $\boldsymbol{M}$ around the effective field in which the component of the magnetization parallel to the effective field, $\boldsymbol{M}_{\parallel}$, is constant in time, while the perpendicular component $\boldsymbol{M}_{\perp}$ varies in time. Since the total angular momentum $\boldsymbol{J} = \boldsymbol{J}_M + \boldsymbol{J}_R$ is conserved, the change in time of the transverse component $\boldsymbol{M}_{\perp}$ is a purely reversible effect between the two subsystems.

The possibility of a change in time of the component $\boldsymbol{M}_{\parallel}$ is related to the presence of damping forces able to relax the system towards equilibrium. The inclusion of these damping forces into the dynamics of $\boldsymbol{M}$ is commonly done by writing the combined equation in the Landau-Lifshitz form or in the Gilbert form [24]. The two forms are equivalent if the modulus of the magnetization is conserved, while they are not if the magnetic moment can also flow from one point of the medium to another one. The issues has been carefully discussed by Saslow [30], who showed that the Landau-Lifshitz form is the appropriate one to be included in a thermodynamic framework.

Within the thermodynamic theory derived in Section 2, we have seen that the internal energy of the system is expressed as a function of a magnetization vector which is the result of the local average of the magnetic moments over a large enough volume Δv^*. Therefore the role of the effective field is taken by the field $\boldsymbol{H}^*$ defined in Eq. (2) (see Appendix A) and we are allowed to take the Larmor term present in the Landau-Lifshitz equation as $-\mu_0\gamma\,\boldsymbol{M}\times\boldsymbol{H}^*$ [42]. The continuity equation for the magnetization derived in Section 2 (see Eq.(1)), which takes into account only relaxation effects, is generalized to include the reversible Larmor term and takes the form [43]

$$\frac{\partial \boldsymbol{M}}{\partial t} + \nabla\cdot\boldsymbol{j}_{\boldsymbol{M}} = -\mu_0\gamma\,\boldsymbol{M}\times\boldsymbol{H}^* + \frac{\boldsymbol{H}^*}{\tau_M}. \tag{6}$$

The left-hand side of Eq.(6) describes the time variation of the magnetization and the divergence of the magnetic moment current $\boldsymbol{j}_{\boldsymbol{M}}$, while at the right-hand side there are the conservative torque and the damping-like term. It is worth to notice that the Larmor term $-\mu_0\gamma\,\boldsymbol{M}\times\boldsymbol{H}^*$, present in Eq.(6), expresses the presence of a reversible exchange of magnetic moment between the magnetic subsystem (M) and the reservoir (R, lattice, applied field, etc.), while the damping-like term, $\boldsymbol{H}^*/\tau_M$, describes the changes of magnetic moment related to the irreversible processes. Because of the presence of the Larmor term, the thermodynamic effective field $\boldsymbol{H}^*$ can be different from zero either because the magnetization is dynamically driven out of the equilibrium state or because the boundary conditions with a side layer are forcing the presence of a non zero magnetic moment current.

It is interesting to project Eq.(6) along different directions. We can choose z as the direction where the symmetry is broken either because of a large applied magnetic field or because of the magnetic moment current injected by a side layer. A particularly simple case is obtained by assuming to deal with an isotropic system, i.e. where $\boldsymbol{H}_{eq} = H_{eq}(M)\boldsymbol{m}$. With the latter condition the projection along z reads

$$\frac{\partial M_z}{\partial t} + \nabla\cdot\boldsymbol{j}_{M_z} = -\mu_0\gamma\;\boldsymbol{M}\times\boldsymbol{H}|_z + \frac{H_z^*}{\tau_M} \tag{7}$$

One can also choose to project Eq.(6) onto the plane perpendicular to the magnetization. In the second case we find the Landau-Lifshitz type equation

$$\frac{\partial \boldsymbol{m}}{\partial t} = -\mu_0\gamma\,\boldsymbol{m}\times[\boldsymbol{H} + \alpha\boldsymbol{m}\times(\boldsymbol{H}^* - \tau_M\nabla\cdot\boldsymbol{j}_{\boldsymbol{M}})] \tag{8}$$

where $\alpha = (\mu_0\gamma M\tau_M)^{-1}$ is the dimensionless damping constant.

