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Abstract

Magnetization instabilities in an insulating ferromagnet can be induced because of the spin Hall effect in a Platinum side layer. In this paper we present a non equilibrium thermodynamic approach to jointly describe magnetization dynamics and magnetic moment transport in the insulating ferromagnet. We employ non equilibrium thermodynamics to derive the magnetic moment current which is injected into the ferromagnet from Pt. We find that the magnetic moment current induces a thermodynamic effective field which depends on the penetration depth of the current. By considering vector magnetization dynamic equation we show that this term is able to compensate the damping and can possibly give rise to instabilities and the onset of self-oscillations. We apply to the case of thin YIG and we derive the corresponding instability. The model predicts the threshold for the onset of self oscillations, without tunable parameters, which are in good agreement with recent experiments .

Key words: spin Hall torque, magnetic moment current, spin waves, nonequilibrium magnetization
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1. Introduction

The capability to inject, manipulate and detect spin polarized currents, represents one of the main objectives of the field of magnon spintronics [1]. A particular relevance is seen in the presence of magnetization torque effects in ferromagnetic insulators (i.e. yttrium iron garnet, YIG) caused by the magnetic moment current induced by a metallic side layer with a large spin Hall effect (i.e. platinum, Pt). In particular, it has recently been shown that when the direction of the injected moment is antiparallel to the magnetization of the ferromagnet, it is possible to generate spontaneous magnetization dynamics [2]. As the magnetization current in the insulating ferromagnet is not carried by conduction electrons, but rather by out-of-equilibrium spin waves or magnons, the effect is different from the well known spin transfer torque [3]. Although the generation of spin currents from magnetization dynamics is well documented [4], the inverse effect is less clear. Here we look at the problem from the point of view of the non equilibrium thermodynamic approach to the transport of magnetic moment originally developed by Johnson and Silsbee [5] and recently applied to the problem of transport of spin waves in insulators [6, 7]. To jointly describe magnetization dynamics and transport, it is necessary has to extend the thermodynamic theory to vector magnetization. In this paper we present such theoretical approach and we discuss the possibility to predict magnetization instabilities and self-oscillations induced by the spin Hall effect.

2. Nonequilibrium thermodynamics

The non equilibrium thermodynamics of fluxes and forces applied to the transport of magnetic moment is based on a continuity equation for the magnetization and on a constitutive relationship between currents and generalized forces. For the scalar magnetization the continuity equation reads $\partial M/\partial t + \nabla \cdot \mathbf{j}_{\mathrm{M}} = H^*/\tau_{\mathrm{M}}$, where $\mathbf{j}_{\mathrm{M}}$ is the magnetic moment current, H^* is the thermodynamic effective field $H^* = H - H_{\mathrm{eq}}(M)$ given by the difference between the magnetic field H and the equation of state at equilibrium $H_{\mathrm{eq}}(M)$ and τ_{M} is a material dependent time constant. This last term corresponds to the presence of sinks and sources for the magnetic moment as expected in a continuity equation. The generalized force associated to the magnetic moment current results to be the gradient of the thermodynamic effective field H^*, then the constitutive relationship reads $\mathbf{j_M} = \mu_0 \sigma_{\mathrm{M}} \nabla H^*$ where σ_{M} is the magnetic moment conductivity, which is a material dependent quantity[5, 6] .

In ferromagnets, characterized by a vector magnetization $\mathbf{M}$, both equations must be generalized. A particularly crucial point is to take into account that the continuity equation becomes a dynamic equation for the magnetization vector and therefore it should also includes the conservative torque term giving rise to the precessional motion of the magnetization vector. By writing this equation in the Landau-Lifshitz-Bloch style [8] we have

$$\frac{\partial \mathbf{M}}{\partial t} + \nabla \cdot \mathbf{j_M} = -\mu_0 \gamma_{\mathrm{L}} \mathbf{M} \times \mathbf{H}_{\mathrm{eff}} + \frac{1}{\tau_{\mathrm{M}}} \mathbf{H}^* \quad (1)$$

