Uncertainty of measurement of calibrated test lengths realised by interferometry in ISO 10360-2 testing

Uncertainty of measurement of calibrated test lengths realised by interferometry in ISO 10360-2 testing

Alessandro Balsamo

Technical Report INRIM 5/2014

April 2014

This Technical Report was written in cooperation with the *Associazione CMM Club Italia*

Disclaimer

Even if this technical report originates from and is rooted into the ISO standardisation work, it expresses solely the author's opinion, and is currently not endorsed by ISO in any form.

The author fully recognises the authority of the ISO/TC213 on this subject and does not intend to question or hinder any current or future standard or document by that Technical Committee in any way. This technical report is published in the absence of any official ISO document on this subject. Its technical content is to be regarded as obsolete and outdated as soon as any relevant ISO document is published. Readers are asdvised to make sure that no updated ISO document has been published on this subject, prior to applying the guidance give in this technical report.

Abstract

The ISO 10360-2 is concerned with testing CMMs (*Coordinate Measuring Machines*) using alternative calibrated test lengths, realised either by material standards or by interferometry. Whatever the calibrated test lengths, the ISO 10360-2 requires the evaluation of the test value uncertainty, and hence of its several components; one of them is the uncertainty in realising the calibrated test lengths.

This technical report gives guidance on how to evaluate the uncertainty incurred in realising calibrated test lengths by interferometry, equivalent to the calibration uncertainty when the calibrated test lengths are realised by material standards.

This uncertainty, $u(\varepsilon_{\mathrm{int}})$, is just a component of the overall test value uncertainty. Other components – such as clamping and overall alignment – are beyond the scope of this technical report. Readers are remembered not to overlook the other uncertainty components.

Contents

Introduction

The ISO 10360-2 [1] is probably the most recognised standard in the field of coordinate metrology, widely applied in industry as well as in research. It is concerned with testing CMMs (*Coordinate Measuring Machines*) according to a standardised procedure, to ascertain whether an individual CMM conforms to a given specification, namely to a *maximum permissible error of length measurement*, $E_{\mathrm{L,MPE}}$ ([1] § 3.6). Typical applications of the test are in acceptance of CMMs at purchase and in periodic reverifications.

In addition, the application of the ISO 10360-2 is deemed a possible path to CMM traceability: even if incomplete – in the true sense of the VIM definition ([2][3] § 2.41), nonetheless it provides documented and traceable evidence that the basic CMM capability of yielding point-to-point distances in space is within specified limits. In recognition of this, documented conformity of CMMs to predefined $E_{\mathrm{L,MPE}}$'s according to the ISO 10360-2 is often considered a basic and essential requirement whenever metrological confirmation of a CMM is pursued, e.g. in Quality Systems. To help in this, numerous calibration laboratories worldwide are officially accredited for performing the ISO 10360-2 test; the accreditation bodies of some countries have even published guidance and/or compulsory documents on this subject (e.g. [4],[5]).

The principle of the ISO 10360-2 is the comparison of CMM indications of point-to-point distances with known corresponding values of calibrated test lengths: the discrepancies are referred to as *length measurement errors*, E_{L}'s ([1] § 3.4), and must be within the corresponding specifications $E_{\mathrm{L,MPE}}$'s for a CMM to pass the test. Clause 7.1.1 specifies that the comparison of each E_{L} with its corresponding $E_{\mathrm{L,MPE}}$ must take "*into account the uncertainty according to ISO 14253-1*". In turn, the ISO 14253-1 [6] involves the measurement uncertainty to prove conformity or non-conformity to specification. As a consequence, an uncertainty value for each length measurement is required to carry out a complete ISO 10360-2 test.

In recognition of the difficulty of – and of the possible controversy in – evaluating the uncertainty of the E_{L}'s measured in ISO 10360-2 tests, the ISO/TS 23165 [7] was published in 2006. It is a guidance document to help testers and tester counterparts evaluating the test value uncertainty, and to prevent controversies between parties at doing. Unfortunately, the previous version of the ISO 10360-2 (revision 2 [8]) was in force when the ISO/TS 23165 was published. As a consequence, the ISO/TS 23165 is currently not aligned with the ISO 10360-2. As the current version [1] is essentially an extension of the previous one [8], the ISO/TS 23165 is still relevant and useful, but incomplete.

To correct this, a revision project of the ISO/TS 23165 was initiated in the competent ISO/TC213/WG10 [9]. While doing, it was realised that some concepts therein were in fact more general than for CMMs only. Then the competent ISO/TC213/WG4 [10] was involved, and is currently at work at a ISO/DIS 14253-5 [11], addressing the evaluation of test value uncertainties of any GPS indicating measuring instrument, including CMMs.

While on one hand the ISO/DIS 14253-5 will lay a sound and unified conceptual ground, on the other hand it has mothballed the revision project of the ISO/TS 23165, hierarchically lower. Meantime, testers and tester counterparts practising the novelties of the ISO 10360-2 have no guidance on the evaluation of the test value uncertainty not covered in the ISO/TS 23165. This problem is particularly acute when the calibrated test lengths are implemented by interferometry: in fact, CMM practitioners are often not familiar with interferometry, and evaluating the uncertainty may be difficult to them. Fortunately, the revision project of the ISO/TS 23165 had time enough to address interferometry before being mothballed: even if the resulting text was not fully worked out and given proper consensus by the ISO/TC213/WG10, still it conveys information that may be valuable for CMM practitioners.

This technical report makes use of and complete the work already done in the ISO/TC213/WG10 about interferometry, currently mothballed and awaiting for the ISO/DIS 14253-5 project completion, with the intention of providing guidance on the evaluation of the test value uncertainty.

1 Scope

This technical report is concerned with giving guidance to CMM testers and tester counterparts, about the evaluation of the measurement uncertainty of calibrated test lengths realised by interferometry, while performing a test on a CMM according to the ISO 10360-2 [1].