A complete solution of the problem would require to jointly solve Eqs.(7) and (8). Here we test the model by first solving the magnetic moment current problem along the specific direction imposed by the layer responsible for the spin Hall effect (i.e. Eq.(7)) and then by considering the dynamics in the ferromagnet (i.e. Eq.(8)).

4. Spin pumping and spin Hall torque effects

In this section we apply the thermodynamic theory derived in Sections 2 and 3 to explore the reciprocity between the spin Hall torque and the spin pumping effects, experimentally found in bilayers made of an insulating ferrimagnet (i.e. YIG) and a non magnetic metal with a large spin Hall effect (i.e. Pt). In spin pumping, the YIG, driven at the ferromagnetic resonance by a RF magnetic field, is able to generate a spin polarized current in Pt and its magnetic damping results to be larger than the one without the Pt layer. In spin Hall torque a magnetic moment current, generated by the spin Hall effect of Pt, causes spontaneous self oscillations of the magnetization in the ferromagnet when the electric current flowing in Pt overcomes a certain threshold [8, 21].

4.1. Spin pumping

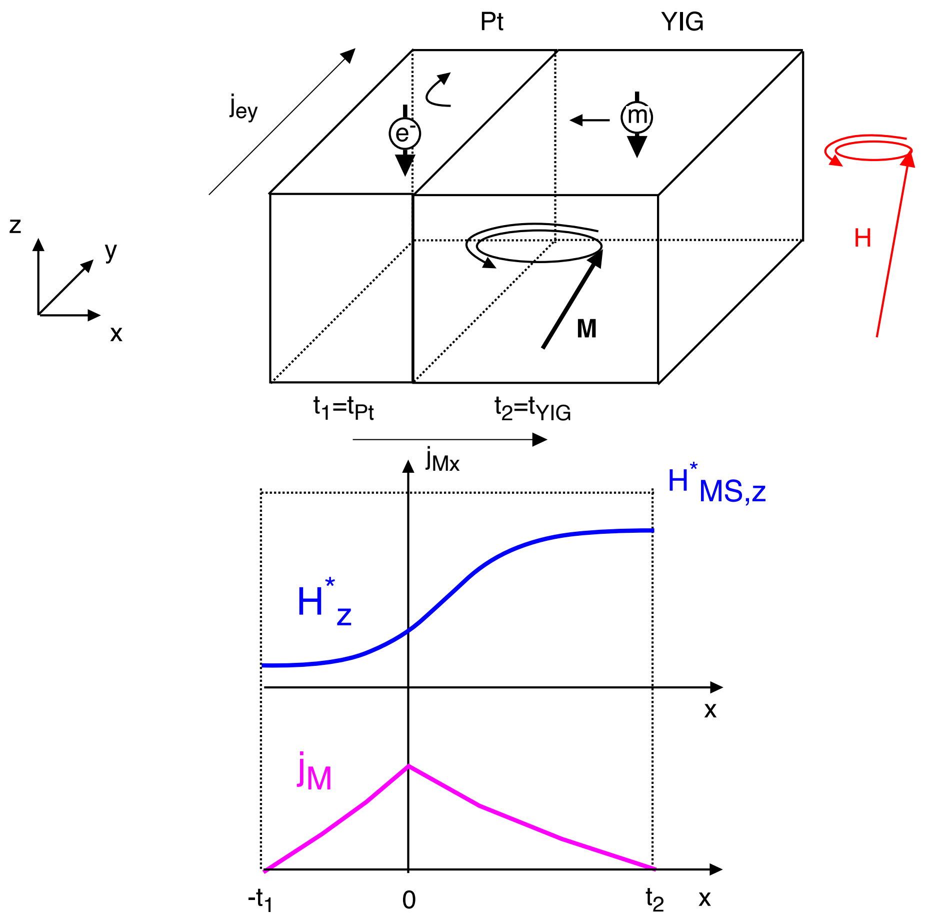

Figure 1. Spin pumping effect. Top: geometry of the bilayer composed by a spin Hall metal (Pt) and an insulating ferromagnet (YIG). The main field and the magnetic moment are oriented along z, the direction of the magnetic moment current is along x and the electric effects are along y. The exciting RF field is perpendicular to z. Bottom: sketch of the solution of the conduction problem (Appendix B.1) for the potential $H_z^*(x)$ and for the current $j_M(x)$. $H_{MS,z}^*$ is the source potential due to the RF driving magnetic field of Eq.(9).