where at the left-hand side of Eq. (1) one finds the time variation of the magnetization and the divergence of the magnetic moment current $\mathbf{j_M}$ (a tensor quantity) and at the right-hand side the conservative torque and the damping term. The precessional torque term depends on the effective field of micromagnetism $\mathbf{H}_{\mathrm{eff}}$, while the damping term depends on the vector thermodynamic effective field

$$\mathbf{H}^* = \mathbf{H} - \mathbf{H}_{\mathrm{eq}}(\mathbf{M}) \quad (2)$$

The last term at the right hand side of Eq.(1) expresses at the same time the sinks and sources for the magnetic moment and the damping for the magnetization dynamics. Eq.(1) contains μ_0, the permeability of vacuum, and γ_{L}, the gyromagnetic ratio for electron, which are known constants, and the time constant τ_{M} which is material dependent. In Eq.(2) $\mathbf{H}$ contains the contributions of both the applied and demagnetizing field, while $\mathbf{H}_{\mathrm{eq}}(\mathbf{M})$ is the vector equation of state at equilibrium. Similarly to the scalar case, the constitutive relation is formally written as $\mathbf{j_M} = \mu_0 \sigma_{\mathrm{M}} \nabla \mathbf{H}^*$ where σ_{M} is the magnetic moment conductivity, which is a now a tensor quantity.

The thermodynamic effective field $\mathbf{H}^*$ can be different from zero either because the magnetization is out of the equilibrium state or because the boundary conditions with a side layer are forcing a magnetic moment current which is absorbed by the ferromagnet. A complete solution of the joint problem would require to consider both the processes. Here we test the model by first solving the magnetic moment current problem along the specific direction imposed by the spin Hall effect layer and then by considering the additional effects of the out-of-equilibrium dynamics in the ferromagnet.

3. Magnetic moment current induced by the spin Hall effect

The spin orbit torque effects are realized in bilayers in which the magnetic moment current is generated in a metal with a large spin Hall effect (such as Pt, Ta and W) and is transmitted into an insulation ferromagnet (i.e. YIG) that will experience an addition torque effect along the direction of the magnetic moment brought by the current (see Fig.1). Here we consider an electric current density in the metal, j_{ey}, direct along y, so that the magnetic moment current induced by the spin Hall effect results to be along x and the direction of the magnetic moment along z. The intensity of the current source is $j_{\mathrm{MS}} = -(\mu_{\mathrm{B}}/e)\theta_{\mathrm{SH}} j_{\mathrm{ey}}$ where μ_{B} is the Bohr magneton, e is the elementary charge. θ_{SH} is the spin Hall angle that quantifies the efficiency of the conversion between charge current and magnetic moment current due to the spin Hall effect in metals with large spin-orbit coupling. The problem of the conduction of the magnetic moment along z through the layers can be solved taking both the continuity equation and the constitutive relationship in steady conditions. Considering both the the magnetic field and the magnetization are along z, Eq.(1) reduces to

$$\frac{\partial M_\mathrm{z}}{\partial t} + \nabla \cdot \mathbf{j}_{\mathrm{M_z}} = \frac{{H_\mathrm{z}}^*}{\tau_\mathrm{M}} \tag{3}$$

For steady states, the result is a diffusion equation for the effective field $l_i^2 \nabla_x^2 H_{i,z}^* = H_{i,z}^*$, that can be solved layer by layer containing as a parameter the diffusion length l_i of each material [6]. We select the following dimensions: material 1 (i.e. spin Hall metal) from $x = -t_1$ to $x = 0$ and material 2 (i.e. ferromagnetic insulator) from $x = 0$ to $x = t_2$ (see Fig. 1). With boundary conditions for the magnetic moment currents (we drop the subscirpt M for simplicity) $j_1(-t_1) = j_2(t_2) = 0$, $j_1(0) = j_2(0) = j_0$ we obtain the following expressions

$$j_1(x) = j_{\mathrm{MS}}\left[1+\frac{\sinh(x/l_1)}{\sinh(t_1/l_1)}\right] + (j_0 - j_{\mathrm{MS}})\frac{\sinh[(x+t_1)/l_1]}{\sinh(t_1/l_1)} \tag{4}$$

