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Freeform scanning on the internal involute waviness measurement standard

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I.N.Ri.M. TECHNICAL REPORT
Abstract

PTB designed and manufactured an internal involute waviness scanning measurement standard (SAFT 2w). The device embodies an internal and an external involute profile both superposed with a certain waviness which enables to characterize the dynamic behavior of probing systems. The measurement standard is designed as a disc with two high accurate reference surfaces (a circle and a plane) to define the datum axis of the workpiece. A precise bore is used to define the x-axis. Both the internal and external involute profiles have been calibrated as unmodified gear profiles according to existing standards and guidelines (e.g. ISO 1328-1), i.e. for both profiles the total deviation $F_a$, the form deviation $f_{fa}$ and the slope deviation $f_{Ha}$ have been calibrated. Moreover, a spectral analysis has been performed using FFT method. The three main components of the spectrum have been calibrated in terms of wavelength and amplitude.

INRIM has investigated the influence of scanning parameter such as 5 different scanning speeds within the range of the machine specification, 3 different workpiece orientations inside the measurement volume and 3 different stylus lengths. The measurement have been carried out on the CMM at INRIM by using the internal involute waviness measuring the calibrated standard SAFT 2w and a model for estimating the measuring uncertainty contribution has been derived from the measurement results. This work is related to the deliverable 5.2.3 of Drive Train Project (ENG56).

Il PTB ha progettato e costruito un campione envolvente (SAFT 2w) per la valutazione degli effetti introdotti dai parametri di scanning nelle misure a coordinate. Il dispositivo comprende un profilo di evolvente interna ed esterna, entrambe sovrapposte con una certa waviness (una lavorazione meccanica che riproduce un andamento ondoso sulla superficie del pezzo con determinate caratteristiche di lunghezza d’onda e ampiezza) che consente di caratterizzare il comportamento dinamico dei sistemi di scansione. Il campione di misura è progettato come un disco con due superfici di riferimento ad alta precisione (un cerchio e un piano) per definire l’asse di riferimento del pezzo. Un foro di precisione viene utilizzato per definire l’asse x. Entrambi i profili di evolvente sono stati tarati come profili di ruote dentate secondo gli standard e le linee guida esistenti (si veda ad esempio ISO 1328-1). Per entrambi i profili sono state tarate la deviazione totale $F_a$, la deviazione modulo $f_{fa}$ e la deviazione della pendenza $f_{Ha}$. Inoltre, è stata effettuata un’analisi spettrale usando il metodo FFT. Le tre componenti principali dello spettro sono state tarate (PTB) in termini di lunghezza d’onda e di ampiezza.

L’INRIM ha analizzato l’influenza dei parametri di scansione, quali: 5 diverse velocità di scansione (da 2 mm /s a 24 mm /s), 3 diversi orientamenti del pezzo all’interno del volume di misura e 3 diverse lunghezze dello stilo (35 mm, 135 mm e 235 mm). Il risultato delle misure è stato analizzato per valutare, per entrambi i profili, le deviazioni $F_a$, $f_{fa}$ e $f_{Ha}$ secondo la ISO 1328-1: 2013 e l’influenza dovuta alle variabili di scansione. Questo lavoro si colloca all’interno del Progetto DriveTrian (ENG56) come deliverable 5.2.3.
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Introduction
Deliverable 5.2.3 (WP 5 - Validation of measurement strategies and determination of achievable measurement uncertainty in industrial environment, Task 5.2 - Determination of the achievable measurement uncertainty) refers to investigation of dynamic behaviour of probing systems due to scanning measurement at CMM of two standard involute profiles, both superposed with a certain waviness. The standard involutes (SAFT 2w) have been manufactured and calibrated according to existing standards and guidelines (e.g. ISO 1328-1) by PTB (see D2.1.1 and D2.1.2). In particular, for both profiles, the total deviation $F$, the form deviation $f_f$, and the slope deviation $f_{H}$, have been calibrated; moreover, a spectral analysis has been performed using FFT method and the three main components of the spectrum have been calibrated in terms of wavelength and amplitude. Result are documented in PTB calibration certificate ref. n. 5.3-2016-014 (D2.1.2)
INRIM investigated the influence of scanning parameter such as 5 different scanning speeds (from 2 mm/s to 24 mm/s), 3 different workpiece orientations inside the measurement volume and 3 different stylus lengths (35 mm, 135 mm and 235 mm). Measurement result have been analysed in order to evaluate, for both profiles, the deviations ($F$, $f_f$, and $f_{H}$) according ISO 1328-1: 2013 and the influence due to the scanning measurement parameters on these result.

1. The measurement standard SAFT 2w
The standard SAFT 2w is a plate with a diameter of 290 mm and a thickness of 20 mm, with 2 polished references on the border (a circle and a plan) in order to determine the reference axis of the workpiece (Fig. 1 and Fig. 2). The standard embodies an internal and an external involute profile both superposed with a certain waviness. The profiles have been manufactured with a wire-cut EDM machine. The machining data points have been obtained by using the function described in Figure 3. The three waviness parameters, which were used, are presented in the Table 1.

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<th>Geometry parameters</th>
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<td>Outer diameter</td>
<td>290 mm</td>
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<td>Involute parameters:</td>
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<td>Radius of base circle</td>
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<td>Range of involute function $\text{inv} (\alpha)$</td>
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<td>ext. Involute:</td>
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<td>Nominal wavelength and amplitude:</td>
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<td>$\lambda_1; A_1$</td>
<td>8 mm; 5 μm</td>
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<tr>
<td>$\lambda_2; A_2$</td>
<td>2.5 mm; 3 μm</td>
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<tr>
<td>$\lambda_3; A_3$</td>
<td>0.8 mm; 1 μm</td>
</tr>
</tbody>
</table>

Fig. 1 SAFT 2W Artefact
Fig. 2: Technical drawing of the internal involute waviness scanning measurement standard.

Fig. 3: Parametric function and the sketch of the involute with waviness.