The YIG/Pt bilayer under investigation is shown in Figure 1. The external RF magnetic field pushes the system in an out-of-equilibrium dynamic steady state which corresponds to the appearance of a non zero thermodynamic potential $\boldsymbol{H}^*$ along z. As any gradient of H_z^* is also the driving force of the magnetic moment current (see Section 2), the presence of an absorbing Pt layer allows to absorb some of this current. In the chosen geometry, such a current will be directed along x and it will carry a magnetic moment pointing along z. Therefore, we are allowed to take the projection of Eq. (6) along z and, in stationary conditions ($\partial M_z/\partial t = 0$), we obtain the diffusion equation:

$$H^*_{MS,z} = H^*_z - l_M^2 \nabla^2 H^*_z \tag{9}$$

where the term $H^*_{MS,z} = \alpha^{-1}\, \boldsymbol{m} \times \boldsymbol{H}|_z$ is the source potential due to the RF driving magnetic field. The amount of current which is absorbed by Pt depends on the geometrical conditions of the space where the flow takes place, on the thicknesses and on the conducting and absorbing properties of both materials. To address the problem, we independently solve Eq.(9) and the analogous diffusion equation for Pt (see Ref.[6]) and we set the boundary conditions. The final solution (see Appendix B.1 for the one dimensional case) provides the space profiles of the potential $H^*_z(x)$ and of the current $j_M(x)$. As part of the magnetic moment is absorbed by Pt, this circumstance corresponds to an enhancement of the damping. To see this fact we take the vector equation (6) and we include all dissipations in the form of an effective local damping

$$\alpha\left(\boldsymbol{H}^* - \tau_M \nabla \cdot \boldsymbol{j_M}\right) \simeq \alpha' < \boldsymbol{H}^* > \tag{10}$$

where $\alpha' = (\mu_0 \gamma M \tau'_M)^{-1}$, τ'_M being an effective time constant describing the effects of the side layers. In the particularly simple case of a one dimensional current flow with an active YIG layer of thickness t_{YIG} and an absorbing Pt layer of thickness t_{Pt} (see Appendix B.1), the effective time constant is found to be

$$\tau'_M = \tau_M \left(1 - \frac{v_{eff}}{v_{YIG}(t_{YIG}/l_{YIG})}\right). \tag{11}$$

The effective magnetic moment conductance v_{eff} of the bilayer appearing in Eq. (11) is given by (see also Eq. (B.6))

$$\frac{1}{v_{eff}} = \frac{1}{v_{Pt}\tanh(t_{Pt}/l_{Pt})} + \frac{1}{v_{YIG}\tanh(t_{YIG}/l_{YIG})} \tag{12}$$

It should be noted that the total conductance is the series of the contributions of YIG and Pt and that each layer contributes with its intrinsic and material dependent conductance $v_M = l_M/\tau_M$, given by the ratio between the diffusion length l_M and the time constant τ_M, and with the ratio between the layer thickness and l_M. For the enhanced damping constant, in the case of thin YIG, i.e. $t_{YIG} \ll l_{YIG}$, we obtain

$$\alpha' - \alpha = \frac{v_{Pt}\tanh(t_{Pt}/l_{Pt})}{\mu_0 \gamma_L M t_{YIG}}. \tag{13}$$

This expression is formally identical to the one for the spin mixing conductance reported in the literature, however in our thermodynamic case it expresses explicitly the dependence on the conductance of Pt, v_{Pt}, and on the ratio t_{Pt}/l_{Pt}. The hyperbolic tangent dependence, present on Eq.(13), implies a saturation of the effect which is indeed found in experiments of spin pumping as a function of the thickness of Pt [14]. Finally we can compute the inverse spin Hall effect in Pt. In order to do this we have to first compute the $H^*_{MS,z}$ source potential by solving the precession equation with enhanced damping. This can be done by taking Eq.(6), using the approximation of Eq. (10) and finally taking the projection of the result in the direction perpendicular to the magnetization. We obtain the Landau-Lifshitz type equation

$$\frac{d\boldsymbol{m}}{dt} = -\mu_0 \gamma_L\, \boldsymbol{m} \times (\boldsymbol{H} + \alpha' \boldsymbol{m} \times \boldsymbol{H}) \tag{14}$$

with the enhanced damping constant of Eq.(13). Eq.(14) can be solved under RF excitation by standard methods [40, 41, 33]. The inverse spin Hall effect in Pt voltage can be immediately computed (see Appendix B.1) and it results:

$$\nabla_y V_e = \frac{\theta_{SH}}{\sigma_e}\left(\frac{e}{\mu_B}\right)\frac{v_{eff} H^*_{MS,z}}{(t_{Pt}/l_{Pt})\coth(t_{Pt}/(2l_{Pt}))}. \tag{15}$$

