A Simple Electrostatic Balance for the Milligram Range

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A simple electrostatic balance for the milligram range

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***Abstract*— We propose a method to realize the kilogram through electrical units, namely the volt. The device is based on a simple plane plate electrostatic actuator used to counterbalance the weight force. The geometry of the electric field is characterized by changing the distance between the plates while maintaining a constant force and measuring the capacitance. This approach leads to the realization of a simple and compact device for the absolute measurement of small masses. The preliminary results show a relative uncertainty of 10^{-3} for mass above 10 mg. A series of uncertainty sources has been pointed out. An improvement of one order of magnitude is expected by correcting these sources.**

***Index Terms*— Definition of the kilogram, electrostatic balance, voltage balance.**

I. Introduction

THE framework for the new definition of the kilogram has been established [1]; it will be based on the Planck's fundamental constant [2] and will overcome the present definition based on the unique artefact standard. The proposed future realizations are based on the Kibble (or watt) balance, where the kilogram is compared with a calculable electromagnetic force, and the silicon sphere, where the kilogram is made from a silicon crystal containing a known number of atoms. Both have their own pros and cons. The first has the advantage of connecting the mass unit to electrical units (namely the volt and the ohm) already realized with quantum devices based on physical constants. On the other hand, the device is extremely complex and few devices in the world have been realized so far [3]. The second has the advantage of being a mass standard itself based on the Avogadro constant and on the lattice constant of silicon, but its realization is quite critical and it suffers from surface effects that must be carefully monitored [4].

A third possible method is the use of an electrostatic force rather than an electromagnetic force i.e. using the attractive force exerted by the electric field between two charged plates rather than the Lorentz's force occurring in a coil immersed in a magnetic field.

Although the first instrument to measure the electrostatic force is the famous Coulomb's torsion balance of 1777, probably the first realization of an electrostatic balance is due to Snow Harris [5] in 1838 where he wanted to treat the novel Faraday's "fine discoveries" with a metrological attitude. However, the first "modern" realization of an electrostatic balance is due to W. Thomson (Lord Kelvin) in 1868 [6]. Since then until recent years, the Thomson's (or Kelvin's) balance, which included a guard ring to reduce the edge effects, has been used to determine the value of the volt. For this reason, it is also referred to as the voltage balance [7,8, 9,10]. Today, with the volt defined through the Josephson effect, the voltage balance is indeed used to link mechanical forces with fundamental constants as it happens in the Kibble balance. Although at a first glance the voltage balance could look simpler, there are two important limitations with respect to the Kibble balance. The first is that in order to generate forces of the order of a kilogram (≈ 9.8 N) we have to apply very high voltages to the capacitor plates. As an example, for a capacitor made of two plates with radius r = 25 mm placed at a distance d = 0.1 mm, in order to achieve a force of 1 kg it is necessary to apply a voltage equal to 3200 V. The second is that it is very difficult to know the absolute value of the electrostatic force with the required accuracy. Indeed, there are many variables like the conductibility of the metal, the geometry of the surfaces the edge effects and electrostatic patches which influence the shape of the electric field so that, even if the voltage can be measured with high accuracy, the capacitance, hence the force cannot.

FIGURE 1

Nevertheless, electrostatic force has been used in different experimental devices to generate a "known" force in torsion balances or pendulum [11], to determine the Planck's constant [12] and, in latest years, to weight small mass standards [13, 14]. In particular, both the PTB [12] and the NIST [13] voltage balances make use of a cylindrical capacitor as the electrostatic actuator. Being the force equal to the derivative of the system energy, in the case of a capacitor the weight force mg is:

$$mg\left(1-\frac{\rho_{air}}{\rho_m}\right)=\frac{1}{2}\frac{\partial C}{\partial z}V^2 \quad (1)$$

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where ρ_{air} and ρ_m are the densities of air and of the mass standard respectively, C is the capacitance, z is the vertical axis and V is the voltage applied to one of the electrodes being the other connected to ground.

At the level of the kilogram, the electrostatic approach is not competitive with the Kibble balance for the reasons above mentioned, but can be competitive for small masses as low as 1 mg (where at the present state of art the best relative uncertainty obtained with classical weighing methods is of the order of 1×10^{-4}) and can extend the weighing needs with a significant improvement of the uncertainty towards even smaller masses. In this range, technological challenges involved in an absolute determination based on electrostatic force are expected to be less demanding than at the kilogram level.