This document is not a full guide on the evaluation of the test value uncertainty, as it focuses on one uncertainty component only, namely the realisation of a calibrated test length by interferometry. Readers are encouraged to separately consider any other relevant uncertainty component, and to propagate them all – together with the one dealt with in this technical report – to obtain the test value uncertainty to be used in deciding conformity or non conformity to specification according to the ISO 10360-2.

Even if the main intent of this technical report is to support the application of the ISO 10360-2, the guidance given can be used in other applications too, whenever an interferometric measurement is made in combination with a moving machine.

2 Measurement uncertainty of calibrated test lengths obtained by interferometry, $u(x_{\mathrm{int}})$

In an interferometric measurement, a retroreflector is moved along a laser beam; this causes the periodic phenomenon of optical interference in the return path, and the displacement is measured by counting the number of cycles and their fractions. The amount of displacement corresponding to a full cycle is a predefined fraction – typically a half – of the wavelength of the laser beam in air; as a consequence, the interferometer effectively works as a ruler whose marks are determined by the laser wavelength in air.

The wavelength in air, λ, depends on the optical frequency of the beam, f, and on its propagation speed in air, v: the former determines the wavelength in vacuo, λ_0, and the latter the refractive index of air, n:

$$\lambda_0 = \frac{c}{f}; \quad n = \frac{c}{v}; \quad \lambda = \frac{v}{f} = \frac{\lambda_0}{n}$$

where c is the speed of light in vacuo.

The value of the wavelength in vacuo is a characteristic of the laser source – usually kept constant by proper stabilisation – and interferometers have this value stored in their firmware.

The refractive index depends on the thermodynamic condition of the air and on its chemical composition; pressure, temperature and humidity are the most important influence quantities. To account for this, interferometers either are equipped with air sensors (so called weather stations) or allow for manual input of the refractive index (sometime referred to as VOL, Velocity Of Light).

In usual circumstances, a recommended error model of an interferometric measurement is

$$\varepsilon_{\mathrm{int}} = (\eta_\lambda - \eta_n) x_{\mathrm{int}} + \Delta n\, l_{\mathrm{DP}} - \frac{\theta^2}{2} x_{\mathrm{int}} + \varphi b$$

where

$\eta_\lambda = \frac{\lambda_{0\mathrm{F}}}{\lambda_0} - 1$	is the relative error of the wavelength in vacuo
$\lambda_{0\mathrm{F}}$	is the value of wavelength in vacuo stored in the interferometer firmware
λ_0	is the actual wavelength in vacuo
$\eta_n = \frac{n_{\mathrm{W}}}{n} - 1 \approx n_{\mathrm{W}} - n$	is the (relative) error of the refractive index of air
n_{W}	is the value of refractive index of air measured by the weather station (or input manually)
n	is the actual refractive index of air
x_{int}	is the quantity measured by interferometry (the measurand)

Δn	is the variation of refractive index of air during the measurement of x_{int}
l_{DP}	is the length of dead path, i.e. the distance of the retroreflector to the beam splitter when the interferometer counter is reset
θ	is the angle of misalignment between the laser beam and the movement of the retroreflector
φ	is the parasitic rotation of the slider, combination of its yaw and pitch
b	is the Abbe arm, i.e. the distance of the retroreflector to the line of the calibrated test length.

(see Figure 1 and Figure 2).

The first two terms in the above equation are specific for interferometry; the other two are the misalignment (cosine) and Abbe (sine) errors, likely to occur in different implementations of the calibrated test lengths, too.

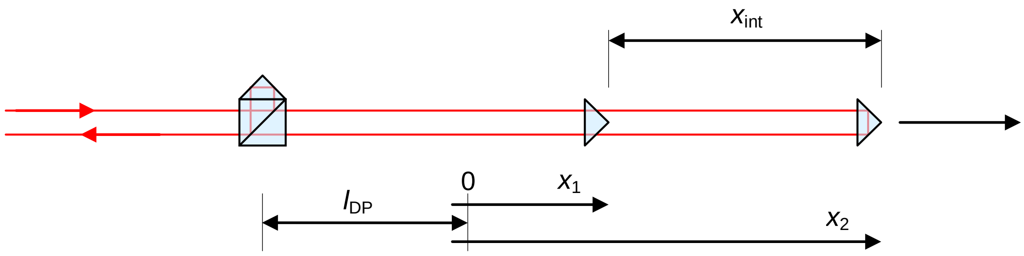

Figure 1 – Scheme of an interferometric measurement

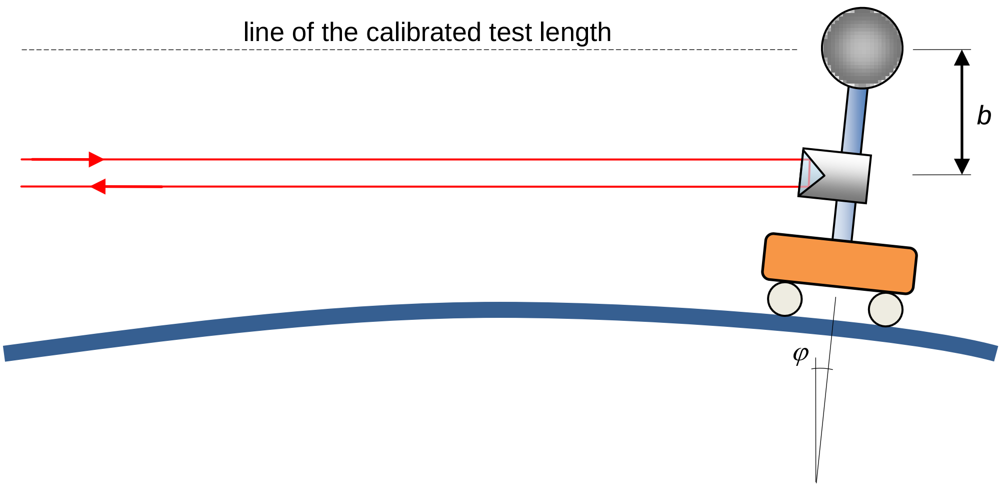

Figure 2 – Scheme of the Abbe error

2.1 Main sources of error

The main sources of error are the following:

a) Laser wavelength in vacuo, $\eta_\lambda x_{\text{int}}$. The actual laser radiation frequency – and hence wavelength in vacuo – is not exactly equal to the value stored in the interferometer firmware. This accounts for the traceability to the metre.

