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Comment on “Physics without determinism: Alternative interpretations of classical physics”

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**Comment on “Physics without determinism:  
Alternative interpretations of classical physics”,  
Phys. Rev. A, 100:062107, Dec 2019**

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## Abstract

The paper “Physics without determinism: Alternative interpretations of classical physics” [Phys. Rev. A, 100:062107, Dec 2019] defines *finite information quantities* (FIQ). A FIQ expresses the available information about the value of a physical quantity. We show that a change in the measurement unit does not preserve the information carried by a FIQ, and therefore that the definition provided in the paper is not complete.

The expression of the state of knowledge about a measurand as a probability distribution (or some summary of it, such as its mean and standard deviation) is the conventional approach for expressing a measurement result [1–4]. However, it does not intuitively parallel the much more immediate concepts of “certain” and “uncertain digits” that every experimentalist feels when taking note of a measurement outcome in the lab notebook.

In [5], Del Santo and Gisin introduce the concept of *finite information quantities* (FIQ). A FIQ ranging in the interval  $[0, 1]$  is expressed by the binary number  $Q = 0.Q_1Q_2Q_3\dots$ , where the individual bits  $Q_k$  are Bernoulli random variables having propensities  $q_k$  for the realisation of the case  $Q_k = 1$ . A specific FIQ  $Q$  is thus defined by the vector of propensities  $\mathbf{q} = [q_1, q_2, \dots, q_k, \dots, q_M, \frac{1}{2}, \frac{1}{2}, \dots]$  of its bits  $Q_k$ ; it is assumed that  $q_k = \frac{1}{2}$  for  $k > M$ , *i.e.*, all bits beyond position  $M$  have a 50 % propensity of being either 0 or 1 and therefore carry no information. Only a finite number  $M$  of propensities are needed to specify  $Q$ .

The FIQ concept is very appealing and it is tempting to adopt it to express the value and uncertainty of a quantity as an alternative to probability distributions. However, for the concept of FIQ to become a practical alternative to the current way of representing the state of knowledge about a quantity, it is mandatory that calculations with them be possible and, hopefully, simple.

Consider for example the expression of the value of a quantity, traditionally written as  $Q = \{Q\}[U]$ , where  $\{Q\}$  is the numerical value and  $[U]$  is the unit. Changing the unit to  $U' = U/L$ ,  $L$  being a constant, implies  $Q = \{Q'\}[U']$ , with  $\{Q'\} = L\{Q\}$ . So, even such an elementary transformation as the change of measurement unit implies the multiplication of a FIQ by a constant.

Indeed, the FIQ definition suggests that it is possible to identify simple, practical calculation rules operating on the finite (and, intuitively, small) number of indeterminate bits and their propensities; rules suitable to be converted in efficient computation algorithms.

The arithmetic relevant to a unit change (Appendix A) shows that the transformation  $Q' = LQ$  generates bits  $Q'_k$  of  $Q'$  which are not mutually independent even if the original  $Q_k$  bits are independent. Therefore, expressing  $Q'$  by providing only the propensities  $q'_k$  of its individual bits deletes some of the original information.

Random variables  $Q$  with independent binary digits  $Q_k$  have been considered in mathematical literature [6–8]. In general,  $Q$  has a ‘reasonable’ probability density function (pdf) only if the  $q_k$  satisfy strict conditions, and in that case the pdf is necessarily an exponential [6]; otherwise, it becomes a fractal [7], hence difficult to associate with a physical quantity.

In conclusion, it appears that a specification of the state of knowledge about a quantity  $Q$  by means of a FIQ should also include information on the dependencies among the  $Q_k$ , and therefore that, although the FIQ concept might be physically sound and useful, its definition as given in [5] is not complete, and deserves further development.

## Appendix A: Minimal FIQ maths

A FIQ arithmetics can be established by generalizing operations on binary numbers. The sum  $S = Q + R = 0.S_1S_2S_3\dots$  of two FIQs,  $Q = 0.Q_1Q_2Q_3\dots$  and  $R = 0.R_1R_2R_3\dots$ , is given by the full adder rule, Tab. I.

$Q_k$	$R_k$	$C_{k+1}$	$S_k$	$C_k$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

TABLE I. Binary full adder truth table.  $C_k$  is the carry bit.

If  $\mathbf{q}$  is the vector of propensities associated with  $Q$ , and  $\mathbf{r}$  with  $R$ , then under the as-

sumption of independence of  $q_k$  and  $r_k$ , the propensity  $s_k$  of each sum bit  $S_k$  can be written as the sum of the four propensities of the  $S_k = 1$  cases in Tab. I:

$$\begin{aligned}
s_k &= (1 - q_k)(1 - r_k)c_{k+1} + (1 - q_k)r_k(1 - c_{k+1}) \\
&\quad + q_k(1 - r_k)(1 - c_{k+1}) + q_k r_k c_{k+1} \\
&= q_k + r_k + c_{k+1} \\
&\quad - 2(q_k r_k + q_k c_{k+1} + r_k c_{k+1}) + 4q_k r_k c_{k+1}
\end{aligned} \tag{A1}$$

and similarly the propensity  $c_k$  of the carry bit  $C_k$  is

$$c_k = q_k r_k + q_k c_{k+1} + r_k c_{k+1} - 2q_k r_k c_{k+1} \tag{A2}$$

For example for the case  $c_{k+1} = \frac{1}{2}$ , we have  $s_k = \frac{1}{2}$  and  $c_k = \frac{1}{2}(q_k + r_k)$ : the information provided by  $q_k$  and  $r_k$  is transferred, through the carry bit  $C_k$ , to bit  $S_{k-1}$ .

Multiplication by a deterministic constant  $L$  can be performed by repeated shifting and addition. Table II gives a simple example. If  $P = LQ$ , where  $\mathbf{q} = [0, 0, q_3, \frac{1}{2} \dots]$  and

	0.	0	0	$Q_3$	...
$\times$			1	1	
	0.	0	0	$Q_3$	...
$+ 0.$	0	0	$Q_3$	$Q_4$	...
$= 0.$	$P_1$	$P_2$	$P_3$	...	

TABLE II. Multiplication table,  $P = LQ$  where  $Q = 0.0Q_2Q_3 \dots$  and  $L = (11)_2 = (3)_{10}$ .

$L = (11)_2 = (3)_{10}$ , then

$$\begin{aligned}
p_1 &= \frac{1}{2}q_3^2 + \frac{1}{4}q_3, \\
p_2 &= q_3 - q_3^2 + \frac{1}{4}, \\
p_3 &= \frac{1}{2}, \quad \dots
\end{aligned} \tag{A3}$$

The propensity of occurrence of specific digit couples can also be computed. For example, denoting as  $p_{12}$  the propensity of the event  $\{P_1 = 1, P_2 = 1\}$  we have  $p_{12} = 0$  (to have  $P_1 = 1$ , it should occur that  $Q_3 = 1$  and  $C_3 = 1$  at the same time, hence  $C_2 = 1$ . However,

the case  $\{Q_3 = 1, C_3 = 1\}$  always generates  $P_2 = 0$ , so  $\{P_1 = 1, P_2 = 1\}$  is never possible). Since  $p_{12} = 0 \neq p_1 p_2$ , bits  $P_1$  and  $P_2$  are not independent.

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