This expression has the same Pt thickness dependence which is found in the literature [10, 11, 15, 19] while the source potential, given by $H^*_{MS,z} = (\alpha'/\alpha)\, \boldsymbol{m} \times \boldsymbol{H}|_z$, is computed from the solution of Eq.(14). By neglecting demagnetizing effects one finds that, at the resonance frequency, $H^*_{MS,z} = H_\perp^2/(\alpha H_z)$.

4.2. Spin Hall torque

Spin Hall torque effects are realized in Pt/YIG systems in which the Pt layer is the spin Hall conductor able to inject a magnetic moment current tensor into YIG (see the scheme of the YIG/Pt bilayer in Fig.2). A constant magnetic field H_z is applied to YIG along the z direction. As a result of the magnetic moment current injected by the Pt layer, a potential H_z^* is generated into the YIG layer by absorption of the current. In this case, to address the problem, we need to solve the transport equation for magnetic moments pointing along z. The solution for the one dimensional case is given in Appendix B.2. In the present paper we limit ourselves to observe what happens if we plug the solution obtained, $\tau_M \nabla \cdot \boldsymbol{j}_{\boldsymbol{M}} = \boldsymbol{H}^*_{SH}$, into Eq.(8). We find:

$$\frac{d\boldsymbol{m}}{dt} = -\mu_0 \gamma_L\, \boldsymbol{m} \times \left[\boldsymbol{H} + \alpha \boldsymbol{m} \times (\boldsymbol{H} - \boldsymbol{H}^*_{SH})\right]. \tag{16}$$

The spin Hall torque term, $\boldsymbol{H}^*_{SH} = H^*_{SH}(x)\mathbf{e}_z$, where $H^*_{SH}(x)$ is given by Eqs.(B.16) and (B.17), is directed along z. Therefore the spin Hall torque term can act either as a damping or an anti-damping contribution depending on whether the magnetic moment injected into YIG is parallel or antiparallel to the magnetization. By taking the approximation of thin YIG thickness, i.e. $t_{YIG} \ll l_{YIG}$, we find that the spin Hall torque field, at zero order in the t_{YIG}/l_{YIG} expansion, is constant though the layer and is equal to

$$H^*_{SH} = -\frac{j_{MS}}{v_{Pt}\coth(t_{Pt}/(2l_{Pt}))}. \tag{17}$$

As the magnetic moment current source of Pt is related to the electric current along y, $j_{MS} = -(\mu_B/e)\theta_{SH} j_{ey}$ (where μ_B is the Bohr magneton, e is the elementary charge and θ_{SH} is the spin Hall angle) from Eq.(17) we obtain a condition for the compensation of the damping expressed as a critical electric current. The same result has been found in the experiments performed on Pt/YIG bilayers reported in Refs.[8, 21]. At currents above the threshold the system is pushed in a far-from-equilibrium state where the approximation of linear systems on which Eqs. (1) and (4) are based, may not be valid anymore. If there are non linear terms in the dependence between the generalized velocity, $\partial \boldsymbol{M}/\partial t$, and the generalized force, $\boldsymbol{H}^*$, then the concepts of linear thermodynamics do not apply anymore. Glansdorff and Prigogine have shown that around a far-from-equilibrium state the behavior of a thermodynamic system is highly influenced by the non-linear terms and it may give rise to spatially organized dynamic patterns called dissipative structures [44, 45].