   It is an error proportional to the measurand x_{int}.

b) Refractive index of air, $-\eta_n x_{\text{int}}$. The refractive index can be either measured by a weather station or input manually. It depends mostly on temperature, pressure and humidity of air. Its value may vary from point to point due to local values of these parameters. The error in the refractive index of air is not only due to the measurement but also to the variability of the measurand along the beam. This variability is a combination of:

- Steady non uniformity along the beam. In particular, vertical variations are expected due to thermal gradients (induced by convection) and pressure gradients (due to weight of the column of air, 12 Pa/m).

- Zero-mean random variations (noise): the mean value over time of the refractive index is uniform along the beam, but the instantaneous value at any point fluctuates. Turbulences and sudden air flow (due to e.g. opening doors or gates) are typical sources of such variations. This variability effectively behaves as measurement noise.

Also, weather stations or other measuring instruments – and particularly pressure sensors – may suffer long term drift: periodical recalibration is needed and a drift uncertainty component should be added.

It is an error proportional to the measurand x_{int}.

c) Dead path, $\Delta n l_{\mathrm{DP}}$ (see Figure 1). At any time, the actual interferometer indication is evaluated in firmware as $x = N\lambda_0 / n$, where N is the interference count, and n is the refractive index of air. The count was reset at a certain point of time, when the retroreflector was l_{DP} away from the beam splitter. If the refractive index drifts during the measurement, Δn, the consequent apparent shift is proportional to the current distance of the retroreflector to the beam splitter, while the compensation accounts up to the reset point only, $N\lambda_0$. Therefore an error occurs, proportional to the remaining part, i.e. the dead path, l_{DP}.

It is a zero error.

d) Misalignment of the laser beam, $-\frac{1}{2}\theta^2 x_{\mathrm{int}}$. The interferometer is sensitive to the displacement of the retroreflector in the direction of the laser beam only. If the retroreflector moves on a slightly different direction, the component orthogonal to the beam is not sensed, and introduces an error. It is often referred to as the cosine error, as the measured quantity is the cathetus while the measurand is the hypotenuse of a triangle.

In principle, it is an error proportional to the measurand x_{int}; however, as the beam alignment is based on centring the beam at the end points of the measured stroke, the residual misalignment angle, θ, is inverse proportional to the measurand x_{int}, resulting in a misalignment error also inverse proportional to the measurand x_{int} (see § 2.3 for more details).

e) Abbe error, φb (see Figure 2). When the slider displacing the retroreflector is affected by yaw and pitch, any distance of the retroreflector to the line of the calibrated test length, b, introduces an error. In the case of a retroreflector directly attached onto the ram, the line of the calibrated test length coincides with the laser beam, and the distance b is null by definition, resulting in no error.

It is an error independent of the measurand x_{int}.

NOTE 1 The rotation angle φ is actually a component only of yaw and pitch, in a plane through the laser beam and the line of the calibrated test length. For the purpose of uncertainty evaluation, this issue is usually disregarded for sake of simplicity: φ is estimated in full, resulting in a slight uncertainty overestimation.

2.2 Recommendations to minimise the errors

The following is recommended to minimise the effects above:

a) Laser wavelength in vacuo. This error is effectively the calibration error of the laser interferometer. It possesses some peculiarities:

- The value of the wavelength in vacuo stated in a calibration certificate, $\lambda_{0\mathrm{cal}}$, can take effect only if the value stored in the interferometer firmware, $\lambda_{0\mathrm{F}}$, (a) can be updated, or (b) is disclosed with full accuracy (i.e. seven or more significant digits) and software means exist to correct the interferometer indication by a factor $\lambda_{0\mathrm{cal}} / \lambda_{0F}$. When neither option is available, the calibration value is ineffective, and can be merely used to assess the amount of error, η_λ.

- Laser sources exhibit an intrinsic reproducibility guaranteed by the physical properties of their laser media. In the most popular case of red lasers with a 633 nm nominal wavelength, the value of $\lambda_{0\mathrm{cal}} = 632.9908$ nm can be used with no need of calibration, with a relative standard uncertainty of $u(\lambda_{0\mathrm{cal}})/\lambda_{0\mathrm{cal}} = 1.5\cdot10^{-6}$ [12].

Further, lasers take a settling time to stabilise their wavelength, and should be turned on sufficiently prior to the test start. The settling time can be derived either from the interferometer technical documentation, or from calibration certificates (when this investigation is carried out at calibration and reported).

b) Refractive index of air. To minimise the effects of steady non uniformity along the laser beam, it is recommended to locate the weather station close to the central point of the measurement line; when this is impractical, at the same height of the central point, as the vertical component is most relevant. Also relevant may be the horizontal components induced by air flow, due to e.g. open doors or gates.

The effect of zero-mean random variations of the air (noise) is immediate, while the response of the weather station (or of other equivalent instruments) attempting to compensate is much slower. This can be mitigated by slowing the interferometer response down and setting a proper integration time, whose amount can be adjusted so that the repeatability of measurement at still is sufficiently small.