Parametric definition:

\[
\begin{aligned}
(x, y) &= \left( r_e \cdot \cos(\text{inv} \alpha) \right) \\
        &= \left( r_e \cdot \sin(\text{inv} \alpha) \right)
\end{aligned}
\]

with

\[
\text{inv} \alpha(y) = \tan \left( \cos^{-1} \left( \frac{r_b}{r_e} \right) \right) - \left( \cos^{-1} \left( \frac{r_b}{r_e} \right) \right)
\]

\[r_e(y) = \sqrt{r_b^2 + y^2}\]

\[r(y) = \sqrt{r_b^2 + y^2}\]

\[y_g(y) = ly + \sum_{n=1}^{3} \left( A_n \sin \frac{2\pi \cdot by}{l_e} \right)\]

\(r_b\): base radius

\(ly, y_b\): roll length

\(A_n\): amplitude of wave \(n\)

\(l_e\): wave length of wave \(n\)
2. Experimental setup and plan of measurement
Free form scanning on the SAFT 2w involute profiles was performed on a CMM Leitz PMM-C 12.10.7 with the following machine specification:
- measuring volume: $12 \times 10 \times 7 \text{ dm}^3$;
- $\text{EMP}_E = 0.6 \text{ \mu m} + 1.7 \cdot 10^{-6} \text{ L}$;
- $P_{\text{FTU}} = 0.6 \text{ \mu m}$;
- Resolution = $0.05 \text{ \mu m}$;
- Stylus model: Leitz trax (tip diameter: 3 mm).

One face of SAFT 2w was equipped with 4 PT100 probes for temperature compensation (Fig 4a). Measurement was performed, for both profiles (Fig. 4b), according to the following scanning measuring parameters:
- workpiece orientations ($\text{WO}$): 0°, 90° and 210° (Fig. 5);
- scanning speed ($\text{SS}$): 2, 8, 14, 20, 24 mm/s;
- stylus length ($\text{SL}$): 35, 135 and 235 mm (135 mm and 235 mm have been obtained by means two titanium extensions of 100 mm and 200 mm, respectively);
- 3 scanning measure repetitions for each parameter set.

Fig 4: a) thermometers arrangement; b) example of internal involute measuring scanning

Fig 5: Workpiece orientations: 0°, 90° and 210° with respect to machine x-axis and y-axis (green)
3. Scanning measurement result and data analysis

According to scanning parameters, a total of 270 measurement profiles have been performed. Measurement required two day of machine functioning during which temperature has ranged from 20.5 °C to 20.7 °C. The coordinate system taken for measuring was strictly in accordance with the artefact calibration certificate issued by PTB [2] (Fig 6). From the measurement data the profiles have been calculated as function of length of roll. Then, the theoretical involute was subtracted from data.

![Fig. 6. Sketch of the SAFT w2 (by PTB report D.2.1.2)](image)

A first evaluation of computed data, evidenced the presence of an unexpected periodic deviation of the profile that seemed to reveal some eccentricity. The reduction of eccentricity by theoretical correction gave as result a profile deviation behaviour very similar to the one showed in PTB certificate (see Fig. 7 and Fig 8).

![Fig. 7. Example of internal involute profile By PTB certificate n. ref. n. 5.3-2016-014](image)

1 For more details on measurement execution (Quindos part-program) see Annex A
2 The presence of periodic deviation has not been evidenced by SAFT 2w calibration certificate [2]- see Annex B.
Despite this positive result, the whole sinusoidal effect seems not to be removed by means of the reduction of a mean eccentricity (see Fig. 9 and Fig. 10), moreover correction induce irregularity in $f_{rta}$ values as function of workpiece orientations (see Tab. 2, 3, 4, and 5); this could suggest the
presence of other effects, as for example thermal effects, that could contribute to the evidenced trend, but these further effects will not be investigated in the present draft. Notice that also the CMI institute (a project partner) during Final meeting declared that they found the same data behaviour during measurement and that they analysed the measurement data by correcting a theoretical eccentricity even if the correction procedure has not been cited (and therefore described) inside of their deliverable report. Considering only the presence of an eccentricity as influence effect, a reduction of data distribution has been necessary for the evaluation of profile deviations. In order to reduce eccentricity effects, a mean eccentricity on three couple of coordinates\(^3\) has been computed and then it has been mathematically removed by all measured profiles.

![Fig. 9: profile deviation before and after the introduction of a polar eccentricity \(e^*\); Internal involute profile](image)

\(^3\) The mean eccentricity \(\bar{e}\) has been evaluated on 1 repetition and 3 workpiece orientations at scanning speed of 2 mm/s and stylus length of 35 mm in the case of internal involute profile. See Annex C for more details.
As previously evidenced, the mathematical correction, contributes to reduce the sinusoidal effect but it does not remove whole effect, especially in the case of external involute profile, that shows higher values of profile deviations with respect to internal one (see Tab. 2, 3, 4 and 5).

Tab. 2 and Tab. 4 show the result of profile deviation analysis conducted on the data distributions reduced by eccentricity. In particular, for both profiles, the total deviation $F_{\alpha}$, the form deviation $f_{\alpha}$ and the slope deviation $f_{H\alpha}$ have been evaluated within the following evaluation ranges:

<table>
<thead>
<tr>
<th>involute</th>
<th>Length of roll</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal involute</td>
<td>20 mm ÷ 120 mm</td>
</tr>
<tr>
<td>External involute</td>
<td>20 mm ÷ 90 mm</td>
</tr>
</tbody>
</table>

Tab. 3 and tab. 5 show the variability of profile deviations with respect to mean profile deviations evaluated at the best measurement conditions; they have been considered as best measurement conditions those conditions that would theoretically guaranteed the lowest effects on measurement result due to scanning speed and stylus length. Therefore, the mean profile deviations have been computed considering the profile deviation values at the following conditions: SL= 35 mm, SS= 2 m/s and WO= 0°, 90°, 210° because it was not possible to distinguish the best workpiece orientation a priori (see Annex C for more details about eccentricity correction).