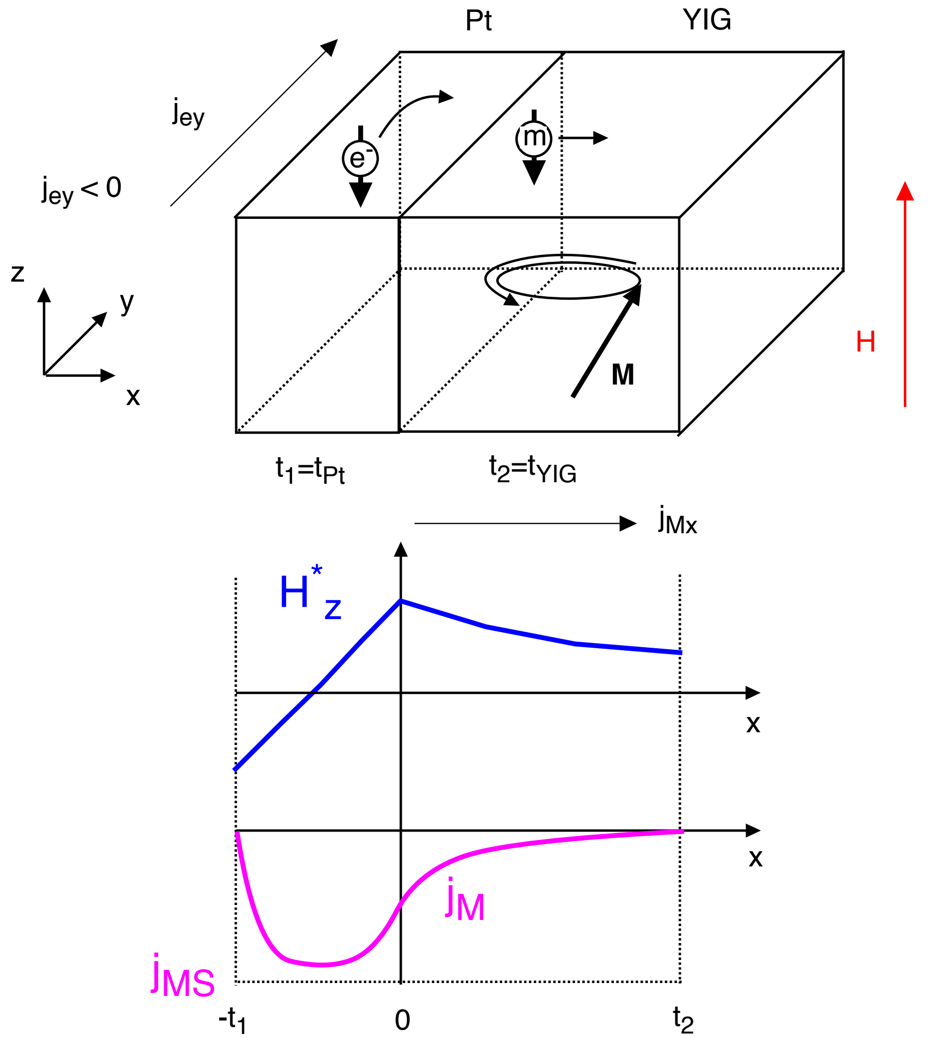

Figure 2. Spin Hall torque effect Top: geometry of the bilayer composed by a spin Hall metal (Pt) and an insulating ferromagnet (YIG). The main field and the magnetic moment are oriented along z, the direction of the magnetic moment current is along x and the electric effects are along y. Bottom: sketch of the solution of the conduction problem (Appendix B.2) for the potential $H_z^*(x)$ and for the current $j_M(x)$. j_{MS} is the magnetic moment source in Pt due to the spin Hall effect.

5. Discussion and conclusions

The reciprocity underlaying the spin pumping and spin Hall torque effects can be tested by using the expression for the enhancement of the damping (Eq.(13) and the equation for the critical current threshold above which self oscillations start to appear. Indeed, in both cases the only unknown parameter of the equations is the conductance of Pt, v_{Pt}. In Ref.[21] both experiments have been performed with the same device, so that the results are amenable to a detailed verification. Experiments were conducted on thin YIG disks (20 nm in thickness, 2μm and 4μm in diameters).