The propagation of pressure in air is very fast, and spurious measured peaks may result from sudden and short perturbations. It is recommended that the test is performed in the absence of significant air flows, or that shields are put around to protect, particularly when the MPE's to test against are very small, and/or the environmental condition is not very favourable.

It is also recommended that the weather station or other instruments are regularly recalibrated, to minimise the effects of long term drift.

c) Dead path. It is recommended that the beam splitter is put as close as possible to the near endpoint of the measuring line, to minimise the length of dead path, l_{DP}.

It is also recommended to reset the interferometer counter when the retroreflector is as close to the beam splitter as possible, possibly even closer than the measurement endpoint. This protects at best from air drifts, by enabling the best match between physical effect and its compensation.

A larger variation in the refractive index of air is expected in long measurements, as the air is given time to drift. It is recommended to speed up measurements, and particularly the portions of them involving the measurement of the point pairs at the extremes of each calibrated test length.

To protect from sudden changes, the same is recommended as in b).

d) Misalignment of the laser beam. When the retroreflector is attached to the ram, the alignment should be of the laser beam to the ram motions. In this case, either the laser is aligned to the motion, or the motion to the laser. In the former case, the CMM part programme is predefined: to align the interferometer, the instructions in the interferometer manual should be followed with great care. In the latter, the laser is directed first, and then the CMM is driven onto the laser beam nearby and far off the beam splitter: two points in space are taken and used to define the straight line of movement.

When the retroreflector is attached to a slider, the alignment should be of the laser beam to the slider motion. In this case, it is recommended to follow the directions given by the slider manufacturer. When an alignment procedure is foreseen, it is recommended to apply it prior of any CMM testing, with the equipment in a handy position, e.g. horizontal on the CMM table.

e) Abbe error. It is fully dependent on the design and/or the mechanical quality of the device implementing the calibrated test length. Given a certain piece of equipment, there is usually little the tester can do to minimise this error. It is recommended that the two relevant parameters – the Abbe arm, b, and the parasitic angle, φ – are made sufficiently small by construction. This error is null by definition when the retroreflector is attached to the ram.

2.3 Evaluation of the uncertainty components

The standard uncertainty of the calibrated test length, $u(\varepsilon_{\mathrm{int}})$, is the quadratic sum of the above uncertainty components:

$$u(\varepsilon_{\mathrm{int}}) = \sqrt{x_{\mathrm{int}}^2\left[u^2(\eta_\lambda) + u^2(\eta_n)\right] + l_{\mathrm{DP}}^2 u^2(\Delta n) + u^2(\varepsilon_{\mathrm{align}}) + u^2(\varepsilon_{\mathrm{Abbe}})}$$

The input uncertainties are estimated as follows:

a) <u>Laser wavelength in vacuo</u>, $u(\eta_\lambda)$. When an uncalibrated red laser with 633 nm nominal wavelength is used, and the value λ_{0cal} = 632.9908 nm is either stored in the interferometer firmware, λ_{0F}, or used in software to correct the interferometer indications, then $u(\eta_\lambda) = 1.5 \cdot 10^{-6}$.

When better accuracy is sought, the laser wavelength should be calibrated; if either the value stored in the firmware, λ_{0F}, can be updated or the interferometer indications are corrected in software accordingly, then $u(\eta_\lambda) = U_{cal} / k$, where U_{cal} is the relative uncertainty of calibration reported in the certificate, and k is the coverage factor, also reported in the certificate (typically $k = 2$). When the calibrated value, λ_{0cal}, is neither input in the firmware nor used in software compensation, any calibration is effectively just a conformity verification. In this case, a tolerance T for the wavelength fit for the specific application should be set prior to the calibration, and the calibration value used to prove that the laser is in specification, taking account of the calibration uncertainty (see ISO 14253-1). If conformity to specification is proved, a uniform distribution of possible wavelength values in the interval [-T/2, T/2] is assumed, resulting in a standard uncertainty $u(\eta_\lambda) = T/\sqrt{12}$.

EXAMPLE 1. If a 633 nm laser is used, and a standard uncertainty of $u(\eta_\lambda) = 1.5 \cdot 10^{-6}$ is deemed as sufficient for the application, then the value of wavelength in vacuo stored in the firmware, λ_{0F}, should be investigated (e.g. by looking up the interferometer data sheet). If a value λ_{0F} = 632.990 743 nm is found, compatible with the predefined calibration value, λ_{0cal} = 632.990 8 nm within its uncertainty, then no action is needed in the measurement and the relative standard uncertainty $u(\eta_\lambda) = 1.5 \cdot 10^{-6}$ is taken.

EXAMPLE 2. If a laser is calibrated with a value of λ_{0cal} = (632.990 801 ± 0.000 013) nm, and the interferometer firmware or software allows to input and effectively use this value, then the relative expanded uncertainty is $U(\eta_\lambda)$ = 0.000 013 / 632.990 801 = $2 \cdot 10^{-8}$, and the relative standard uncertainty $u(\eta_\lambda) = U(\eta_\lambda) / k = 1 \cdot 10^{-8}$.

EXAMPLE 3. No provisions are given to modify or compensate the value of wavelength in vacuo stored in the firmware, which is known to be λ_{0F} = 632.990 743 nm. A tolerance on the wavelength is then set, fit for the application. For example, a tolerance $T = 4 \cdot 10^{-7} = \pm 2 \cdot 10^{-7}$ is deemed as acceptable for the application, as is negligible to – or at most comparable with – other uncertainty contributors in the overall budget. Then the laser is calibrated, and the value of λ_{0cal} = (632.990 801 ± 0.000 013) nm is obtained. This is found to be compatible with the tolerance (see ISO 14253-1): $|\lambda_{0cal} / \lambda_{0F} - 1| < T / 2 - U_{cal}$ or |632.990 801 / 632.990 743 - 1| = $0.9 \cdot 10^{-7} < 2 \cdot 10^{-7}$ - (0.000 013 / 632.990 801) = $1.8 \cdot 10^{-7}$. The relative standard uncertainty is then imposed by the tolerance, $u(\eta_\lambda) = T/\sqrt{12} = 4 \cdot 10^{-7}/\sqrt{12} = 1.2 \cdot 10^{-7}$.

b) <u>Refractive index of air</u>, $u(\eta_n)$. Three uncertainty components should be considered, and evaluated depending on the circumstances (see Figure 3):

1) $u(i_{cal})$, due to the instrument(s) just after recalibration;

2) $u(i_{drift})$, due to the instrument drift;

3) $u(v_{noise})$, due to the zero-mean random variability of the air (noise).

