On the linearisation of a measurement model – An objective legitimacy criterion

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In the JCGM 101 [JCGM101] framework, the available knowledge about the true value of a physical quantity is expressed by a probability distribution. For convenience of expression, the distribution can be summarized by stating its parameters: typically, the mean and the standard deviation. If the quantity of interest is measured indirectly, from a set of input quantities with a measurement model, the probability distribution associated to the measurand is computed by propagating the probability distributions of all input quantities through the measurement model (the possible correlations are taken into account by joint probability distributions). The distribution propagation problem is usually dealt with by means of computer-intensive numerical calculations.

In the GUM'95 [GUM] framework, the parameters of the input quantity distributions, the mean and the standard deviation, are propagated through a linearized version of the measurement model, with a simple analytical expression, to obtain the mean and the standard deviation of the distribution associated with the measurand.

It is broadly agreed that the JCGM 101 approach is the consistent one, and that the GUM'95 approach is an approximation that is suitable only when the model nonlinearity is "small". To the best of our knowledge, only empirical procedures have been suggested so far to decide case by case whether the GUM'95 procedure yields acceptable results [JCGM101, Sec. 8.1; GUM-6, Annex F].

We propose an objective criterion to validate the use of the GUM '95 procedure.

Let us first consider for simplicity a model with one input quantity. The zero- and first-order terms of the Taylor series expansion of the model give the linearised model employed in the GUM'95. The criterion consists in comparing the absolute magnitude of the first-order term with that of the remainder (all orders from the second), over an interval where the probability of the input quantity is significantly different from zero.

We show that if the remainder of the Taylor expansion is smaller than the first-order term, the corresponding difference between the GUM'95 and the JCGM 101 estimates of the mean value of the measurand distribution is smaller than the measurand standard deviation predicted by the linearized model. That is, the GUM'95 estimation bias is small and so its approach is legitimate for the specific situation.

The criterion can be extended to models having more than one input quantity.
We'll discuss simple but practical examples of the application of the criterion during the presentation.

[JCGM101] Joint Committee for Guides in Metrology JCGM 101:2008 "Evaluation of measurement data — Supplement 1 to the *Guide to the expression of uncertainty in measurement* — Propagation of distributions using a Monte Carlo method", 2008. Online: www.bipm.org.

[GUM] Joint Committee for Guides in Metrology JCGM 100:2008 (GUM 1995 with minor corrections) "Evaluation of measurement data — Guide to the expression of uncertainty in measurement", 2008. Online: www.bipm.org.

[GUM-6] Joint Committee for Guides in Metrology JCGM GUM-6:2020 "Guide to the expression of uncertainty in measurement — Part 6: Developing and using measurement models", 2020. Online: www.bipm.org