Spin pumping experiments report a damping enhancement of $\alpha' - \alpha \simeq 15 \cdot 10^{-4}$. By taking the saturation magnetization of the YIG film as $\mu_0 M_s = 0.215$ T and the Pt layer thickness as $t_{Pt} = 8$ nm, the only two remaining free parameters are the diffusion length of Pt, l_{Pt}, and its conductance, v_{Pt}. Spin Hall torque experiments were conducted by an applying a magnetic field, $H_{a,z}$, along a diameter, so that the total magnetic field $\boldsymbol{H}$ is the sum of the applied and of the demagnetizing fields. It can be assumed the demagnetizing coefficien to be t $N_{d,x} \simeq 1$, so that the condition for the compensation of the damping is expressed as $H^*_{SH} = (H_{a,z} + (1/2)M_s)C_0$, where $C_0 = 1 + (\Delta H_0/\alpha')/\sqrt{H_{a,z}(H_{a,z} + M_s)}$ is a correction factor related to the presence of an inhomogeneous contribution to the linewidth ΔH_0 [21]. The threshold current for the appearance of self oscillations in the ferromagnet, in this approximation, can be predicted by using the spin diffusion parameters of Pt only and it results to be $j_{ey} = (e/(\mu_B \theta_{SH}))v_{Pt} \coth(t_{Pt}/(2l_{Pt}))H^*_{SH}$. By taking $\theta_{SH} = -0.056$ from Ref.[21] (the spin Hall angle for the magnetic moment is opposite with respect to that of the spin), the only free adjustable parameters are again l_{Pt} and v_{Pt}. By using the experimental data from Ref.[21] we find that they evaluate to $l_{Pt} \simeq 10$ nm and $v_{Pt} \simeq 2$ m/s. These results provide a diffusion length slightly larger than the value determined in previous measurements ($l_{Pt} = 7.3$ nm from [15]) and a conductance which is slightly lower than those found from the spin Seebeck experiments ($v_{Pt} = 3$ m/s from Ref.[6]).
In conclusion, in this paper we have extended the non-equilibrium thermodynamics of Johnson and Silsbee [32] to describe vector magnetization. The resulting model is able to jointly describe the magnetization dynamics and the magnetic moment transport between different layers. The application of the model to the spin pumping and spin Hall torque effects arising in YIG/Pt bilayers has shown that the two effects occur because of diffusion processes allowing for the transport of currents between the two layers. In the case of thin YIG the two effects are related by two parameters only: the diffusion length of Pt, l_{Pt}, and the conductance of Pt, v_{Pt}. Both these parameters can be determined by comparison with experimental data. Future efforts will be devoted to derive the details of the magnetization dynamics beyond the instability threshold by taking into account possible non linear effects in the framework of the Glansdorff and Prigogine non linear thermodynamics [44, 45].

Appendix A. The thermodynamic potential $\boldsymbol{H}^*$ for a ferromagnet

A central role in the energetics of ferromagnets is played by the effective field which is defined as the derivative of the enthalpy of the magnetic system. In the thermodynamic theory of Section 2 the magnetization $\boldsymbol{M}$ is defined as the result of a statistical average over a volume Δv^* small enough to permit the description of the system as a continuum but large enough to allow for all the thermal effects, including thermal spin waves, to be averaged out. If we assume small deviations from a given direction (i.e. we aim to study the uniform mode and the magnetostatic modes) we can express the enthalpy as

$$u_{e,L} = u_{e,T}(s, \boldsymbol{M}) + K_{AN} f_{AN}(\boldsymbol{m}) - \frac{1}{2}\mu_0 \boldsymbol{H}_M \cdot \boldsymbol{M} - \mu_0 \boldsymbol{H}_a \cdot \boldsymbol{M} \quad \text{(A.1)}$$

where $\boldsymbol{m} = \boldsymbol{M}/|\boldsymbol{M}|$ is the magnetization unit vector and the terms at the right hand side represent the thermodynamic, anisotropy, magnetostatic, and applied field term, respectively. $u_{e,T}(s, \boldsymbol{M})$ is the thermodynamic energy term. The dimensionless function $f_{AN}(\boldsymbol{m})$ is an appropriately normalized function describing the angular dependence of local anisotropy effects and K_{AN} is the anisotropy constant. The vector $\boldsymbol{H}_a$ is the applied field whereas $\boldsymbol{H}_M$ is the magnetostatic field, solution of magnetostatic Maxwell equations:

$$\nabla \cdot \boldsymbol{H}_M = -\nabla \cdot \boldsymbol{M} \tag{A.2}$$

$$\nabla \times \boldsymbol{H}_M = 0 \tag{A.3}$$

As the Landau enthalpy $u_{e,L}$ does not contains the derivatives of the magnetization, the effective field is given by the simple derivative with respect to the magnetization vector $\boldsymbol{M}$ and it is found to be $\mu_0 \boldsymbol{H}^* = -\delta u_{e,L}/\delta \boldsymbol{M}$ with $\boldsymbol{H}^* = \boldsymbol{H} - \boldsymbol{H}_{eq}$ (i.e. equation 2) where $\boldsymbol{H} = \boldsymbol{H}_M + \boldsymbol{H}_a$ and $\boldsymbol{H}_{eq} = (1/\mu_0)\partial u_{e,T}(s, \boldsymbol{M})/\partial \boldsymbol{M} - \boldsymbol{H}_{AN}$ is the equation of state at equilibrium with the anisotropy field being $\boldsymbol{H}_{AN} = H_{AN}\, \partial f_{AN}/\partial \boldsymbol{m}$ and $H_{AN} = 2K_{AN}/(\mu_0 M)$. In micromagnetics [24], the effective field is defined as the functional derivative of

$$u_{e,L} = u_{e,M}(s, M) + A\,(\nabla \boldsymbol{m})^2 + K_{AN} f_{AN}(\boldsymbol{m}) - \frac{1}{2}\mu_0 \boldsymbol{H}_M \cdot \boldsymbol{M} - \mu_0 \boldsymbol{H}_a \cdot \boldsymbol{M} \tag{A.4}$$

where the first two terms at the right hand side of equation(A.4) differs from the first terms at the right hand side of equation(A.1). The two terms are the thermodynamic and exchange terms. As the magnetization vector field $\boldsymbol{M}$ is the result of a statistical average over a volume Δv_M much smaller than Δv^*, the energy $u_{e,M}(s, M)$ is characterized by a minimum at $M = M_s$ so deep that one can safely assume the magnetization modulus to be the constant value M_s. In the exchange term, $(\nabla \boldsymbol{m})^2$ is a short-hand notation for $|\nabla m_x|^2 + |\nabla m_y|^2 + |\nabla m_z|^2$ and A is the exchange stiffness constant. Therefore from the definition $\mu_0 \boldsymbol{H}_{eff} = -\delta u_{e,L}(s, \boldsymbol{M})/\delta \boldsymbol{M}$ we have

$$\boldsymbol{H}_{eff} = l_{EX}^2 \nabla^2 \boldsymbol{M} - \boldsymbol{H}_{AN} + \boldsymbol{H}_M + \boldsymbol{H}_a \tag{A.5}$$

where the length parameter l_{EX} is the so-called exchange length $l_{EX} = \sqrt{2A/(\mu_0 M_s^2)}$. It is worth to observe that the potential $\boldsymbol{H}^*$ appears as a coarse grained version of the micromagnetic effective field $\boldsymbol{H}_{eff}$.

Appendix B. Solutions of the diffusion equation in one dimension

Appendix B.1. Spin pumping

In spin pumping, material 1 is a non magnetic metal (from $x = -t_1$ to $x = 0$) while material 2 is a ferromagnet (from $x = 0$ to $x = t_2$), see Figure 1. We solve the diffusion equations with the following boundary conditions (we drop the subscript M for simplicity) $j_1(-t_1) = j_2(t_2) = 0$, $j_1(0) = j_2(0) = j_0$. The equation for $H_2^*(x)$ is $H_2^* - l_2^2 \nabla_x^2 H_2^* = H_{MS,2}^*$ where $H_{MS,2}^*$ is a parameter. The boundary condition for the potential at $x = 0$ is $H_2^*(0) = H_1^*(0)$ which corresponds to a transparent interface. The currents are

$$j_1(x) = j_0 \frac{\sinh((x + t_1)/l_1)}{\sinh(t_1/l_1)} \tag{B.1}$$

$$j_2(x) = j_0 \frac{\sinh((x - t_2)/l_2)}{\sinh(t_2/l_2)} \tag{B.2}$$

and the potentials are

$$H_1^*(x) = \frac{j_0}{v_1}\frac{\cosh((x+t_1)/l_1)}{\sinh(t_1/l_1)} \tag{B.3}$$

$$H_2^*(x) = H_{MS,2}^* - \frac{j_0}{v_2}\frac{\cosh((x-t_2)/l_2)}{\sinh(t_2/l_2)} \tag{B.4}$$

where the parameter $v_i = l_i/\tau_i$, with the dimensions of a velocity, represents the conductance of each layer. The current at the interface is

$$j_0 = v_{eff} H_{MS,2}^* \tag{B.5}$$

where

$$\frac{1}{v_{eff}} = \frac{1}{v_1 \tanh(t_1/l_1)} + \frac{1}{v_2 \tanh(t_2/l_2)} \tag{B.6}$$

is the total magnetic moment conductance of the bilayer. The average value of the field $H_2^*(x)$ over the layer is

$$< H_2^*(x) >= H_{MS,2}^* \left(1 - \frac{v_{eff}}{v_2(t_2/l_2)}\right) \tag{B.7}$$