A fourth component – the steady non uniformity along the beam – is disregarded hereafter because it is negligible in usual practical cases, provided that the index of refraction is measured close to the midpoint of the full measured stroke.

The refractive index of air can be measured by means either of a weather station or of individual instruments, typically a thermometer, a barometer and a hygrometer. Proper calibration is recommended, and assumed hereafter. The instrument(s) may or may not be corrected to match the most recent calibration values, depending on whether the instrument software or firmware allows for this.

If the instrument(s) can be and are corrected, then the remaining error immediately after calibration is only that of calibration: the uncertainty $u(i_{cal})$ is the calibration uncertainty. In addition, the instrument(s) may drift over time, and a component $u(i_{drfit})$ should be added. To evaluate that, the amount of drift should be estimated first, based e.g. on actual drifts occurred between previous calibrations. Then, the standard uncertainty can be derived by assuming a uniform distribution and by dividing that amount by √3. $u(i_{cal})$ and $u(i_{drfit})$ are to be summed in quadrature.

If the instrument(s) cannot be corrected, then $u(i_{cal})$ should be based on the MPE stated in the data sheet. The calibration does not affect the indication, and is used just as a verification of conformity of the instrument(s) to the MPE. An uncertainty component is then derived by assuming a uniform distribution of the error of indication, and by dividing its MPE half-width by √3: if MPE = ± A, then $u(i_{cal}) = A / \sqrt{3}$.

NOTE 1 The MPE used to evaluate the uncertainty may not be taken from a data sheet. In particular, when no data sheet is available (or it is but reports no MPE), or when the specification of the instrument(s) has changed over time: in the former case, a reasonable value can be set discretionally, in the latter the latest value can be used. In either case, the MPE value must be stated prior to the latest calibration, and the conformity verified against such MPE.

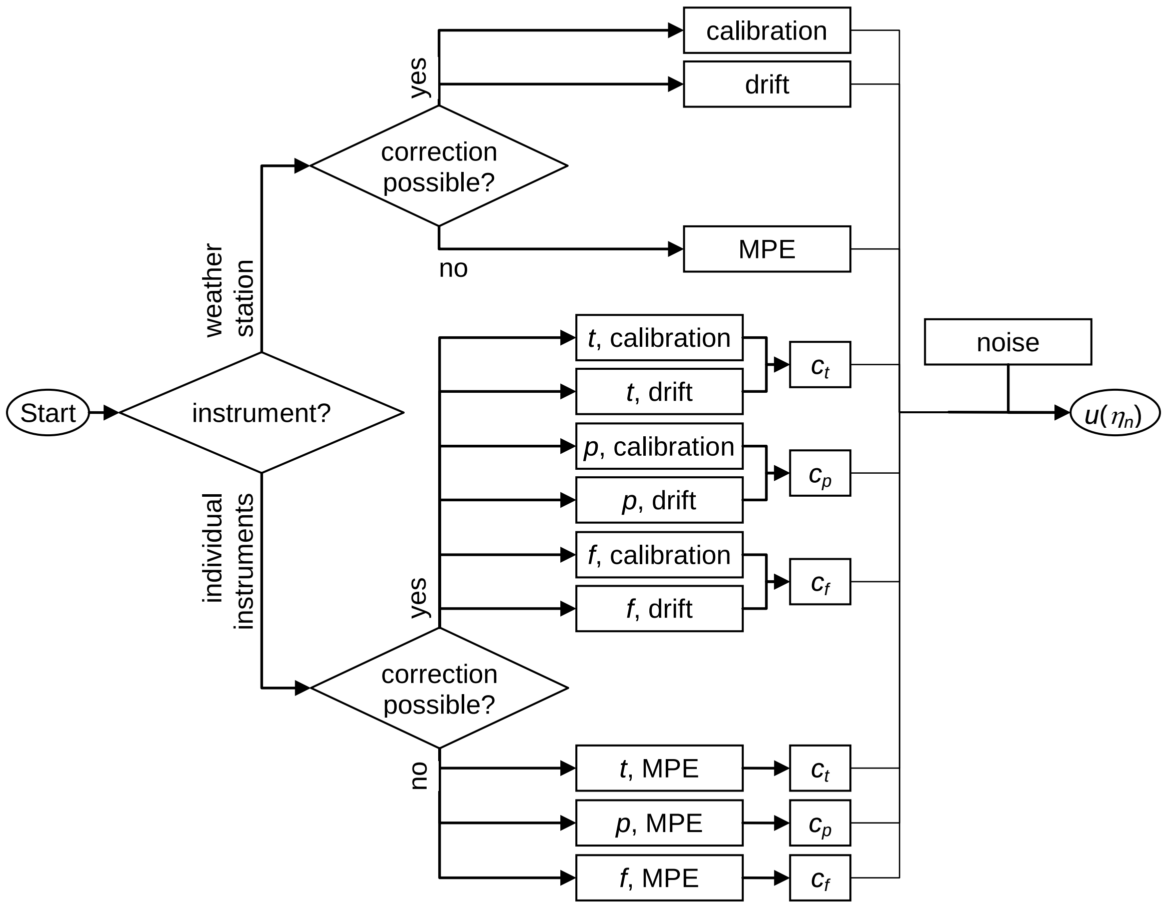

Figure 3 – Flow chart for evaluating the uncertainty of the refractive index of air. Joining lines imply sums in quadrature. See the text for details on the meaning of individual boxes.