$$j_2(x) = -j_0 \frac{\sinh[(x-t_2)/l_2]}{\sinh(t_2/l_2)} \tag{5}$$

The corresponding potential is $H^*(x) = \tau_i \nabla_x j(x)$. We have therefore

$$H_1^*(x) = \frac{j_{\mathrm{MS}}}{v_1}\frac{\cosh(x/l_1)}{\sinh(t_1/l_1)} + \frac{j_0 - j_{\mathrm{MS}}}{v_1}\frac{\cosh[(x+t_1)/l_1]}{\sinh(t_1/l_1)} \tag{6}$$

$$H_2^*(x) = -\frac{j_0}{v_2}\frac{\cosh[(x-t_2)/l_2]}{\sinh(t_2/l_2)} \tag{7}$$

where the parameter $v_i = l_i/\tau_i$, with the dimensions of a velocity, represents the conductance of the layer. By using the boundary condition for the fields at $x = 0$, $H_1^*(0) = H_2^*(0)$, corresponding to a negligible interface resistance, we finally obtain the value of the current at the interface j_0 as

$$j_0 = j_{\mathrm{MS}} \frac{v_{\mathrm{eff}}}{v_1 \coth(t_1/(2l_1))} \tag{8}$$

where v_{eff}, defined as

$$\frac{1}{v_{\mathrm{eff}}} = \frac{1}{v_1 \tanh(t_1/l_1)} + \frac{1}{v_2 \tanh(t_2/l_2)} \tag{9}$$

is the total magnetic moment conductance of the bilayer. The sought potential in the ferromagnet (2) is therefore

$$H_2^*(x) = H_{2,0}^* \frac{\cosh((x-t_2)/l_2)}{\cosh(t_2/l_2)} \tag{10}$$

where

$$H_{2,0}^* = -\frac{j_{\mathrm{MS}}}{v_1 \coth(t_1/(2l_1))}\frac{v_{\mathrm{eff}}}{v_2 \tanh(t_2/l_2)} \tag{11}$$

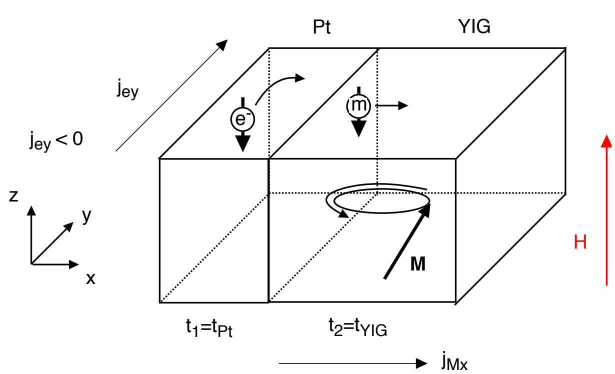

Figure 1: Scheme of the bilayer composed by (1) a metal with a large spin Hall effect (such as Pt, Ta and W) and (2) an insulating ferromagnet (i.e. YIG). The magnetic moment currents flows along x, the electric current is along y and the magnetic field is along z.

4. Magnetization dynamics

In view of the application of the non equilibrium thermodynamic theory to magnetization dynamics problems it is appropriate to derive the projections of Eq.(1) onto the plane perpendicular to the magnetization direction. By assuming negligible anisotropy, we take $\mathbf{H}_{\mathrm{eq}}$ of Eq.(2) along the magnetization direction. Then, by defining $\mathbf{m} = \mathbf{M}/M$, Eq.(1) reduces to

$$\frac{\partial \mathbf{m}}{\partial t} = -\mu_0\gamma_{\mathrm{L}}\mathbf{m}\times[\mathbf{H}_{\mathrm{eff}} - \alpha\mathbf{m}\times(\mathbf{H} - \tau_{\mathrm{M}}\nabla\cdot\mathbf{j_M})] \tag{12}$$

where $\alpha = (\mu_0\gamma_{\mathrm{L}}M\tau_{\mathrm{M}})^{-1}$ is the damping constant. It is worth to notice that the term $\nabla\cdot\mathbf{j_M}$ describes the absorption of magnetic moment in the ferromagnet which is generated in the other layer, then the term can be is written as an additional field $\mathbf{H}_{\mathrm{sot}} = \tau_{\mathrm{M}}\nabla\cdot\mathbf{j_M}$ representing the spin orbit torque. If $\mathbf{H} = \mathbf{H}_{\mathrm{sot}}$ one has the compensation of the damping and the magnetization dynamics reduces to the precessional equation $\partial\mathbf{m}/\partial t = -\mu_0\gamma_{\mathrm{L}}\mathbf{m}\times\mathbf{H}_{\mathrm{eff}}$.