So far, among the capacitor geometries, the cylindrical one allowed to obtain the best results compared to the parallel plates solution because of the more uniform behavior with displacement along the z-axis. On the other side, parallel plates capacitor allows to obtain larger forces at lower voltages in a simpler and more compact geometry.

In the present work, we propose the realization of a voltage balance based on a parallel plates capacitor where the beam of the balance is kept fixed while the distance between the plates is changed by a controlled amount in order to accurately determine the actuator parameters.

Preliminary tests have been carried out in the range from 5 mg to 20 mg, and experimental data have been compared to the results obtained by the finite element method (FEM) analysis.

The structure of this paper is as follow. In section II we introduce the measurement procedure and the main quantities of influence. Section III describes the measurement system. In section IV the computation of the capacitance by FEM and the model for the capacitor are given. The results are shown in section V. Conclusions and an outlook for the future improvements are given in section VI.

II. Measurement procedure

The measurement procedure can be described in four steps, a schematic representation is given in Fig. 2:

1) determination of the capacitance *vs* distance function over a specific range centered about at a reference position d_0, with the balance unloaded (null load reading L_0);
2) determination of the capacitance gradient at distance d_0
3) loading of the mass standard m to be calibrated;
4) compensation of the deadweight force F_m applying the voltage V to the electrodes, so that the reading of the balance L is again equilibrated at null load L_0.

A sensitive bridge circuit measures the capacitance, and a linear actuator controls the relative distance between the electrodes. In the present work, the distance of the two electrodes was from about 50 µm to 100 µm, and the relevant capacitance between 40 pF and 80 pF. From the function $C=f(d)$, the gradient, or local slope, is determined.

FIGURE 2

Comparing this approach against the cylindrical coaxial electrodes, the main advantage is to obtain higher capacitance and capacitance gradient, in addition, the off-axis problem due to the motion of the electrodes is less significant, because of the very short displacement of the electrodes. However, with the cylindrical electrodes, the capacitance is in principle a linear function of the distance, whereas with plate electrodes the real function is approximately hyperbolic.

For this reason, the fitting curve should be accurately determined. In this work the model of the curve fitting has been selected accordingly to the results evaluated by the FEM analysis.

An important effect to be considered is the environmental temperature variation. The temperature dependence of the capacitance is explained by the thermal expansion of the structure of the system. The drift of the capacitance produces also effects on the gradient, for this reason to compensate the effect of linear drift in the measurement, temperature values are recorded for each single measurement.

Finally, it is important to consider that the capacitance *vs* distance function must be evaluated with the balance unloaded because of the hysteresis of the balance pan suspension. Indeed, we have observed that the feedback control loop cannot maintain zero deflection of the suspension when the deadweight is loaded and unloaded from the balance, as described in detail in section V.

III. Practical realization

The practical realization is based on a Mettler Toledo mod. UMX5 (range 5 g, resolution 0.1 µg) as the unbalance sensor and on two 25 mm diameter aluminum parallel plates as the electrodes. The pan of the balance has been modified by adhering on it the bottom plate electrode. The upper electrode is fixed to a piezoelectric actuator, PI-P753.2 (displacement range 25 µm, accuracy 2 nm), which allows a fine adjustment of the gap between the two electrodes. The actuator is calibrated with an uncertainty less than 1 nm through the INRIM interferometric facility made to the purpose. The two electrodes are both isolated by a Kapton film from the balance and from the piezo actuator, which is mounted on a tilting system, based on fine pitch INVAR screws, to allow the alignment of the upper electrode with respect to the pan balance. The procedure used to find the parallelism between the electrodes consists in acting on the screws to reduce the distance while finding the maximum capacitance value before the two electrodes come into contact. This guarantees a parallelism better than an arcminute. The piezoactuator itself has a straightness of few arcseconds measured with an autocollimator. The two electrodes are connected with fine wires of 0.1 mm diameter to the AH-2500A 1 kHz capacitance bridge. The voltage is applied to the electrodes by a DC voltage generator and measured by a Agilent 3458A voltmeter. Both the voltmeter and the capacitance bridge are traceable to the respective electrical standards at INRIM. An automatic system is used for loading and unloading the mass standard.

Fig. 3 shows a schematic representation of the measurement system and Fig. 4 a picture of the whole experiment.

FIGURE 3

FIGURE 4

IV. FEM ANALYSIS

We propose the realization of a preliminary physical model of the voltage balance and its numerical solution by the finite element method (FEM), in order to evaluate the actual dependence between capacitance and electrode separation distance.