When the refractive index is measured by individual instruments, the directions above are to be applied to each of them, and the derived contributions propagated. The refractive index of air depends mostly on temperature, pressure, and humidity of air. The standard uncertainty, $u(\eta_n)$, is then the quadratic sum of the measurement uncertainties of these quantities, each taken with its sensitivity coefficient:

$$u(n) = \sqrt{c_t^2 u^2(t) + c_p^2 u^2(p) + c_f^2 u^2(f)}$$

where $u(t)$, $u(p)$, $u(f)$ are the standard uncertainty of temperature, pressure and humidity, respectively, and c_t, c_p, c_f their sensitivity coefficients. This equation holds when the input standard uncertainties are the combination of the calibration and drift uncertainties (when the instruments are corrected), as well as when they are derived from the MPE's (when the instruments are not corrected). The sensitivity coefficients are subject to negligible variation in ordinary air conditions: the values reported in Table 1 can be taken in most practical cases.

Table 1 – Sensitivity coefficients of the refraction index of air.

temperature	c_t	$-0.93 \cdot 10^{-6}$ °C^{-1} = $-0.93 \cdot 10^{-6}$ K^{-1}
pressure	c_p	$2.7 \cdot 10^{-9}$ Pa^{-1} = $0.27 \cdot 10^{-6}$ hPa^{-1}
humidity	c_f	$-0.36 \cdot 10^{-9}$ Pa^{-1} = $-8.5 \cdot 10^{-9}$ $\%_{RH}^{-1}$
- For the humidity, the first value is relative to the partial pressure of vapour, the second to the relative humidity. - All values calculated for a red laser (λ_0 = 633 nm), standard reference temperature (t = 20 °C), atmospheric pressure (p = 101 325 Pa), dry air (f = 0).		

Due to the relatively low sensitivity of the refractive index of air to humidity, c_f, this parameter is sometimes not measured but just guessed among few predefined values, e.g. 25% (dry), 50% (medium), 75% (moist). In this case, the uncertainty can be evaluated assuming a uniform distribution about these values: $u(f) = \Delta f / \sqrt{12}$ where Δf is the (maximum) interval among the predefined values ($u(f) = 25\% / \sqrt{12} = 7\%$ in the above example case).

In all cases, a component $u(v_{\text{noise}})$ should be considered to account for zero-mean random variability of the air refractivity. It can be evaluated quantitatively by keeping the CMM still and recording the interferometer indications for a period of time comparable with that of actual interferometric measurement: the standard deviation of the recorded values is taken as $u(v_{\text{noise}})$. If non uniformity of the measuring volume is suspected, it is recommended to repeat this evaluation several times with the retroreflector in different positions, and to take the quadratic mean of the standard uncertainties (mean of variances)[1]. This component is expected to be small if a proper integration time is set in the interferometer counter.

c) <u>Dead path</u>, $u(\Delta n)$. If the environmental values have been recorded for a sufficient time prior to the test, the amount of drift can be estimated with a drift test, similar to that described in the ISO/TR 16015 [13]: a time window as long as each interferometric measurement, is moved along the recorded data, the maximum value is taken, and the standard uncertainty is evaluated as the maximum value divided by √3, in the assumption of a uniform distribution.

If the recorded quantity is the refractive index itself, than the standard uncertainty $u(\Delta n)$ is derived immediately. If temperature, pressure and humidity are recorded instead, each standard uncertainty should be summed in quadrature, $u(\Delta n) = \sqrt{c_t^2 u^2(\Delta t) + c_p^2 u^2(\Delta p) + c_f^2 u^2(\Delta f)}$, where the sensitivity coefficients are the same as in b), see Table 1.

If no record is available, a type B evaluation is required, based on experience and/or technical documentation of e.g. the conditioning system.

d) <u>Misalignment of the laser beam</u>, $u(\varepsilon_{\text{align}})$. A typical and recommended way of aligning an interferometer is by observing the signal strength – as indicated by a power meter usually embedded in the interferometer unit – while moving the retroreflector along the beam. In the presence of misalignment, the motion has a non null component transverse to the beam, which slightly offsets the return beam off the receiver, making the signal strength change. On the contrary, when no or minimum strength variation is observed, alignment is deemed as achieved.

In addition to true misalignment, the motion may be not perfectly straight, because of the geometrical and control errors of the CMM (when the retroreflector is attached to the ram) or of the guideway straightness (when it is attached to a slider).

Misalignment and straightness errors are usually observed together at the power meter. After proper alignment, the residual error and its standard uncertainty are

$$\varepsilon_{\text{align}} = (\cos\theta - 1)x_{\text{int}} \approx -\frac{\theta^2}{2} x_{\text{int}}, \qquad \theta \approx \frac{\|\boldsymbol{p}_2 - \boldsymbol{p}_1\|}{x_{\text{int}}}$$

$$u(\varepsilon_{\text{align}}) = u\left(\frac{\theta^2}{2}\right) x_{\text{int}} = \sqrt{\frac{5}{12}} \frac{a^2}{x_{\text{int}}}$$

where

---

[1] The simple mean of variances is an unbiased estimator of the *pooled variance* when the number of observations in each set – at each retroreflector position in our case – is equal. When sets contain different numbers of observations, then the simple mean should be substituted with a weighted mean according to the individual number of statistical degrees of freedom (see [2][3] § H.3.6 Note).