In the present paper we limit to observe what happens if we just take, for the divergence of the magnetic moment current, the solution previously obtained in section 3 for the bilayer. We have $\mathbf{H}_{\mathrm{sot}}(x) = H^*_{\mathrm{YIG}}(x)\mathbf{e}_z$ where the space dependence is given by Eqs.(10) and (11). The direction of the moment injected is along z, then the spin orbit torque term is also along z and it can be either a damping or an anti-damping contribution depending on whether the magnetic moment injected is parallel or antiparallel to the magnetization of the ferromagnet. By taking the approximation of thin YIG thickness, $t_{\mathrm{YIG}} \ll l_{\mathrm{YIG}}$, we find at zero order in a $t_{\mathrm{YIG}}/l_{\mathrm{YIG}}$ expansion, the spin orbit torque field constant though the layer

$$H_{\mathrm{sot}} = -\frac{j_{\mathrm{MS}}}{v_{\mathrm{Pt}}\coth(t_{\mathrm{Pt}}/(2l_{\mathrm{Pt}})).} \tag{13}$$

The condition of the compensation of damping can be used to predict the critical current as a function of the magnetic field and can be compared with experimental data on the bilayer Pt/YIG of Ref.[2]. Experiments were conducted on thin YIG disks (20 nm thickness, 2μm and 4μm diameters) with applied magnetic field along a diameter ($H_{\mathrm{a,z}}$). The magnetic field $\mathbf{H}$ is due to the sum of the applied and the demagnetizing field. Assuming the demagnetizing coefficient $N_{d,x} \simeq 1$, we derive the condition for instability as $H_{\mathrm{az}} + M_s = H_{\mathrm{sot}}$. In this condition the magnetization $\mathbf{m}$ may be driven to an instability leading to self oscillations. The threshold current, in this approximation, can be predicted by using only the spin diffusion parameters of Pt

$$j_{\mathrm{ey}} = \frac{e}{\mu_{\mathrm{B}}}\frac{1}{\theta_{\mathrm{SH}}}v_{\mathrm{Pt}}\coth(t_{\mathrm{Pt}}/(2l_{\mathrm{Pt}}))(H_{\mathrm{az}} + M_s) \tag{14}$$

By taking the saturation magnetization of the YIG film $\mu_0 M_s = 0.215$ T, the Pt layer of thickness $t_{\mathrm{Pt}} = 8$ nm, $\theta_{\mathrm{SH}} = -0.056$ from [2] (the spin Hall angle for the magnetic moment is opposite with respect to the spin one) and by using the Pt values $l_{\mathrm{Pt}} = 7.3$ nm and $v_{\mathrm{Pt}} = 3$ m/s [6] we obtain the plot of Fig.2. The nice agreement at large magnetic field shows that the magnetic moment conductivity between layers can be well described by the thermodynamic approach presented here. At low magnetic fields, the fact that magnetization of the YIG may not be perfectly parallel to z due to incomplete saturation then giving a different type of magnetic moment conductivity, can explain the differences between model end experiments. This point can be improved by considering the injection of moments perpendicular to the magnetization.

