Charge relaxation in biological tissues with extremely high permittivity

Charge relaxation in biological tissues with extremely high permittivity – A subtle aspects of motion-induced fields in MRI

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Abstract— The paper presents a phenomenological model to describe charge relaxation (and the corresponding screening effect) in biological tissues, i.e., lossy dielectrics where a non-negligible conductivity and an extremely high relative permittivity (on the order of $50{\cdot}10^6$) may coexist. This situation is very peculiar and is not treated in textbooks. The model is used to study an open issue in electromagnetic dosimetry, namely the effect of charge relaxation on the motion-induced electric fields in the body of the operators working in Magnetic Resonance Imaging environments.

Index Terms—Biomagnetics, Motion-induced fields, Magnetic Resonance Imaging, Dielectric properties of biological tissues

I. INTRODUCTION

It is well known that charge separation occurs in conductors placed within an external electric field $\boldsymbol{E}_0$. In this case, free charges rearrange themselves over the surface of the body, giving rise to surface charge densities and creating a Coulombian field $\boldsymbol{E}_Q$ that tries to counteract $\boldsymbol{E}_0$. For good conductors, like copper, this charge relaxation process requires a time interval on the order of 10 fs [Ashby 1975, Ohanian 1983, Gauthier 1985, Bochove 1990, Elliott 1993]. Thus, if $\boldsymbol{E}_0$ is static or quasi-stationary, a complete screening (with a net field $\boldsymbol{E}_N = 0$ inside the body) is obtained in a time that, for many practical purposes, is negligible. In case of perfect dielectrics, the material polarizes and the dielectric dipoles oppose to $\boldsymbol{E}_0$ their own polarization field $\boldsymbol{E}_p$. So, depending on the relative permittivity ε_r, a partial shielding is obtained ($\boldsymbol{E}_N \neq 0$). Since most dielectrics exhibit a dispersive behavior with frequency, the screening capability depends on the rate of change of the applied field. The slower the dynamic is, the more the polarization mechanisms develop [Elliott 1993, Martinsen 2002]. Between these two extremes, we find lossy dielectrics, where the relaxation process occurs with a time constant τ given by the ratio between the absolute permittivity ε and the electrical conductivity σ of the material [Ohanian 1983]. A special case for these materials is given by biological tissues, to which, at very low frequency (around 1 Hz), an extremely high permittivity (EHP) $\varepsilon_r \approx 50{\cdot}10^6$ has been assigned [Polk 1996, Gabriel 1996a, Gabriel 1996b, Gabriel 1996c, Kuang 1998, Martinsen 2002, Stoneman 2010, Cundin 2011]. These astonishing values come from experiments affected by some technical difficulties (in particular, the polarization of the electrodes used to perform the measurement of the impedance of the biological sample) [Gabriel 1996a, Kuang 1998, Stoneman 2010], leading to high uncertainty levels. Nevertheless, extrapolations obtained through dispersion models (that fit very well the more accurate measurements in the radiofrequency band) are consistent with the experiments [Gabriel 1996c]. Moreover, some interpretative theories, which could explain the behavior (e.g. through the "counterion mechanism"), are available and the data do not infringe the bond given by Kramers-Kronig relations [Polk 1996, Kuang 1998, Martinsen 2002, Greenebaum 2006, Cundin 2011]. Thus, even if the due caution must be adopted before trusting such EHP, this feature becomes of some interest when studying human exposure to electromagnetic fields, and some technical standards have already adopted the EHP of biological tissues (for example, Standard IEC 62226-3-1 [2007] suggests an average $\varepsilon_r = 10^5$ for a human body at 50 Hz). The impact of the dielectric permittivity on the development of electromagnetic induction inside a human body at low frequency has been specifically investigated previously [Barchanski 2005, Zilberti 2015], finding that the dielectric phenomena do not provide a significant contribution when computing the faradic induced electric field. However, as mentioned by Jokela and Saunders [2011], a lack of knowledge affects charge relaxation in biological tissues, because an EHP leads to a non-vanishingly small time constant (around 50 ms, considering a conductivity on the order of 10 mS/m) and it might delay the screening effect due to the rearrangement of free charges against an external electric field. On the other hand, thanks to the strong polarization, EHP may at the same time act as a shield against the external field, hence giving rise to two opposite effects. This situation is very peculiar and it has never been analyzed, to the author's knowledge, in the literature, where the attention is typically focused on good conductors or insulators having "ordinary" permittivities.