θ is the misalignment angle of the line through the two points taken to compute the point-to-point distance, to the laser beam

$\boldsymbol{p}_2$, $\boldsymbol{p}_1$ are the displacement vectors normal to the laser beam, of the retroreflector to the laser beam (combined effect of misalignment and motion straightness error), at the two points taken to compute the point-to-point distance

a is the maximum admissible distance of the retroreflector to the beam, as detected by the power meter all along the motion.

NOTE 2 The equations above assume that the distribution of each displacement vector, $\boldsymbol{p}_1$ or $\boldsymbol{p}_2$, is uniform within a circle (normal to the laser beam) of radius a. This reflects a typical situation where a threshold a is set to the allowed displacement, all along the motion, i.e. the retroreflector path is all inside a cylinder about the laser beam with radius a.

NOTE 3 The equations above also assume that the two displacement vectors, $\boldsymbol{p}_1$ or $\boldsymbol{p}_2$, are uncorrelated to each other. This assumption is appropriate when the separation of the two, x_{int}, is large, while is rather conservative for small x_{int}'s (e.g. at the shortest calibrated test length). In fact, the continuity of the motion implies some short-distance correlation. As a consequence, the above equation is conservative and safe to use.

NOTE 4 The misalignment (cosine) error is always negative and hence cannot be zero mean. A rigorous application of the GUM would require a systematic correction ([14][15] § F.2.4.4) of the resulting bias, which is usually impractical in the case of ISO 10360-2 testing. The above equations assume no correction, and resort to the rms (root mean square) of the predicted error, that is the quadratic sum of the standard deviation and of the uncorrected bias. Refraining from correcting the known bias increases the standard uncertainty.

NOTE 5 As the alignment error is inversely proportional to the separation of the two points delimiting a calibrated test length, x_{int}, the most critical situation is expected for the shortest calibrated test length.

The alignment contributor $u(\varepsilon_{\text{align}})$ is tabulated in Table 2 for illustration.

Table 2 – Examples of values of the misalignment uncertainty component (in micrometres).

	x_{int} / mm			
a / µm	5	100	250	500
50	0,3	0,0	0,0	0,0
100	1,3	0,1	0,0	0,0
500	32,3	1,6	0,6	0,3

To evaluate the standard uncertainty, $u(\varepsilon_{\text{align}})$, the value of the maximum admissible distance of the retroreflector to the beam, a, must be determined. This can be done based on the power meter indication; unfortunately, the meter scale is often in arbitrary units, seldom in length units. As a first step then, a local calibration of the scale is recommended. This can be done easily by imposing a known small later movement to the retroreflector while observing the power meter. If the retroreflector is attached to the ram, the movement can be done by the CMM itself. If instead the retroreflector is attached to a slider, then lateral movements may not be allowed by the guideway. In this case, an auxiliary retroreflector may be attached temporarily onto a micrometric stage mounted on the slider, to impose the later movement.

NOTE 6 The conversion of the power meter scale to length units attains to the interferometer only. Its calibration can be done once in a time, possibly prior to and separately from the actual ISO 10360-2 testing, and not necessarily in the presence of a CMM.

Once the power meter is calibrated and its indications converted to actual length units, the value a can be determined as:

— The displacement at which the interferometer counting is broken, usually indicated by the interferometer unit with an alarm. This choice is most conservative: if counting is maintained along the full stroke during a measurement, then certainly the retroreflector never moved farther than a off the beam.

— Alternatively, when the value of a as above is deemed too large, a can be determined as the actual maximum indication of the power meter along a full stroke. The reduction in value is achieved at the cost of observing the meter indication dynamically along the stroke, and – if the retroreflector is attached onto the ram – of repeating the determination of *a* for each calibrated test length, as the CMM motion errors may be different along different lines.

NOTE 7 In the case of the retroreflector attached to a slider, the value of *a* is a joint characteristic of the sliding device and of the interferometer. Its determination can be done once in a time, possibly prior to the actual ISO 10360-2 testing, and not necessarily in the presence of a CMM.

e) <u>Abbe error</u> $u(\varepsilon_{\text{Abbe}})$. The Abbe error is null by definition when the retroreflector is attached to the ram: $u(\varepsilon_{\text{Abbe}}) = 0$ in this case.

When instead it is attached to a slider, the two relevant parameters – the Abbe arm, *b*, and the parasitic angle, φ – must be estimated and combined as

$$\varepsilon_{\text{Abbe}} = b\varphi$$

$$\begin{cases} u(\varepsilon_{\text{Abbe}}) = bu(\varphi) & (b > 0) \\ u(\varepsilon_{\text{Abbe}}) = u(b)u(\varphi) & (b = 0) \end{cases}$$

NOTE 8 Any retroreflector has got a finite size; the Abbe arm, *b*, is the distance to the line of the calibrated test length of the retroreflector effective point.

NOTE 9 The retroreflector effective point is that of which the interferometer effectively measures the displacement. Small rotations of the retroreflector about its effective point do not affect the interferometric measurement. Effective points of typical retroreflectors are: the vertex of the cube for a cube corner, the sphere centre for a cat's eye.

NOTE 10 The line of the calibrated test length is that through the points probed by the CMM and used to compute the point-to-point distance.

The design of the sliding device sets the Abbe arm, *b*. When it is nominally non null, $b > 0$, then the derivation is straightforward, and the first uncertainty equation above holds. Even when, on the contrary, the Abbe arm is nominally null, $b = 0$, – i.e. the Abbe principle holds, that is good design practice – still the actual Abbe arm may be not exactly null, and some uncertainty should be accounted for. In this case, the second uncertainty equation above holds, which involves the standard uncertainty $u(b)$, a measure of how well the nominal geometry is implemented in practice.

NOTE 11 This latter equation is a case of second order approximation in the propagation of the input to the output uncertainties (see GUM [14][15] § 5.1.2 NOTE and H.1.7).