As example of uncorrected profile deviation, Tab. 6 shows the profile deviations for the internal involute profile without eccentricity correction whereas Tab. 7 shows the corrected sample standard deviations of deviation profiles by varying the scanning speed.

The total deviation $F_{\alpha}$, the form deviation $f_{\alpha}$ and the slope deviation $f_{H\alpha}$ in case of uncorrected condition (values in Tab. 6) are also shown in Fig. 11, 12 and 13.
# INTERNAL involute - SAFT w2

<table>
<thead>
<tr>
<th>Profile Deviations/μm</th>
<th>Stylus length / mm</th>
<th>Orientation / °</th>
<th>Scanning speed / mm s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>35</td>
<td>0</td>
<td>4.457, 4.456, 4.383, 4.175, 4.227</td>
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<tr>
<td></td>
<td></td>
<td>90</td>
<td>-0.198, 0.002, -0.049, -0.163, -0.075</td>
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<td>-4.236, -4.385, -4.553, -4.880, -4.890</td>
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<tr>
<td>profile slope deviation</td>
<td>135</td>
<td>0</td>
<td>6.852, 6.955, 6.963, 6.087, 6.332</td>
</tr>
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<td></td>
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<td>90</td>
<td>0.155, 0.360, 0.294, 0.117, -0.820</td>
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<td>210</td>
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<td>6.424, 6.162, 6.156, 3.259, 3.322</td>
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<td>22.840, 23.503, 24.016, 24.788, 24.190</td>
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<td>19.660, 21.055, 22.005, 22.530, 21.861</td>
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<td>23.890, 24.692, 27.264, 25.032, 23.852</td>
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Tab 2: internal involute profile deviations
### Table 3: Internal profile deviations variability with respect to mean profile deviations at best scanning conditions (SS= 2 mm/s, SL= 35 mm, for all orientations)

<table>
<thead>
<tr>
<th>Profile deviations variability/μm</th>
<th>Stylus length/mm</th>
<th>Orientation/°</th>
<th>Scanning speed/mm s⁻¹</th>
<th>2</th>
<th>8</th>
<th>14</th>
<th>20</th>
<th>24</th>
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<tr>
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<tr>
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</tr>
</tbody>
</table>

**Mean profile deviations**

| SS= 2 mm/s; SL= 35 mm | $\bar{f}_{Hx}$ = 0.008 μm | $\bar{f}_{fo}$ = -21.028 μm | $\bar{F}_{α}$ = 22.184 μm |
## EXTERNAL involute - SAFT w2

<table>
<thead>
<tr>
<th>Profile Deviations / μm</th>
<th>Stylus length / mm</th>
<th>Orientation / °</th>
<th>Scanning speed / mm s⁻¹</th>
</tr>
</thead>
<tbody>
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Tab 4: External involute profile deviations
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Tab. 5: External profile deviations variability with respect to mean profile deviations evaluated at best scanning conditions (SS= 2 mm/s, SL= 35 mm, for all orientations)
### Tab. 6: Internal profile deviations without eccentricity correction

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### Tab. 7: Corrected sample standard deviations of deviation profiles by varying the scanning speed.

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Fig. 11: Total deviation at different scanning speeds, workpiece orientations and Stylus lengths – internal involute

Fig. 12: Form deviation at different scanning speeds, workpiece orientations and Stylus lengths – internal involute
After the profile deviation analysis, measurement data have been spectral analysed by means of FFT method in order to determine the superimposed waviness as a function of scanning parameters; furthermore the three largest amplitudes within the evaluation range have been determined. The found wavelengths and amplitudes indicates the presence of a waviness consistent with the superposed nominal waviness (see section 1). Tables 8, 9, 10 and 11 summarize result obtained at different measurement conditions (workpiece orientations, scanning speeds and stylus lengths). Since data analyses show that scanning conditions do not influence significantly the FFT result and that detection of superposed waviness is very repeatable with scanning speeds (SS) (see Tab 8 and Tab. 10), it has been decided to show the result at different stylus lengths (SL) and orientations (WO) only in the cases of SS= 2 mm/s.

Finally, Fig. 14 and Fig. 15 show two examples of conducted FFT analysis.

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<th>\lambda_1 / mm</th>
<th>A_1 / \mu m</th>
<th>f_2 / mm^{-1}</th>
<th>\lambda_2 / mm</th>
<th>A_2 / \mu m</th>
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Tab. 8. result of FFT analysis for the internal involute profile at different SS and at SL= 35 mm, WO= 0°
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<th>$A_1 / \mu\text{m}$</th>
<th>$f_2 / \text{mm}^{-1}$</th>
<th>$\lambda_2 / \text{mm}$</th>
<th>$A_2 / \mu\text{m}$</th>
<th>$f_3 / \text{mm}^{-1}$</th>
<th>$\lambda_3 / \text{mm}$</th>
<th>$A_3 / \mu\text{m}$</th>
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<td>0.4001</td>
<td>2.499</td>
<td>2.976</td>
<td>1.2504</td>
<td>0.800</td>
<td>1.002</td>
</tr>
</tbody>
</table>

Tab. 9: result of FFT analysis for internal involute profile at SS= 2 mm/s and different SL and WO

<table>
<thead>
<tr>
<th>SS / mm</th>
<th>$f_1 / \text{mm}^{-1}$</th>
<th>$\lambda_1 / \text{mm}$</th>
<th>$A_1 / \mu\text{m}$</th>
<th>$f_2 / \text{mm}^{-1}$</th>
<th>$\lambda_2 / \text{mm}$</th>
<th>$A_2 / \mu\text{m}$</th>
<th>$f_3 / \text{mm}^{-1}$</th>
<th>$\lambda_3 / \text{mm}$</th>
<th>$A_3 / \mu\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.1200</td>
<td>8.333</td>
<td>4.274</td>
<td>0.4000</td>
<td>2.500</td>
<td>2.969</td>
<td>1.2499</td>
<td>0.800</td>
<td>0.990</td>
</tr>
<tr>
<td>135</td>
<td>0.1201</td>
<td>8.330</td>
<td>4.293</td>
<td>0.4002</td>
<td>2.499</td>
<td>2.981</td>
<td>1.2505</td>
<td>0.800</td>
<td>0.996</td>
</tr>
<tr>
<td>235</td>
<td>0.1201</td>
<td>8.330</td>
<td>4.310</td>
<td>0.4002</td>
<td>2.499</td>
<td>2.983</td>
<td>1.2505</td>
<td>0.800</td>
<td>1.014</td>
</tr>
</tbody>
</table>

Tab. 10. result of FFT analysis for the external involute profile at different SS and at SL= 35 mm, WO= 0°

<table>
<thead>
<tr>
<th>SS / mm s$^{-1}$</th>
<th>$f_1 / \text{mm}^{-1}$</th>
<th>$\lambda_1 / \text{mm}$</th>
<th>$A_1 / \mu\text{m}$</th>
<th>$f_2 / \text{mm}^{-1}$</th>
<th>$\lambda_2 / \text{mm}$</th>
<th>$A_2 / \mu\text{m}$</th>
<th>$f_3 / \text{mm}^{-1}$</th>
<th>$\lambda_3 / \text{mm}$</th>
<th>$A_3 / \mu\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.1286</td>
<td>7.777</td>
<td>4.799</td>
<td>0.4001</td>
<td>2.500</td>
<td>2.928</td>
<td>1.2430</td>
<td>0.804</td>
<td>0.857</td>
</tr>
<tr>
<td>8</td>
<td>0.1286</td>
<td>7.779</td>
<td>4.806</td>
<td>0.4000</td>
<td>2.500</td>
<td>2.935</td>
<td>1.2570</td>
<td>0.796</td>
<td>1.063</td>
</tr>
<tr>
<td>14</td>
<td>0.1286</td>
<td>7.776</td>
<td>4.799</td>
<td>0.4001</td>
<td>2.500</td>
<td>2.970</td>
<td>1.2574</td>
<td>0.795</td>
<td>0.993</td>
</tr>
<tr>
<td>20</td>
<td>0.1286</td>
<td>7.776</td>
<td>4.767</td>
<td>0.4001</td>
<td>2.499</td>
<td>3.069</td>
<td>1.2432</td>
<td>0.804</td>
<td>0.890</td>
</tr>
<tr>
<td>24</td>
<td>0.1286</td>
<td>7.777</td>
<td>4.779</td>
<td>0.4001</td>
<td>2.500</td>
<td>3.142</td>
<td>1.2430</td>
<td>0.804</td>
<td>0.873</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SS / mm</th>
<th>$f_1 / \text{mm}^{-1}$</th>
<th>$\lambda_1 / \text{mm}$</th>
<th>$A_1 / \mu\text{m}$</th>
<th>$f_2 / \text{mm}^{-1}$</th>
<th>$\lambda_2 / \text{mm}$</th>
<th>$A_2 / \mu\text{m}$</th>
<th>$f_3 / \text{mm}^{-1}$</th>
<th>$\lambda_3 / \text{mm}$</th>
<th>$A_3 / \mu\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.1286</td>
<td>7.777</td>
<td>4.799</td>
<td>0.4001</td>
<td>2.500</td>
<td>2.928</td>
<td>1.2430</td>
<td>0.804</td>
<td>0.857</td>
</tr>
<tr>
<td>135</td>
<td>0.1286</td>
<td>7.777</td>
<td>4.799</td>
<td>0.4001</td>
<td>2.500</td>
<td>2.936</td>
<td>1.2430</td>
<td>0.804</td>
<td>0.871</td>
</tr>
<tr>
<td>235</td>
<td>0.1286</td>
<td>7.777</td>
<td>4.806</td>
<td>0.4001</td>
<td>2.500</td>
<td>2.932</td>
<td>1.2430</td>
<td>0.804</td>
<td>0.880</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SS / mm</th>
<th>$f_1 / \text{mm}^{-1}$</th>
<th>$\lambda_1 / \text{mm}$</th>
<th>$A_1 / \mu\text{m}$</th>
<th>$f_2 / \text{mm}^{-1}$</th>
<th>$\lambda_2 / \text{mm}$</th>
<th>$A_2 / \mu\text{m}$</th>
<th>$f_3 / \text{mm}^{-1}$</th>
<th>$\lambda_3 / \text{mm}$</th>
<th>$A_3 / \mu\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.1286</td>
<td>7.775</td>
<td>4.781</td>
<td>0.4001</td>
<td>2.499</td>
<td>2.927</td>
<td>1.2432</td>
<td>0.804</td>
<td>0.865</td>
</tr>
<tr>
<td>135</td>
<td>0.1286</td>
<td>7.776</td>
<td>4.770</td>
<td>0.4001</td>
<td>2.499</td>
<td>2.935</td>
<td>1.2432</td>
<td>0.804</td>
<td>0.873</td>
</tr>
<tr>
<td>235</td>
<td>0.1286</td>
<td>7.776</td>
<td>4.770</td>
<td>0.4001</td>
<td>2.499</td>
<td>2.938</td>
<td>1.2431</td>
<td>0.804</td>
<td>0.915</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SS / mm</th>
<th>$f_1 / \text{mm}^{-1}$</th>
<th>$\lambda_1 / \text{mm}$</th>
<th>$A_1 / \mu\text{m}$</th>
<th>$f_2 / \text{mm}^{-1}$</th>
<th>$\lambda_2 / \text{mm}$</th>
<th>$A_2 / \mu\text{m}$</th>
<th>$f_3 / \text{mm}^{-1}$</th>
<th>$\lambda_3 / \text{mm}$</th>
<th>$A_3 / \mu\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.1286</td>
<td>7.778</td>
<td>4.793</td>
<td>0.4000</td>
<td>2.500</td>
<td>2.934</td>
<td>1.2428</td>
<td>0.805</td>
<td>0.860</td>
</tr>
<tr>
<td>135</td>
<td>0.1286</td>
<td>7.776</td>
<td>4.782</td>
<td>0.4001</td>
<td>2.500</td>
<td>2.930</td>
<td>1.2431</td>
<td>0.804</td>
<td>0.862</td>
</tr>
<tr>
<td>235</td>
<td>0.1286</td>
<td>7.776</td>
<td>4.772</td>
<td>0.4001</td>
<td>2.500</td>
<td>2.924</td>
<td>1.2431</td>
<td>0.804</td>
<td>0.891</td>
</tr>
</tbody>
</table>
Tab. 11: Result of FFT analysis for external involute profile at SS = 2 mm/s and different SL and WO.

Fig. 8: FFT for internal involute profile (SL = 35 mm, SS = 2 mm/s, WO = 0°); dashed lines mark the evaluation range. Wavelength: 0.77 mm – 20 mm

Fig. 9: FFT for external involute profile (SL = 35 mm, SS = 2 mm/s, WO = 0°); dashed lines mark the evaluation range. Wavelength: 0.77 mm – 20 mm
Conclusion
Freeform scanning on an internal involute profile measurement standard designed and manufactured by PTB has been conducted. First evaluations on measurement data evidenced the presence of unsuspected effects as a possible eccentricity and some possible thermal effects not deeply investigated, yet.

Result after eccentricity reduction, suggest a not very significant trend of the slope deviation \( f_{\alpha} \) as a function of scanning speed or stylus length; variation of \( f_{\alpha} \) values at different orientations seems to suggest a not adequate evaluation of secondary effect rather than a real effect of orientation on measurement result (this aspect has to be investigated in more detail). About total deviations \( F_{\alpha} \) and form deviations \( f_{\alpha} \) is not present a significant trend as function of scanning speed, orientation or stylus length; limited differences seem to suggest a modest worsening of performances but magnitudes of these differences do not allow to define a clear trend.

On the contrary, spectral analysis of data, suggest high and stable performances of the machine. Actually, result show that evaluations of wavelength and amplitude are very repeatable and that they are not influenced by workpiece orientation, scanning speed or stylus length.

In particular, negligible variations in evaluation of wavelength and amplitude as function of orientations and stylus length could means respectively an adequate compensation of machine geometrical error and a valid probing system qualification, whereas the analysis in term of scanning speed (in the scanning speed range considered with respect to the waviness investigated) allows to confirm the maintenance of these high performances also at the most critical measuring conditions.

Therefore, generally, the waviness analysis result allows to give an overall positive evaluation in term of performances for the CMM used in this work.

Reference
[1] PTB Report - deliverable D 2.1.1
[2] PTB Report - deliverable D 2.1.2

Annexes
A - Quindos part-program
B – TR 17/2017 – Annex B: SAFT w2 calibration certificate by PTB
C - Eccentricity correction (formulary)
Annex A - T.R. 17/2017:

Q7 part program “DriveTrain evolvente.wdb” – 01/08/2017 for the scanning of the internal involute waviness standard performed on the INRIM CMM Leitz PMM- C 12.10.7

! ------------------------- JRP DriveTrain -------------------------
! ---------------------- Campione d’evolvente ----------------------

! Geometria del campione
! Distanza dal bordo esterno del colletto per la presa punti del piano superiore
BORDO=5
SEMI_SPESSORE=10
SEMI_INGOMBRO=145

! TEMPERATURE -------------
! Acquisizione temperature scale CMM
TMPCOMP (COE=0.000000, AUT=Y, TEL=TEMP, DEL=Y)

! -------------------------
! LEGGO SOLO

! --------------
GETVALS (OBJ=TEMP, TYP=ELE, RDS=(X,Y,Z), REA=(X,Y,Z))

! Acquisizione temperature campione evolvente
! Chiamo la procedura
DELCHS (NAM=CHS:~IMP2EVA_*, CNF=N, TYP=CHS)

! --------------
IMP2EVA_FILE = ~TEMPER_FILE
IMP2EVA_X = "TERM_1"
IMP2EVA_Y = "TERM_2"
IMP2EVA_Z = "TERM_3"
IMP2EVA_A = "TERM_4"
IMP2EVA_B = ""
IMP2EVA_D = ""

! --------------
IMP2EVA_E = ''
IMP2EVA_F = ''

INDPRC (NAM=IMP2EVA)
! copia elemento creato dalla procedura e poi lo cancella
CPYOBJ (FRM=IMP2EVA_ELE, TO =TEMP_CAL_A)
DELELE (NAM=IMP2EVA_ELE, CNF=N)
! leggo le 4 temperature
GETVALS (OBJ=TEMP_CAL_A, TYP=ELE, RDS=(X,Y,Z,A), REA=(T1,T2,T3,T4))
MEDIEL=(T1+T2+T3+T4)/4
TMPMEDI=(X+Y+Z+A)/4

DO
(NAM=I, BGN=1, END=5)
! muove in posizione inizio scansione interno
MoveCmmInmm (TYP=ABS, DST=(16,26,20), CSY=CSY_AUT)
! imposta la velocità di scansione
PUTVALS (OBJ=CONTORNO_INT.NOM.PTS(3), TYP=ELE, RDS=A, VAL=SPEED(I))
! scansione bordo interno
ME2DE (NAM=CONTORNO_INT, CSY=CSY_AUT, INO=O)
! compone stringa per output
~Vel=Int
CVREACHS (NAM=~/VELOCITA, VAL=SPEED(I), FM1=2, INT=Y, ANG=N, SPZ=Y, RLS=Y, RTZ=Y)
CONCAT (NAM=/~/PERCORSO, STR=(~/RADICE_FILE,~/VELOCITA,~/VEL,~/RIPETIZIONE), Ini=Y)
! esporta la scansione
FMTOBJ (FIL=~/PERCORSO, NAM=CONTORNO_INT, STA=NEW, TYP=ELE, STY=APT, DSC=(X,Y,Z), DEL=Y)
! muove in posizione
MoveCmmInmm (TYP=ABS, DST=(17,-112,20), CSY=CSY_AUT)
MoveCmmInmm (TYP=ABS, DST=(16,26,20), CSY=CSY_AUT)
! imposta velocità di scansione
PUTVALS (OBJ=CONTORNO_EST.NOM.PTS(3), TYP=ELE, RDS=A, VAL=SPEED(I))
! scansione bordo esterno
ME2DE (NAM=CONTORNO_EST, CSY=CSY_AUT)
! compone stringa per output
~Vel=Est
CVREACHS (NAM=~/VELOCITA, VAL=SPEED(I), FM1=2, INT=Y, ANG=N, SPZ=Y, RLS=Y, RTZ=Y)
CONCAT (NAM=/~/PERCORSO, STR=(~/RADICE_FILE,~/VELOCITA,~/VEL,~/RIPETIZIONE), Ini=Y)
! esporta la scansione
FMTOBJ (FIL=~/PERCORSO, NAM=CONTORNO_EST, STA=NEW, TYP=ELE, STY=APT, DSC=(X,Y,Z), DEL=Y)
MoveCmmInmm (TYP=ABS, DST=(17,-112,20), CSY=CSY_AUT)
MoveCmmInmm (TYP=ABS, DST=(16,26,20), CSY=CSY_AUT)
ENDDO

! ------ TEMPERATURE.R-------- !
! --------------------------!

! acquisizione Temperature campione evolvente
! chiama la procedura
DELCHS (NAM=CHS:~IMP2EVA_*, CNF=N, TYP=CHS)
~IMP2EVA_FILE = ~/TEMPER_FILE
~IMP2EVA_X = "TERM_1"
~IMP2EVA_Y = "TERM_2"
~IMP2EVA_Z = "TERM_3"
~IMP2EVA_A = "TERM_4"
~IMP2EVA_B = ""
~IMP2EVA_D = ""
~IMP2EVA_E = ""
INDPRC (NAM=IMP2EVA)
! copia elemento creato dalla procedura e poi lo cancella
CPYOBJ (FRM=IMP2EVA_ELE, TO =TEMP_CAL_R)
DELELE (NAM=IMP2EVA_ELE, CNF=N)
! ------ !
! Report !
! ------ !
! si riportano i due elementi sottostanti secondo il CMMA$CSY
TRALE (NEV=CENTRO_ASSOLUTO, TRA=CMMA$CSY, OLD=CENTRO_AUT, TYP=CSY, RPL=Y, EVA=N)
TRALE (NEV=ASSE_X_ASSOLUTO, TRA=CMMA$CSY, OLD=ASSE_X_AUT, TYP=CSY, RPL=Y, EVA=N)

DELOUE (NAM=~/RO, CNF=N, TYP=QUE)
ADDEVA (NAM=~/PRB.TEMP_CAL_A.TEMP_CAL_R)
ADDEVA (NAM=(CENTRO_ASSOLUTO,ASSE_X_ASSOLUTO))
FLEXREPORT (LAY=VICI, PRI=N, XFL=C:\Users\LNPC775\Desktop\DriveTrain evolvente\MISURE\report)
STOP
Kalibrierschein

Calibration Certificate

Gegenstand: Wellenbehafetes Evolventen-Scanning-Normal
Object: Involute waviness scanning measurement standard

Hersteller: Physikalisch-Technische Bundesanstalt (PTB)
Manufacturer: Bundesallee 100
PTB
D-38116 Braunschweig

Typ: Scanningartefakt mit internem und externem Evolventenprofil mit überlagertes Welligkeit
Type: Scanning artifact with internal and external involute profile with superposed waviness

Kennnummer: SAFT 2w
Serial No.: SAFT 2w

Auftraggeber: Physikalisch-Technische Bundesanstalt (PTB)
Applicant: Bundesallee 100
PTB
D-38116 Braunschweig

Anzahl der Seiten: 11
Number of pages: 11

Geschäftszeichen: 5.3-2016-014
Reference No.: 5.3-2016-014

Kalibrierzeichen: 50574 PTB 16
Calibration mark: 50574 PTB 16

Datum der Kalibrierung: 2016-03-22
Date of calibration: 2016-03-22

Im Auftrag
On behalf of PTB

Dr. rer. nat. M. Stein

Im Auftrag
On behalf of PTB

Dipl.-Ing. (FH) A. Wedmann

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Kalibriergegenstand

Calibration standard

Wellenbehafetes Evolventen-Scanning-Normal

Involute waviness scanning measurement standard


The involute waviness scanning measurement standard embodies an internal and an external involute profile with superposed waviness. Moreover it is equipped with reference surfaces to determine a reference circle and a reference plane. A precise bore is used to define the x-axis.

Evolventenparameter:

Involute parameters:

<table>
<thead>
<tr>
<th></th>
<th>Interne Evolvente internal involute</th>
<th>Externe Evolvente external involute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grundkreisradius</td>
<td>20 mm</td>
<td>20 mm</td>
</tr>
<tr>
<td>Radius of base circle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bereich der Evolventenfunktion (\alpha)</td>
<td>0° - 270°</td>
<td>0° - 200°</td>
</tr>
<tr>
<td>Range of involute function (\alpha)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Wellenbehauftetes Evolventen-Scanning-Normal samt Auflagevorrichtung

Involute waviness scanning measurement standard with subbase

**Kalibrierverfahren**

*Calibration procedure*


The profiles of the measurement standard were calibrated on a coordinate measuring machine for which traceability has been proved. The individual measurement values were determined by a multiple orientation measurement procedure. For this purpose, the measurement standard was measured in four positions displaced by 90°. The measurement results are the averaged values from all measurements.
Bezüge

References


The datum face of the measurement standard is marked by the engraving. The reference axis of the measurement standard was numerically determined. For this purpose, a reference circle and a reference plane on the measurement standard (see sketch of the measurement standard) were measured and determined by least squares method. The reference axis of the measurement standard was fixed through the center of the reference circle and perpendicular to the reference plane. In order to determine the workpiece coordinate system the reference axis was fixed as z-axis. The x-direction reference bore was measured with 36 single points and its center determined by least squares method. By this center point the x-axis is defined.

1. Bezugsseite (Höhenbezug)  
   datum face (height reference)
2. Bezugskreis  
   reference circle
3. Bezugs ebene  
   reference plane
4. x-Richtungsbohrung  
   x-direction reference bore
5. Internes Evolventenprofil  
   Internal involute profile
6. Externes Evolventenprofil  
   External involute profile
7. Grundkreisbogen mit Radius 20 mm  
   base circle arc with radius 20 mm

Skizze des Normals
Sketch of the measurement standard
Umgebungsbedingungen
Environmental conditions

Temperatur während der Messung
Temperature during the measurement

(20 ± 0,2) °C

Normative Verweise
Normative references

Die Bezeichnung am Evolutennormal und die Auswertungen erfolgten, sofern nicht explizit anders beschrieben, unter Berücksichtigung der folgenden Richtlinien und Normen:

For identification and evaluations on the involute artifact the following guidelines and standards were taken into account unless otherwise explicitly noted:

ISO 1328-1, September 2013 (E);
VDI/VDE 2607, Februar 2000;
VDI/VDE 2612, Mai 2000;
DIN ISO 21771, August 2014;
DIN ISO 21772, Juli 2012;
DIN ISO 21773, August 2014;
DIN 3999, November 1974;

Messunsicherheit
Measurement uncertainty


The uncertainty stated is the expanded measurement uncertainty obtained by multiplying the standard measurement uncertainty by the coverage factor $k = 2$. It has been determined in accordance with the “Guide to the Expression of Uncertainty in Measurement (GUM)”. The value of the measurand then normally lies with a probability of 95 % within the attributed coverage interval.
Messparameter
Measurement parameters

Die Messungen wurden auf der Grundlage der auf Seite 2 aufgelisteten und der folgenden Angaben durchgeführt.

The measurements were based on the parameters listed on page 2 with the following additional parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antastkugeldurchmesser</td>
<td>3 mm</td>
</tr>
<tr>
<td>Stylus sphere diameter</td>
<td></td>
</tr>
<tr>
<td>Punktedichte auf Wälzlänge</td>
<td>20 mm⁻¹ Scanning (v = 3 mm/s)</td>
</tr>
<tr>
<td>Point density on length of roll</td>
<td></td>
</tr>
<tr>
<td>Lage des Profils (z-Wert)</td>
<td>-10 mm</td>
</tr>
<tr>
<td>Position of the profile (z-value)</td>
<td></td>
</tr>
<tr>
<td>Punkteanzahl für Bezugskreis</td>
<td>1872 Scanning (v = 10 mm/s)</td>
</tr>
<tr>
<td>Number of points at reference circle</td>
<td></td>
</tr>
<tr>
<td>Punkteanzahl für BezugsEbene</td>
<td>72 Einzelpunktantastung</td>
</tr>
<tr>
<td>Number of points at reference plane</td>
<td></td>
</tr>
<tr>
<td>Punkteanzahl für x-Richtungsbohrung</td>
<td>36 Scanning (v = 10 mm/s)</td>
</tr>
<tr>
<td>Number of points at x-direction reference bore</td>
<td></td>
</tr>
</tbody>
</table>
Ergebnisse Profil

Results profile

Aus den Messdaten wurden die Profile im Verhältnis zur Wälzlänge berechnet. Dazu wurde die theoretische Evolvente abgezogen und anschließend innerhalb der angegebenen Auswertebereiche die Profil-Gesamtabweichung \( F_a \), Profil-Formabweichung \( f_{ia} \) und Profil-Winkelabweichung \( f_{is} \) ermittelt.

From the measurement data the profiles have been calculated as function of length of roll. Therefore, the theoretical involute was subtracted from the data. Then, within the evaluation ranges the total deviation \( F_a \), the form deviation \( f_{ia} \) and the slope deviation \( f_{is} \) have been determined.

### Profil Auswertungsbereich

**Profile evaluation range**

<table>
<thead>
<tr>
<th></th>
<th>Interne Evolvente</th>
<th>Externe Evolvente</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start der Auswertung</td>
<td>( L_{\text{Start}} ) (in Wälzlänge)</td>
<td>20 mm</td>
</tr>
<tr>
<td>Start of evaluation</td>
<td>( L_{\text{Start}} ) (length of roll)</td>
<td></td>
</tr>
<tr>
<td>Ende der Auswertung</td>
<td>( L_{\text{End}} ) (in Wälzlänge)</td>
<td>120 mm</td>
</tr>
<tr>
<td>End of evaluation</td>
<td>( L_{\text{End}} ) (length of roll)</td>
<td></td>
</tr>
</tbody>
</table>

### Ergebnisse Profil

**Results profile**

<table>
<thead>
<tr>
<th>Profil-Gesamtabweichung</th>
<th>Profil-Formabweichung</th>
<th>Profil-Winkelabweichung</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_a ) in ( \mu m )</td>
<td>( f_{ia} ) in ( \mu m )</td>
<td>( f_{is} ) in ( \mu m )</td>
</tr>
<tr>
<td>Int ( \pm 1,0 )</td>
<td>( \pm 1,0 )</td>
<td>( \pm 0,7 )</td>
</tr>
<tr>
<td>Ext ( \pm 1,0 )</td>
<td>( \pm 1,0 )</td>
<td>( \pm 0,8 )</td>
</tr>
</tbody>
</table>

Abkürzungen in der Tabelle: Int: Internes Evolventenprofil, Ext: Externes Evolventenprofil

Abbreviations in the table: Int: Internal involute profile, Ext: External involute profile
Diagramm

Bei dem folgenden Diagramm zu den Profilergebnissen ist zu beachten, dass ein Diagramm aus den Mehrlagenmessungen ausgesucht wurde, das den Kalibrierwerten am nächsten kommt.

At the following diagram relating to the profile results please note that one diagram was selected from the multiple orientation measurement procedure which best approaches the calibrated values.
Spektrale Analyse
Spectral analysis

Mittels FFT (Fast Fourier Transform) wurde das Profil spektral analysiert. Aus dem transformierten Profil wurden die 3 größten Amplituden in dem angegeben Auswertebereich ermittelt.