The purpose of this paper is to propose a phenomenological model able to describe the dynamics of charge relaxation in non-magnetic lossy dielectrics with EHP, putting in evidence the evolution of the net electric field inside the material. As an example of application, the model is used to study an open problem in electromagnetic dosimetry: the effect of charge relaxation on the electric fields induced in the body of the workers moving through the stray stationary magnetic field of a Magnetic Resonance Imaging (MRI) scanner.

II. PHENOMENOLOGICAL MODEL

The proposed model relies on a crucial assumption: the relevant polarization mechanisms (as well as the changes of polarization across time) occurs on a time scale much shorter than the relaxation time constant τ in EHP materials. Having postulated this requirement, the following "step-by-step" procedure is adopted:

1) Given the harmonic spectrum (supposed to be quite narrow) of the driving term $\boldsymbol{E}_0$, the corresponding values of permittivity and conductivity for the material under examination are chosen. Note that EHP is typically exhibited at very low frequency, where parameter dispersion is almost negligible.

2) At the starting instant, when $\boldsymbol{E}_0$ is switched on, the material is considered as a perfect dielectric. Thus, the polarization of bound charges (quantified through ε) occurs instantaneously as a reaction to $\boldsymbol{E}_0$, creating the polarization field $\boldsymbol{E}_p$. This can be done because, thanks to the previous step, the value of ε has been selected as the effective permittivity exhibited by the material at the frequency of interest. Hence, within the object (domain Ω), free charges feel now exposed to the net field $\boldsymbol{E}_N = \boldsymbol{E}_0 + \boldsymbol{E}_p$.

3) The conduction current density $\boldsymbol{J} = \sigma\boldsymbol{E}_N$ is computed through Ohm's law. Its normal component, J_n, is then evaluated along the inner side of the bounding surface $\partial\Omega$ of the object.

4) The distribution of the surface charge density ς stored by J_n along $\partial\Omega$, is computed, after a suitably small time step Δt, according to the boundary condition $J_n = d\varsigma/dt$ [Redžić 2004a].

5) The Coulombian electric field $\boldsymbol{E}_Q$ produced by ς is calculated *in vacuo* and summed to the value of $\boldsymbol{E}_0$ (which, in the meantime, may have changed, according to its own time-behavior), to get the "total" field $\boldsymbol{E}_T$, which takes into account both the externally applied field and the field due to the migration of free charges inside Ω.

6) An updated value of the net field $\boldsymbol{E}_N$ is computed, as for point 2, considering the dielectric material exposed now to $\boldsymbol{E}_T$, instead of $\boldsymbol{E}_0$.

7) The procedure (points 3 to 6) is repeated for increasing values of time. Of course, when repeating point 4, the computation of ς does not start afresh, but the value obtained at the previous step is updated by adding (algebraically) the "new" charges stored along $\partial\Omega$ by J_n.

Note that, in addition to the basic hypothesis of different time scales for polarization and charge relaxation, the model assumes that the induced currents are confined within Ω ($\boldsymbol{J} = 0$ in the external air).

III. ANALYTICAL SOLUTION

The proposed procedure could be implemented in a numerical way. However, the availability of an analytical solution always provides useful insights into the main features of the phenomenon. In order to get such an analytical solution, we consider a homogeneous sphere exposed to an external field $\boldsymbol{E}_0$, which, in absence of the sphere itself, would be uniform and directed along the z-axis. The material surrounding the sphere is vacuum, with permittivity ε_0. Under these circumstances, it is well known [Stratton 1941] that, in case of a perfect dielectric material, a uniform net field

$$\boldsymbol{E}_N = \frac{3\boldsymbol{E}_0}{2+\varepsilon_r}, \tag{1}$$

parallel to the "original" field $\boldsymbol{E}_0$, establishes inside the sphere.

On the other hand, in case of a sphere composed of a perfect electric conductor, the redistribution of free charges gives rise to a surface charge density that produces a field equal and opposite to $\boldsymbol{E}_0$ inside the sphere and nullifies $\boldsymbol{E}_N$. Such charge density is [Stratton 1941]

$$\varsigma(\theta) = 3\varepsilon_0 E_0 \cos\theta \tag{2}$$

where angle θ indicates the colatitude, whose cosine is given by the inner product between the unit vector of the z-axis, $\hat{z}$, and the outward unit vector normal to the surface in the considered point. Note that ς is negative in the hemisphere where the external field enters the surface, positive in the other hemisphere and null on the equatorial plane. By reversing the reasoning, a generic surface charge density with the same spatial distribution (i.e., $\varsigma = \varsigma_M\cos\theta$) produces, within the spherical volume, a uniform Coulombian field directed toward the negative z-axis (i.e., opposite to $\boldsymbol{E}_0$)

$$\boldsymbol{E}_Q = -\frac{\varsigma_M}{3\varepsilon_0}\hat{z}\,. \tag{3}$$

If the sphere is composed of a non-magnetic material with conductivity σ and extremely high relative permittivity ε_r (leading to a relatively long relaxation time constant τ), the previous equations can be exploited to carry on the proposed procedure analytically. At the starting instant t_1, when the external field is switched on, polarization takes place instantaneously, while charge relaxation has a negligible influence. Thus, the net electric field $\boldsymbol{E}_N(t_1) = E_N(t_1)\hat{z}$ is computed via (1) and results to be much weaker than $\boldsymbol{E}_0(t_1)$. If the conductivity and the rate of change of the field quantities are small enough to avoid any skin effect, this net electric field drives a uniform current density, whose component normal to the sphere surface is $J_n(t_1) = \sigma E_N(t_1)\cos\theta$. Now, assuming that such current density operates for a small time increment Δt (much shorter than τ), the surface charge density stored along the surface of the sphere results to be

$$\varsigma(t_2,\theta) = J_n(t_1)\Delta t = \varsigma_M(t_2)\cos\theta\,, \tag{4}$$

where $t_2 = t_1 + \Delta t$. This charge density produces a Coulombian field $\boldsymbol{E}_Q(t_2)$ (in vacuo), parallel but opposite to $\boldsymbol{E}_0$, as indicated by (3). So, at the second time step t_2, the bound charges of the sphere experiences a total field $\boldsymbol{E}_T(t_2) = \boldsymbol{E}_0(t_2) + \boldsymbol{E}_Q(t_2)$, and a new step of computation can start, replacing $\boldsymbol{E}_0$ with $\boldsymbol{E}_T$ when applying relation (1). Note that the balance $\boldsymbol{E}_N = \boldsymbol{E}_T + \boldsymbol{E}_p$ (instead of $\boldsymbol{E}_N = \boldsymbol{E}_0 + \boldsymbol{E}_p$) holds for $t > t_1$, and the surface charge density must be updated as

$$\varsigma(t_i) = \varsigma(t_{i-1}) + J_n(t_{i-1})\,\Delta t\,. \tag{5}$$

IV. APPLICATION TO MOTION-INDUCED FIELDS

As already mentioned, the analytical solution here developed is used to study an open issue in the evaluation of motion-induced electric fields due to the movement of a human body through the stray field of an MRI scanner. At a first view, such problem has nothing to do with the situation described above (exposure to a magnetic field, instead of an electric field). However, the link among this practical problem and the phenomenon of charge relaxation becomes evident if we recall that, in a reference frame co-moving with the body, the current-carrying wires producing the magnetic field appear to be charged [Rosser 1968]. Hence, the body feels exposed not only to a magnetic field, but also to an electric field (actually, it would be also exposed to the electric fields due to the voltages applied to the current-carrying wires, which have a capacitive coupling; in our analysis, we do not focus the attention on these electric fields, as if they were shielded). This situation can be reminded through a simple example of relativistic electromagnetism.

Let us consider two very long wires, parallel to the *x*-axis, laid at distance *D*, carrying equal but opposite electric currents of amplitude *I*. Now consider a non-magnetic sphere, composed of a lossy dielectric with EHP, located in the middle between the wires and translating along the *x*-direction, at speed *v*. If the size of the sphere is small with respect to *D*, in a first approximation we can assume that it is exposed to a uniform magnetic flux density [Feynman 1963]

$$\boldsymbol{B} = 2\left[\mu_0 \frac{I}{2\pi\left(D/2\right)}\right]\hat{y}, \tag{6}$$

where μ_0 is the magnetic permeability of vacuum and $\hat{y}$ is the unit vector of the *y*-direction. Hence, in this example, $\boldsymbol{B}$ is perpendicular to $\boldsymbol{v}$. The current density flowing along the wires is uniform on their cross section *s* (under the assumption of negligible Hall effects mutually produced by the conductors). Its magnitude is

$$J_w = \frac{I}{s} = \frac{B}{\mu_0}\frac{\pi\left(D/2\right)}{s}. \tag{7}$$

(The author is aware that a self-induced Hall effect is also present within each wire [Matzek 1968, Peters 1985, Hernàndez 1988, Gabuzda 1993, Redžić 2012], but, for the purpose of the paper, it would not introduce any significant modification to the discussion).

As prescribed by relativity [Rosser 1968], the two wires appear to be charged when observed from the reference frame of the moving sphere. Despite the very low speeds of interest for this paper, the volume charge density is not negligible and can be computed as

$$\rho = \frac{vJ_w}{c^2} = \frac{vB\pi\left(D/2\right)\varepsilon_0}{s}. \tag{8}$$

Note that we do not restrict the analysis to constant value of *v*, because, if the acceleration is relatively low, it is possible to work with the "instantaneous rest frame" of the sphere [Van Bladel 1984]. The corresponding charge density per unit of length is

$$\lambda = \rho s = vB\pi\varepsilon_0\left(D/2\right). \tag{9}$$

The sign of these charge densities is positive for the wire where the electric current flows in the opposite direction with respect to $\boldsymbol{v}$, and negative for the other wire. In the reference frame of the moving sphere, the two charged wires produce an "environmental" electric field. In the middle between the two wires, the magnitude of this electric field is [Feynman 1963]

$$\boldsymbol{E}_0 = 2\left[\frac{\lambda}{2\pi\varepsilon_0\left(D/2\right)}\right]\hat{z} = \boldsymbol{v}\times\boldsymbol{B}, \tag{10}$$

which, according to our approximation (sphere small with respect to *D*), can be considered uniform in correspondence to the sphere.

This simple example reminds that, in its co-moving reference frame, the sphere experiences an externally-applied electric field numerically equal to the term $\boldsymbol{v}\times\boldsymbol{B}$ (i.e., the motional part of the Lorentz force per unit of charge) computed by an observer at rest with respect to the wires. Thus, the description of charge separation developed for the exposure to an external electric field can be adopted, *ceteris paribus*, also in case of motion through a stationary magnetic field. Note that, in our analysis, we have implicitly made use of the so-called Galilean Magnetic Limit approximation, which does not prescribe any transformation for $\boldsymbol{B}$ when passing from the reference frame of the wires to that of the moving body [Le Bellac 1973, de Montigny 2007, Heras 2010].

The deviation from local neutrality in a body moving through a stationary magnetic field has been pointed out in some seminal articles describing systems at equilibrium [Lorrain 1990, Lorrain 1998, Redžić 2001, Lorrain 2001]. Then, this aspect has been deepened in some very interesting papers by Bringuier [2003, 2004], who discussed the steady-state solution for conductors and perfect dielectrics, and by Redžić [2002, 2004a, 2004b, 2010], who focused the attention on good conductors, where dielectric currents are negligible and charge separation occurs almost instantaneously. However, none of these papers deals with lossy dielectrics having EHP, where the mixed screening effects of the strong permittivity and of charge separation must be accounted for at the same time.

Concerning the motion of a human body through a stationary magnetic field, specific computational techniques have been developed to evaluate the induced electric field with a rotational nature (i.e., associated to a magnetic field that, due to motion, appears to be time-varying in the reference frame of the body) [Cobos-Sànchez 2009, Chiampi 2011, Cobos-Sànchez 2013, Chiampi 2013, Laakso 2013, Trakic 2014]. These approaches are also able to compute, for any time instant, the distribution of charge density along the surface of the body, but they assume that charge separation occurs instantaneously. In other words, these computational tools would predict a net electric field equal to zero in our example involving the sphere translating in a uniform magnetic field (see Table 1 in [Cobos-Sànchez 2012]). This solution describes correctly the steady-state situation of the sphere translating at constant speed, when charge separation has concluded and the Coulombian field due to the rearrangement of the free charges counterbalances the Lorentz force perfectly. However, as already explained, τ may not be completely negligible in human tissues [Jokela 2011] and the corresponding delay implies a non-zero net electric field under dynamic conditions. A very preliminary attempt to quantify such a net field was proposed in [Zilberti 2013], but without taking into account the screening effect due to polarization. Another step towards a realistic description of motion-induced electric fields was done in [Zilberti 2015, Zilberti 2016], which described formulations written in the co-moving frame of the body taking into account the dielectric part of the induced currents. However, also in these cases the effect of charge separation was not observable, because the driving term was restricted to the magnetic field (i.e. the source of the faradic induced field), without considering that, in the adopted co-moving frame, the body feels exposed to an additional electric field due to the "relativistic charges".

To show a quantitative evaluation of $\boldsymbol{E}_N$ in a biological tissue with EHP, and, at the same time, to discuss the contribution of charge relaxation to motion-induced electric fields in MRI, we refer to a sphere, filled with a material corresponding to the gray matter of a human brain. We imagine that the sphere translates along the *x*-axis, through a stationary and uniform magnetic field (with B = 1 T) directed along the *y*-axis. As shown with (6)-(10), in its own reference

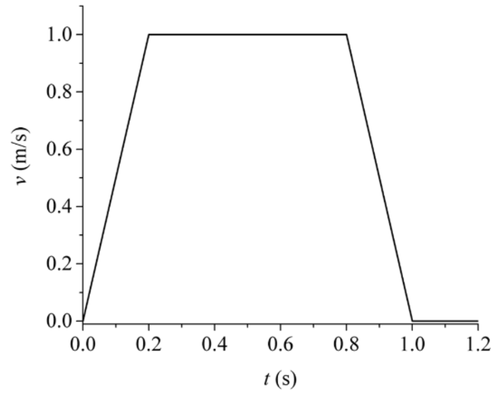

Fig. 1. Speed profile adopted in the computation, involving an acceleration, a phase at uniform speed, and a deceleration that reduces the speed to zero.

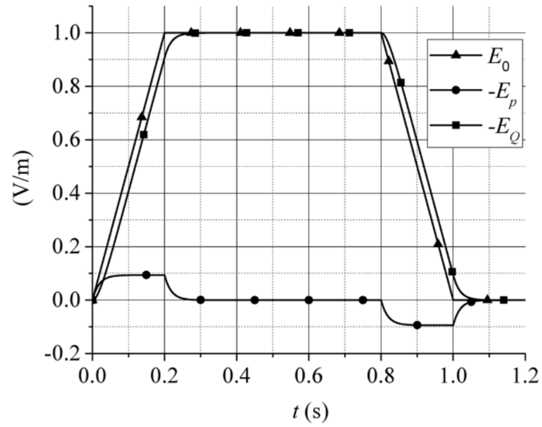

Fig. 2. Time evolution of the field due to external sources (E_0) and of the fields due to bound and free charges (E_p and E_Q, respectively). These latter are plotted with reversed sign to facilitate the comparison.

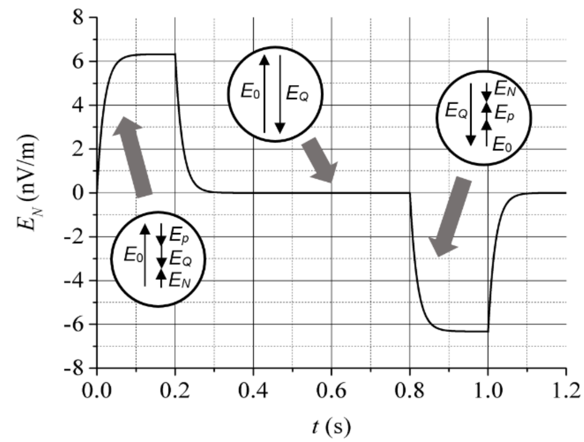

Fig. 3. Time evolution of the net electric field E_N due to the superposition of the other electric fields (sketched qualitatively in the insets).

frame the sphere experiences an electric field $\boldsymbol{E}_0 = E_0\hat{z}$, whose magnitude follows the time behavior of v. This happens because $\boldsymbol{B}$ is stationary and we assume that the currents induced in the biological material are too low to perturb the distribution of $\boldsymbol{B}$ impressed by the external sources [Cobos-Sànchez 2009, Chiampi 2011, Cobos-Sànchez 2012, Cobos-Sànchez 2013, Chiampi 2013, Laakso 2013, Trakic 2014, Zilberti 2015, Zilberti 2016].

For this example, the speed profile has been chosen to be compatible with a normal human movement (Fig.1). A Fourier decomposition applied to $v(t)$ easily reveals that the harmonic content of this signal (and therefore of the driving term $\boldsymbol{E}_0$) is almost completely concentrated within 5 Hz. For this narrow frequency band, the 4th-order Cole-Cole dispersion model typically used to obtain the dielectric properties of biological tissues shows negligible dispersion of parameters and provides $\varepsilon_r \approx 44.5\cdot10^6$ and $\sigma \approx 21$ mS/m for gray matter [Hasgall 2015], leading to $\tau \approx 18.8$ ms.

Figure 2 shows the z-component (i.e., the only non-zero component) of $\boldsymbol{E}_0$ (which simply reflects the behavior of v), of the polarization field $\boldsymbol{E}_p = E_p\hat{z}$ and of the Coulombian field $\boldsymbol{E}_Q = E_Q\hat{z}$, obtained by applying the proposed computational procedure to the current example (with Δt set to $\tau/100$). In order to facilitate the comparison with the magnitude of $\boldsymbol{E}_0$, the z-component of both $\boldsymbol{E}_p$ and $\boldsymbol{E}_Q$ is plotted with a reversed sign in this figure.

At the very first instants, $\boldsymbol{E}_p$ reacts to $\boldsymbol{E}_0$ instantaneously, whereas some inertia is exhibited by $\boldsymbol{E}_Q$. This latter quantity follows almost perfectly the typical response of a 1st-order dynamical system, with a time constant equal to τ (note that this behavior was not explicitly imposed in the phenomenological model). After the first instants, during the acceleration phase, $|E_Q|$ chases E_0, while E_p tends to saturate. Then, during the phase at uniform speed, the magnitude of $\boldsymbol{E}_Q$ practically reaches the one of $\boldsymbol{E}_0$, and E_p reduces to zero. This situation complies with a steady-state (possible because the sphere, which has a finite extent, translates at constant speed through a uniform field, and does not experience any variation of $\boldsymbol{B}$) when a perfect screening of the external field is obtained within the body and both the conduction current density and the polarization must vanish [Redžić 2004a]. Finally, during the deceleration phase, the free charges accumulated along the external surface of the sphere go back to restore local neutrality, but, owing to their delay with respect to the change of $E_0(t)$, the direction of the total field $\boldsymbol{E}_T$ is now reversed (i.e. it points toward the negative z-axis) and therefore $\boldsymbol{E}_p$ is reversed too.

Figure 3 shows the time evolution of the z-component of the net field $\boldsymbol{E}_N = E_N\hat{z}$ within the sphere. From a qualitative viewpoint, its behavior is the same as for the polarization (and also for the conduction current density), with equilibrium plateaus joined by transient evolutions having a time constant τ. What is interesting to note is that, due to the combined shielding effect of free and bound charges, the magnitude of the net field is always extremely small with respect to E_0. In particular, even at the quite high flux density considered here (B = 1 T), the amplitude of $\boldsymbol{E}_N$ results to be many orders of magnitude smaller than the typical faradic electric fields induced inside a human body moving through the stray field of an MRI scanner (around 100 mV/m, for the medical staff moving within the stray flux density of a real MRI scanner, on the order of some hundreds of millitesla) [Cobos-Sànchez 2009, Chiampi 2011, Cobos-Sànchez 2012, Cobos-Sànchez 2013, Chiampi 2013, Laakso 2013, Trakic 2014, Zilberti 2015]. The results of the model scales linearly with the value of the magnetic flux density and therefore these considerations would not change by increasing the value of B.

In conclusion, the proposed example proves that, despite the relatively long time interval needed to obtain the migration of free charges, the extremely high permittivity attributed to biological materials makes this net electric field a second order effect when dealing with motion-induced electric fields in homogeneous objects. Further investigations are required to check the validity of these results in case of heterogeneous structures, like a human body, where significant parameter discontinuities may be present at internal boundaries.

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