The computation of the capacitance by FEM allows taking into account the edge effect of the capacitor, reproducing the real configuration of the system. Then it is possible to evaluate an appropriate expression for the capacitance, which should be used to fit the measurement data.

The computation of the capacitance is carried out by solving the Laplace equation for the electrostatic potential φ in axisymmetric cylindrical coordinates, considering Dirichlet and Neumann boundary conditions.

$$\begin{cases} \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{\partial^2\varphi}{\partial z^2} = 0 \\ \varphi = f \qquad \text{on } \Omega_D \\ \boldsymbol{\nabla}\varphi = \boldsymbol{g} \qquad \text{on } \Omega_N \end{cases} \tag{2}$$

where r and z are, respectively, the radial and axial coordinates, Ω_D and Ω_N are, respectively, the parts of the boundary with Dirichlet and Neumann boundary conditions, f and $\boldsymbol{g}$ are the functions which represent the boundary conditions.

The capacitance C can be calculated from the distribution of the electrostatic potential as:

$$C = -\frac{1}{V}\int_{\Omega} \varepsilon\, \boldsymbol{\nabla}\varphi \cdot \boldsymbol{n}\, d\Omega \tag{3}$$

where V is voltage difference applied to the electrodes, ε is the dielectric constant of air and $\boldsymbol{n}$ is the unit vector normal to the boundary surface Ω. The numerical solution of the Laplace equation for the electrostatic potential has been computed by the FEM software FreeFem++ [15]. The axisymmetric physical domain of the voltage balance is sketched in Fig. 5.

FIGURE 5

The thickness of the two parallel plate electrodes is 2.5 mm for the bottom electrode and 4 mm for the upper one, being their radius equal to 12.5 mm. The piezo-capacitive actuator has been assumed to be 30 mm height and 10.5 mm radius, while the stem of the balance pan has been considered as 30 mm height and 2 mm radius.

The axisymmetric computational domain has been defined setting the boundary conditions as shown in Fig. 6; the overall dimension of the domain, which has been considered for the FEM analysis, is that of the border of the system connected to ground.

FIGURE 6

Fig. 7 shows an example of the results for the distribution of the electrostatic potential.

FIGURE 7

The numerical model has been solved and the capacitance has been computed for different distances between the parallel plate electrodes, in order to evaluate an appropriate expression for the capacitance, whose mathematical form should be used to fit the measurement data.

In Table I, the capacitances computed by FEM, for different electrode separation distances, are listed and compared with the corresponding values of the ideal capacitances C_{id}, defined as:

$$C_{id} = \frac{\varepsilon\, A}{d} \tag{4}$$

where A is the base surface of the plate electrodes.

TABLE I

From Table I, it can be observed that the actual capacitance is higher than the ideal one because of the edge effect and the higher the electrodes separation distance (at constant radius of the plate electrodes), the greater the ratio δ between actual and ideal capacitance.

The ratio δ computed by FEM has been compared against the one calculated by the model described in [16] for values of the ratio between distance and radius of the parallel plate capacitor lower than 0.1 and deviations within 3% have been obtained. Such deviations can be due to the different configuration of the disk capacitors, which entails different edge effects.

The regression analysis of the numerical results has allowed deriving the following expression for the capacitance as function of the electrode separation distance:

$$C = \frac{K_1}{d} + K_2 d^{K_3} \tag{5}$$

where K_1, K_2 and K_3 are the fitting coefficients. Fig. 8 shows the regression curve obtained from the numerical results with the values of the fitting coefficients. The residuals of the fit are within 0.02 pF.

FIGURE 8

The gradient of the capacitance has been calculated by FEM and compared against the one calculated from measurement data; the maximum deviation was about 0.1 pF/µm at a capacitance around 80 pF. Such a deviation can be due to both the approximations adopted for the physical modelling of the voltage balance (geometry of the electrodes, identification of the computational domain, boundary conditions, axisymmetric approximation, etc) and the numerical discretization of the problem (size of the mesh, mesh refinement at corners of the electrodes).

V. Preliminary results and discussion

We have used three test masses of 5 mg, 10 mg and 20 mg nominal value. For each mass, we have tested the procedure for different distances between the electrodes ranging from 50 µm to about 100 µm for a capacitance value ranging from 80 pF to about 40 pF. Because of the high sensitivity of the capacitance to temperature changes (relative variation of about 0.05 $°C^{-1}$), the measurement cycle is optimized to minimize the duration and is repeated several times.