The Inverse spin Hall effect in Pt (1) can be computed by taking the equation for the spin Hall effect relation the electric current along y and the magnetic moment current tensor along x [6]

$$j_{ey}(x) = -\sigma_e \nabla_y V_e + \theta_{SH}\left(\frac{e}{\mu_B}\right) j_1(x) \tag{B.8}$$

As the thickness is small we consider the potential uniform and, for an open circuit measurement, we ask that integral, over the layer 1, of the electric current density $j_{ey}(x)$ to be zero. Then we find

$$\nabla_y V_e = \frac{\theta_{SH}}{\sigma_e}\left(\frac{e}{\mu_B}\right) < j_1 > \tag{B.9}$$

where $< j_1 >= t_1^{-1}\int_{-t_1}^{0} j_1(x)dx$. From the magnetic moment current, equation (B.1) we find the gradient of the voltage as

$$\nabla_y V_e = \frac{\theta_{SH}}{\sigma_e}\left(\frac{e}{\mu_B}\right) \frac{v_{eff} H_{MS,2}^*}{(t_1/l_1)\coth(t_1/(2l_1))} \tag{B.10}$$

Appendix B.2. Spin Hall torque

In spin Hall torque (see Figure 2) an electric current density j_{ey} is driven into the metal (1) along y, so that the magnetic moment current induced by the spin Hall effect results to be along x and the direction of the magnetic moment along z. The intensity of the current source is $j_{MS} = -(\mu_B/e)\theta_{SH} j_{ey}$ where μ_B is the Bohr magneton, e is the elementary charge. θ_{SH} is the spin Hall angle that quantifies the efficiency of the conversion between charge current and magnetic moment current due to the spin Hall effect. With boundary conditions for the magnetic moment currents $j_1(-t_1) = j_2(t_2) = 0$, $j_1(0) = j_2(0) = j_0$ we obtain the following expressions for the currents

$$j_1(x) = j_{MS}\left[1 + \frac{\sinh(x/l_1)}{\sinh(t_1/l_1)}\right] + (j_0 - j_{MS})\frac{\sinh[(x+t_1)/l_1]}{\sinh(t_1/l_1)} \quad \text{(B.11)}$$

$$j_2(x) = -j_0\frac{\sinh[(x-t_2)/l_2]}{\sinh(t_2/l_2)} \quad \text{(B.12)}$$

and the corresponding expressions for the potentials

$$H_1^*(x) = \frac{j_{MS}}{v_1}\frac{\cosh(x/l_1)}{\sinh(t_1/l_1)} + \frac{j_0 - j_{MS}}{v_1}\frac{\cosh[(x+t_1)/l_1]}{\sinh(t_1/l_1)} \quad \text{(B.13)}$$

$$H_2^*(x) = -\frac{j_0}{v_2}\frac{\cosh[(x-t_2)/l_2]}{\sinh(t_2/l_2)} \quad \text{(B.14)}$$

By using the boundary condition for the potenatials at $x = 0$, $H_1^*(0) = H_2^*(0)$, corresponding to a negligible interface resistance, we obtain the value of the current at the interface j_0 as

$$j_0 = j_{MS}\frac{v_{eff}}{v_1 \coth(t_1/(2l_1))} \quad \text{(B.15)}$$

where v_{eff}, given by equation(B.6). The sought potential in the ferromagnet (2) is therefore

$$H_2^*(x) = H_{2,0}^*\frac{\cosh((x-t_2)/l_2)}{\cosh(t_2/l_2)} \quad \text{(B.16)}$$

where

$$H_{2,0}^* = -\frac{j_{MS}}{v_1 \coth(t_1/(2l_1))}\frac{v_{eff}}{v_2 \tanh(t_2/l_2)} \quad \text{(B.17)}$$

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