The profile has been interpolated in order to obtain data with higher point density. The profile has been spectral analysed with FFT method (Fast Fourier Transformation). The three largest amplitudes within the evaluation range have been determined.

Parameter für spektrale Analyse:
Parameters for spectral analysis:

<table>
<thead>
<tr>
<th>Auswertbereich Wellenlänge</th>
<th>Evaluation range wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.77 mm - 20 mm</td>
</tr>
</tbody>
</table>

Ergebnisse spektrale Analyse
Results spectral analysis

<table>
<thead>
<tr>
<th>Wellenlänge in mm</th>
<th>Amplitude in μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength in mm</td>
<td>Amplitude in μm</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Abkürzungen in der Tabelle: Int: Internes Evolventenprofil, Ext: Externes Evolventenprofil
Abbreviations in the table: Int: Internal involute profile, Ext: External involute profile

Diagramm
Diagram

Bei dem folgenden Diagramm zu den Ergebnissen der spektralen Analyse ist zu beachten, dass ein Diagramm aus der FFT Analyse ausgesucht wurde, das den Kalibrierwerten am nächsten kommt.

At the following diagram relating to the results of spectral analysis please note that one diagram was selected from the FFT analysis which best approaches the calibrated values.
Die Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig und Berlin ist das nationale Metrologieinstitut und die technische Oberbehörde der Bundesrepublik Deutschland für das Messwesen. Die PTB gehört zum Geschäftsbereich des Bundesministeriums für Wirtschaft und Energie. Sie erfüllt die Anforderungen an Kalibrier- und Prüflaboratorien auf der Grundlage der DIN EN ISO/IEC 17025.

Zentrale Aufgabe der PTB ist es, die gesetzlichen Einheiten in Übereinstimmung mit dem Internationalen Einheitensystem (SI) darzustellen, zu bewahren und weiterzugeben. Die PTB steht damit an oberster Stelle der metrologischen Hierarchie in Deutschland. Die Kalibrierscheine der PTB dokumentieren eine auf nationale Normale rückgeführte Kalibrierung.

Zur Sicherstellung der weltweiten Einheitlichkeit der Maßeinheiten arbeitet die PTB mit anderen nationalen metrologischen Instituten auf regionaler europäischer Ebene in EURAMET und auf internationaler Ebene im Rahmen der Meterkonvention zusammen. Dieses Ziel wird durch einen intensiven Austausch von Forschungsergebnissen und durch umfangreiche internationale Vergleichsmessungen erreicht.

The Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig and Berlin is the National Metrology Institute and the supreme technical authority of the Federal Republic of Germany for metrology. The PTB comes under the auspices of the Federal Ministry of Economics and Energy. It meets the requirements for calibration and testing laboratories as defined in DIN EN ISO/IEC 17025.

The central task of PTB is to realize, to maintain and to disseminate the legal units in compliance with the International System of Units (SI). PTB thus is at the top of the metrological hierarchy in Germany. The calibration certificates issued by PTB document a calibration traceable to national measurement standards.

PTB cooperates with other national metrology institutes - at the regional European level within EURAMET and at the international level within the framework of the Metre Convention - with the aim of ensuring the worldwide coherence of the measurement units. This aim is achieved by an intensive exchange of the results of research work and by comprehensive international comparison measurements.
Annex C - T.R. 17/2017:

Eccentricity correction and calculation of the profile deviation parameters ($f_{Ha}$, $f_{fa}$ and $F_a$) of the standard involutes (SAFT 2W): procedure and formulary

The procedure is synthetically articulated in the following steps:

1. Determination of the components of the polar eccentricity vector \( \bar{e} \) by means of the Microsoft Excel built-in optimization tool \textit{"Solver"} in the best scanning conditions ($v = 2$ mm s\(^{-1}\), stylus length = 35 mm), averaging the obtained values among the 3 different workpiece orientation ($0^\circ$, $90^\circ$ and $210^\circ$).

2. Creation of an OriginLab Origin batch working on the suitable roll length ranges for both external and internal involute profiles to generate, for all the scanning conditions (combinations of 5 variable speeds, 3 stylus lengths and 3 workpiece orientations), the following ordered quantities:
   a. the \textit{eccentric coordinates} \( X_{ecc} \) and \( Y_{ecc} \) of an arbitrary point on the involute profile by correction of the the CMM acquired points with the previously calculated \( \bar{e} \) components (\( e_x \) and \( e_y \));
   b. the \textit{module of the position vector} of the same point: \( \rho_{ecc} = \sqrt{X_{ecc}^2 + Y_{ecc}^2} \);
   c. the \textit{"eccentric" anomaly}: \( \varphi_{ecc} = \arctan \left( \frac{Y_{ecc}}{X_{ecc}} \right) \);
   d. the \textit{roll angle}: \( \theta_{ecc} = \varphi_{ecc} + \arccos \left( \frac{R_b}{\rho_{ecc}} \right) \), where \( R_b = \) base radius;
   e. the \textit{"observed" roll length}: \( l_{obs} = \sqrt{\rho_{ecc}^2 - R_b^2} \pm R_p \) (being \( R_p = \) radius of the probe tip, the \textit{"+"} sign for the internal involute profile and the \textit{"-"} sign for the external involute profile);
   f. the \textit{"expected" roll length}: \( l_{exp} = R_b \theta_{ecc} \);
   g. the difference between the observed and the expected roll length, that is the \textit{profile deviation} \( \Delta_l \).

3. Performance of linear regressions on the \( \Delta_l \) vectors of data, so that the obtained slopes define the $f_{Ha}$ parameter. The other profile parameters are calculated in the following way: being \textit{residuals} = \textit{difference between regression line and profile deviation for each experimental point}, \( f_{fa} = \max(\text{residuals}) - \min(\text{residuals}) \); \( F_a = \max(\Delta_l) - \min(\Delta_l) \).