The simplest cycle consists in setting the piezo actuator in its central position (elongation 12.5 µm over 25 µm), zeroing the balance and loading the mass to the balance, compensating the balance by applying the proper voltage and finally measuring the capacitance for at least four displacements of the piezo with respect to the central position . A power law function is used to fit the three points and the value of the derivative of the curve is calculated for the central position. This value combined with the recorded voltage gives the estimated value for the mass. The measure is repeated and the average and the standard deviation are calculated.

In a more accurate approach, we evaluate the capacitance gradient just before and after the electrostatic weighing of the mass, in order to compensate the linear drift. For a given set of positions of the piezo actuator, the capacitance values are recorded, so that we obtain pairs of values of distance and capacitance from which the gradient $\partial C/\partial d$ can be calculated.

The capacitance gradient $\partial C/\partial d$ at the weighing position is assessed by the regression analysis of measurement data of capacitance C and relative displacement Δd of the piezo-capacitive actuator with respect to the distance d_0 between the electrodes. According to (5), the fitting model for the capacitance can be expressed as follows:

$$C = \frac{c_1}{d_0 + \Delta d} + c_2 (d_0 + \Delta d)^{c_3} \tag{7}$$

where c_1, c_2, c_3 and d_0 are the values of fitting parameters.

Table II shows an example of the measurements obtained for a mass standard of 10 mg, where the gradient of the capacitance has been calculated from the fitting curve obtained by the regression analysis of the measurement data.

TABLE II

Fig. 9 shows the capacitance measurements (square points, left scale) and the applied voltage to balance the 10 mg mass standard (circular points, right scale), as function of the distance between electrodes (the full range of the piezo actuator divided into 5 equal intervals 5 µm each).

FIGURE 9

In Fig. 10 is reported the capacitance measured as a function of the distance in four different runs. First, with the balance unloaded the distance between the electrodes is divided into 10 intervals. For each position the capacitance has been measured first decreasing the distance down to 77 µm, than increasing it until the initial value of 102 µm. This is repeated with the balance loaded with a 20 mg mass. The yellow curve shows the average capacitance versus the distance (in fF, right scale). The four other curves represent the residuals from the average of the four runs. From this it is evident the effect of thermal drift always occurring in our measurements (difference between back and forth curves) and the difference between loaded and unloaded conditions. From the latter we have estimated a deviation of 135 nm of the plate position from the loaded and unloaded conditions. Both the effects of thermal drifts and the effect of the plate position change have been taken into account in the data analysis.

similarly with the variation of capacitance when the deadweight is loaded and when the voltage V is applied to equilibrate the balance at null load. Typical values of this displacement are between 55 and 135 nm for masses from 5 to 20 mg.

FIGURE 10

The repeatability of the various sets of measurements carried out with the two methods mentioned, is typically between 1 µg and 2 µg, while the error between the estimated value and the reference value has a systematic value around -10 µg (i.e. the estimated value is always less than the reference one). In the case of the 20 mg mass the relative error is less than 10^{-3}.

Systematic effects such as: calibration of the piezo actuator, parasitic capacitance, contact potentials, thermal drifts must be carefully addressed.

VI. Conclusion and perspectives

We have realized a prototype of a simple voltage balance based on parallel plates capacitor suitable for measuring small mass standards up to 1 g. The characterization of the actuator is made by changing the distance between the electrodes while keeping fixed the beam of the balance (constant force) and measuring the capacitance changes of the same. This approach greatly simplifies the mechanical design with respect to classical cylindrical capacitor actuators at the expenses of a more complex model for the calculation of the force. Preliminary results carried out in the milligram range show a promising uncertainty of the order of 10^{-3}. We have identified several critical aspects that have to be addressed for the next version of the experiment.

In particular we will:

- Reduce the roughness of the capacitor plates;
- Take care of the edge shape (both in the plate manufacturing and in the model) in order to reduce the effect on the electric field;
- Automatize the compensation procedure to avoid the presence of the operator;
- Implement an optical alignment system to guarantee the parallelism of the plates;
- Improve the thermal control of the measurement environment;
- Study the influence of edge effects due to the lateral capacitance gradient.

Relying on the improvement listed above and with the help of refined theoretical simulations, we foresee to obtain an uncertainty better than 10^{-4} at the milligram scale, which is close to the state of the art limit of classical scales.

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