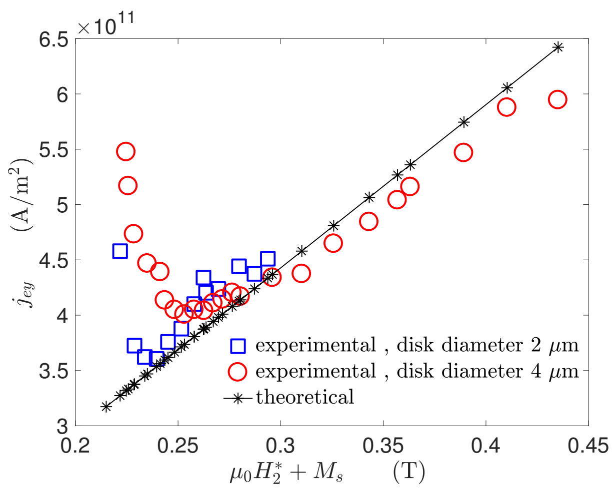

Figure 2: Threshold electric current density j_{ey} of Pt corresponding to the onset of self oscillations in YIG due to the spin orbit torque effect. Points: experimental data after Ref.[2] on YIG disks (20 nm thickness, 2μm and 4μm diameters). Line: theory, Eq.(14) with parameters given in the text. The magnetic field is $H = H_{az} + M_s$

5. Conclusions

In this paper we have extended the non equilibrium thermodynamic transport approach of Johnson and Silsbee [5] to describe vector magnetization. The resulting model is able to jointly describe magnetization dynamics and transport property. In the present study we have applied this model to the prediction of the magnetization instabilities that can be induced in an insulating ferromagnets because of the spin Hall effect in an adjacent Platinum layer. We have first solved the magnetic moment transport problem for the bilayer Pt/YIG with magnetization aligned along the z axis. Next we have projected the full equation into the plane perpendicular to the magnetization and we have used the previous result on the transport as a constraint. In this way the effect of the magnetic moment current injected in YIG can be interpreted as an extra thermodynamic field which depends on the penetration depth and may have the direction opposite to the damping. We have applied the model to the case of thin YIG with a uniform thermodynamic effective field and we have derived the critical values corresponding to the instability of YIG in good agreement with experimental results of Ref.[2]. Future efforts will be devoted to derive the details of the magnetization dynamics beyond the instability threshold.

References

[1] B. Lenk, H. Ulrichs, F. Garbs, M. Münzenberg, The building blocks of magnonics, Physics Reports 507 (2011) 107. doi:10.1016/j.physrep.2011.06.003.

[2] M. Collet, X. de Milly, O. d/'Allivy Kelly, V. V. Naletov, R. Bernard, P. Bortolotti, J. Ben Youssef, V. E. Demidov, S. O. Demokritov, J. L. Prieto, M. Munoz, V. Cros, A. Anane, G. de Loubens, O. Klein, Generation of coherent spin-wave modes in yttrium iron garnet microdiscs by spin-orbit torque, Nat Commun 7. doi:DOI:10.1038/ncomms10377.

[3] D. Ralph, M. Stiles, Spin transfer torques, Journal of Magnetism and Magnetic Materials 320 (7) (2008) 1190 – 1216. doi:https://doi.org/10.1016/j.jmmm.2007.12.019.
URL http://www.sciencedirect.com/science/article/pii/S0304885307010116

[4] Y. Tserkovnyak, A. Brataas, G. E. W. Bauer, Enhanced gilbert damping in thin ferromagnetic films, Phys. Rev. Lett. 88 (2002) 117601. doi:10.1103/PhysRevLett.88.117601.
URL http://link.aps.org/doi/10.1103/PhysRevLett.88.117601

[5] M. Johnson, R. H. Silsbee, Thermodynamic analysis of interfacial transport and of the thermomagnetoelectric system, Phys. Rev. B 35 (1987) 4959.

[6] V. Basso, E. Ferraro, A. Sola, A. Magni, M. Kuepferling, M. Pasquale, Nonequilibrium thermodynamics of the spin seebeck and spin peltier effects, Phys. Rev. B 93 (2016) 184421. doi:10.1103/PhysRevB.93.184421.

[7] V. Basso, E. Ferraro, M. Piazzi, Thermodynamic transport theory of spin waves in ferromagnetic insulators, Phys. Rev. B 94 (2016) 144422. doi:10.1103/PhysRevB.94.144422.
URL https://link.aps.org/doi/10.1103/PhysRevB.94.144422

[8] D. A. Garanin, Fokker-planck and landau-lifshitz-bloch equations for classical ferromagnets, Phys. Rev. B 55 (1997) 3050–3057. doi:10.1103/PhysRevB.55.3050.
URL https://link.aps.org/doi/10.1103/PhysRevB.55.3050