The values of $u(\varphi)$ and either *b* or $u(b)$, should be derived from the manufacturer's technical specifications of the sliding device. When these are not available,

— and nominally $b > 0$, measure *b* with e.g. a handhold instrument such as a calliper; considering that *b* is to be multiplied by the uncertainty $u(\varphi)$, and any error on *b* results in a second order term, the measurement of *b* is not required to be accurate (5% to 10% is sufficient).

— and nominally $b = 0$, estimate the maximum distance of the retroreflector effective point to the line of the calibrated test length, b_{max}, and divide by $\sqrt{3}$, $u(b) = b_{\text{max}}/\sqrt{3}$. A type B evaluation (see GUM [14][15] § 4.3) of b_{max} can be based on the estimated geometrical quality of the slider-retroreflector assembly and on the CMM accuracy in targeting the point of the reference feature. As to the latter, if a sphere is probed, the accuracy of the intended point – the sphere centre – is that of the CMM *when measuring*; if a gauge block is probed instead, the accuracy of the intended point – the face centre – is that of the CMM *when approaching the target point*.

— estimate $u(\varphi)$ either type A or B: in the former case, measure the parasitic angle with a goniometric instrument – such as an autocollimator or an interferometer with goniometric accessories – and evaluate the experimental standard deviation of all measured angles; in the latter case, base the evaluation on educated appraisal.

According to the PUMA method [16], and considering that the evaluation may result difficult and time consuming, a first conservative but quick evaluation is recommended: if another uncertainty contributor is dominating, there would be no point in spending further time and effort, and a rough but conservative trial would be as good. More specifically, interferometric measurement uncertainties are often dominated by the refractive index of air, $u(\eta_n)$, which is recommended to consider and compare first.

When the interferometer is calibrated in full as a system, i.e. including the laser head, the counter and the weather station, the components a) and b) are included in the calibration uncertainty and do not need any further assessment.

Typical values of the uncertainty components above are listed in Table 3.

Table 3 – Typical values of uncertainty components in interferometric measurement.

		Input quantities			Components, x_{int} = 50 mm [µm]		Components, x_{int} = 1 000 mm [µm]	
			Min	Max	Min	Max	Min	Max
wavelength	η_λ		$1\cdot10^{-8}$	$1.5\cdot10^{-6}$	0.0	0.1	0.0	1.5
air refractivity	η_n	t p RH	0.025 °C 0.25 hPa 1%	0.5 °C 2.5 hPa 25%/√3 = 7%	0.0	0.0	0.1	0.8
dead path	$\Delta n, l_{DP}$	t p RH l_{DP}	0.01 °C 0.1 hPa 0% 10 mm	0.25 °C 1 hPa 5% 500 mm	0.0	0.2	0.0	0.2
misalignment	θ	a	50 µm	0.5 mm	0.0	3.2	0.0	0.2
Abbe	φ, b	φ $u(b)$, b	5 µrad 0.1 mm	100 µrad 50 mm	0.0	5.0	0.0	5.0
				$\boldsymbol{u(x_{int})}$	**0.0**	**6.0**	**0.1**	**5.3**
				$k = 2$, $\boldsymbol{U(x_{int})}$	**0.1**	**11.9**	**0.1**	**10.6**

3 References

[1] ISO 10360-2:2009 Geometrical product specifications (GPS) – Acceptance and reverification tests for coordinate measuring machines (CMM) – Part 2: CMMs used for measuring linear dimensions

[2] JCGM 200:2012 International vocabulary of metrology – Basic and general concepts and associated terms (VIM)

[3] ISO/IEC Guide 99:2007 International vocabulary of metrology – Basic and general concepts and associated terms (VIM)

[4] SIT/Tec-016/09 rev.01:2010 Accreditamento SIT di verifiche di prestazioni di CMM secondo la UNI EN ISO 10360-2:2005 - requisiti tecnici (hyperlink)

[5] ENAC NT-15:2012 Calibración de medidoras por coordenadas. Criterios de trazabilidad (hyperlink)

[6] ISO 14253-1:2013 Geometrical product specifications (GPS) – Inspection by measurement of workpieces and measuring equipment – Part 1: Decision rules for proving conformity or nonconformity with specifications

[7] ISO/TS 23165:2006 Geometrical product specifications (GPS) – Guidelines for the evaluation of coordinate measuring machine (CMM) test uncertainty

[8] ISO 10360-2:2001 Geometrical Product Specifications (GPS) – Acceptance and reverification tests for coordinate measuring machines (CMM) – Part 2: CMMs used for measuring size

[9] ISO/TC213/WG10 Technical Committee on Dimensional and Geometrical Product Specification and Verification – Working Group on Coordinate Measurement Machines

[10] ISO/TC213/WG4 Technical Committee on Dimensional and Geometrical Product Specification and Verification – Working Group on Uncertainty of measurement and decision rules

[11] ISO/DIS 14253-5 Geometrical Product Specifications (GPS) – Inspection by measurement of workpieces and measuring equipment – Part 5: Uncertainty in testing indicating measuring instruments

[12] J.A. Stone, J.E. Decker, P. Gill, P. Juncar, A. Lewis, G.D. Rovera and M. Viliesid, *Advice from the CCL on the use of unstabilized lasers as standards of wavelength: the helium–neon laser at 633* nm, Metrologia **46** (2009) 11–18.

[13] ISO/TR 16015:2003 Geometrical product specifications (GPS) – Systematic errors and contributions to measurement uncertainty of length measurement due to thermal influences

[14] JCGM 100:2008 Evaluation of measurement data – Guide to the expression of uncertainty in measurement

[15] ISO/IEC Guide 98-3:2008 Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)

[16] ISO 14253-2:2011 Geometrical product specifications (GPS) – Inspection by measurement of workpieces and measuring equipment – Part 2: Guidance for the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification