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Practical experimental design of task-specific uncertainty evaluation for coordinate metrology*

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This research describes a strategy for evaluating the measurement uncertainty of the feature dimensions and forms of products using a coordinate measuring machine, which is widely used in manufacturing industry. The proposed strategy outputs a task-specific measurement uncertainty, where the task includes the specification of the distribution of measurement points. Being executable with a small number of measurement trials, the strategy is practical for industrial use.

Keywords: Dimensional metrology, Measurement uncertainty, Design of experiment, Analysis of variance, Coordinate measuring machine.

1. Introduction

When sharing product information among the concerned parties in connected industries, the products must be unambiguously defined. Product shapes are specified by their dimensions and geometrical tolerances [1].

Coordinate measuring machines (CMMs) and other three-dimensional measuring systems validate the actual forms of products against their respective designed forms [2]. When verifying the dimensions of products, the measurement uncertainties must be considered [3]. A task-specific measurement uncertainty is

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evaluated by combining the standard uncertainties of the contributors [4]. However, it is hard to estimate the uncertainty contributions of individual dimensional measurements using a CMM, because measurement results are affected by many factors, such as deformations of the machine frame and environmental temperature changes [5][6]. In some studies, the uncertainty is instead assessed by design of experiment (DOE) [7-9], but this method is also difficult to perform on actual industrial floors.

The authors of the present study are developing an experimental procedure for task-specific measurement uncertainty evaluation using a CMM. The proposed uncertainty estimation applies a DOE and the analysis of variance (ANOVA) approach. The development is part of the EURAMET project titled Standards for the Evaluation of the Uncertainty of Coordinate Measurements in Industry (EUCoM).

This article describes the evaluation of the measurement uncertainties of the feature dimensions and forms of products, which are derived from the designated distributions of measurement points on the product surfaces. From a statistical viewpoint, the distribution of the measurement points used for feature extraction [10] should be non-patterned to reduce the systematic-error contribution to the measurement. Unfortunately, applying a random-points sampling strategy on typical industrial CMMs is impractical, because these machines are numerically controlled by a computer. The proposal strategy outputs a task-specific measurement uncertainty, where the task includes the specification of the distribution of measurement points.

The experiment designed in this article is similar to Taguchi's L4 orthogonal array method [11].

2. Uncertainty contributors of a CMM measurement task

Essentially, a CMM measures single-point coordinates. The dimensions and forms of the features are then computed from the set of measurement points sampled by the CMM. Therefore, the uncertainty in the dimensions or forms is derived from the uncertainty in the single-point coordinates: u_P. The uncertainty u_P is a combination of various uncertainty contributors, which are difficult to evaluate individually because of the complex factors in a CMM's measurement task.

In practice, combined uncertainties of measurements are dominated by random errors and systematic errors. The contributions of these respective factors on the measurement uncertainty propagate and are modularized to their contributors such as the items of an uncertainty budget. By proper design of the

measurement experiment, the unknown systematic errors can be treated as random errors.

2.1. *Randomizing the unknown systematic-error sources*

When a product is measured on a CMM in one orientation, the measured coordinates of a single point in the product's coordinate system (x, y, z) include random and systematic errors. The errors derived from the contributors are propagated to the machine coordinates of the CMM. The systematic-error contribution depends on the location of the single point in the machine coordinate system and on the orientation of the product coordinate system relative to the machine coordinate system.

If the measurement is made in a specified position and orientation, the measurement result is biased by systematic error. Even if the measurement is repeated and the measurement results are averaged, the effect of the bias is neither eliminated nor reduced. To evaluate the magnitude of the bias, extra measurements are necessary.

In the present study, a product is measured at different positions and orientations on a CMM. This approach randomizes the systematic errors in the respective measurements by varying the magnitude and coefficient of the error propagation to the measured results. The effect of the systematic error is then reduced by averaging the repeated measurement results. The respective contributions of the systematic errors are evaluated from the multi-orientation measurements using the ANOVA technique.

3. Design of experiment

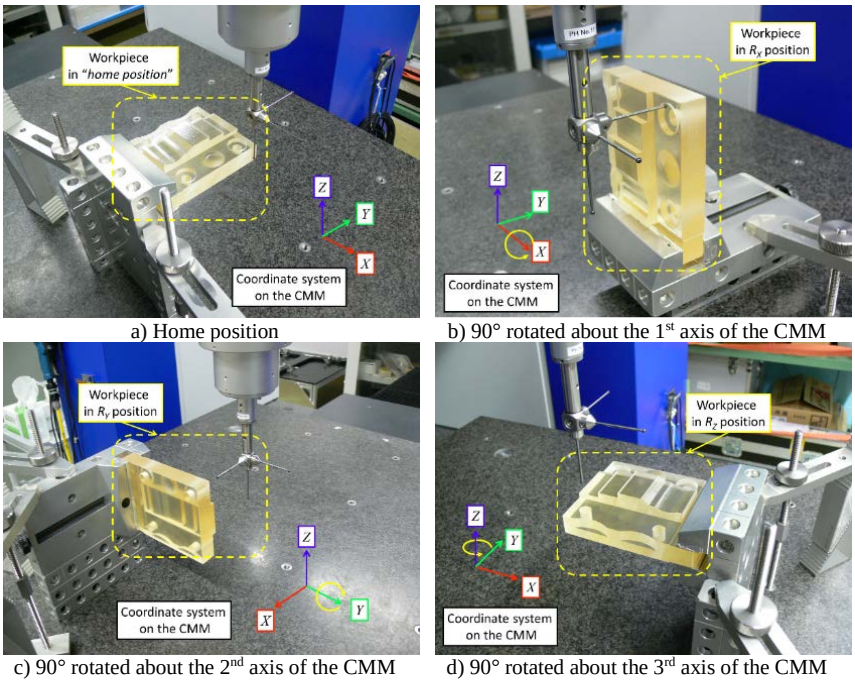
To accurately estimate the systematic-error effect, the measurements should be performed in the maximum possible number of positions and orientations. Moreover, the positions and orientations should be randomly distributed. However, increasing the number of measurement repetitions and trials raises the measurement cost. Therefore, the DOE should provide the uncertainty contribution of the unknown systematic error in the CMM within a small number of measurement trials. In this research, the effect of the systematic error was evaluated from the measurement results of only four significantly different orientations on a CMM.

The systematic error in the CMM is observed as the deformation of the cubic region of interest (ROI) of the CMM, which holds the workpiece to be measured [12][13]. The influence of the ROI deformation is approximated by a linear transformation function. To evaluate the effect of the systematic error in the ROI,

it is sufficient to perform the measurements for uncertainty estimation in a few but essential number of orientations.

Figure 1 illustrates the four essential orientations for randomizing the systematic error on a CMM [14]. The primal position (home position) is the ordinal orientation of the workpiece selected by the operator. The other three positions are obtained after rotating the coordinate system of workpiece by 90° about the 1st, 2nd, and 3rd axes of the CMM machine's coordinate system.

The systematic errors caused by the *X*-, *Y*-, and *Z*-axis of the CMM in the ROI are unchanged throughout the measurements, but their propagations onto the coordinates of the workpiece coordinate system differ among the four positions. After averaging the measurement results at the four positions, the effect of the unknown systematic errors is reduced. The contribution of the systematic error in the averaged output is evaluated along with the variance of the measurement results at the four positions.



a) Home position b) 90° rotated about the 1st axis of the CMM

c) 90° rotated about the 2nd axis of the CMM d) 90° rotated about the 3rd axis of the CMM

Fig. 1. Setup for randomizing and measuring the unknown systematic error.

4. Simulation

To demonstrate that the uncertainty evaluated by the measurement results at the four orientations closely approximates the true uncertainty, a Monte Carlo Simulation (MCS) was performed. As mentioned in Sec. 2, the essential measurements by a CMM are single-point coordinates. To simplify the measurement task, the measurement workpiece in the following simulation was a tetrahedron.

The coordinates of the four vertices of the tetrahedron were measured on the CMM in several orientations. The workpiece coordinate system was constructed from the coordinates of three vertices (the 1st, 2nd, and 3rd vertices) of the tetrahedron. The coordinates of the remaining vertex (the 4th vertex) were reserved for checking the simulation.

Table 1 shows the MCS conditions. The validation set consisted of 500 randomly generated tetrahedron sets and deformed ROIs. Each side of each tetrahedron was about100 mm long. In each ROI, the tetrahedron was measured in 200 random orientations and four specified orientations (see Fig. 1).

Table 1. Conditions of the Monte Carlo simulations

Measurement object	Vertices of tetrahedrons (side length about 100 mm)
Orientations in one ROI	200 at random
Trials of ROI deformation	500 at random
Deformation of the ROI	
Scale factor	5 μm/m
Perpendicularity between axes	2" (10 μrad)

Figure 2 presents a simulation result in one ROI. The red dots show the deviations between the averaged coordinates of the 4th vertex and the measured coordinates of the 4th vertices of the 200 randomly orientated tetrahedrons in the workpiece coordinate system. The blue crosses show the deviations between the averaged coordinates of the 4th vertex and the measured coordinates of the 4th vertices of the four specified orientated tetrahedrons. As shown in Fig. 2, the measured positions of the 4th vertex in the ROI deviated more widely among the orientations than the distribution of vertex positions given by the random orientations in the MCS.

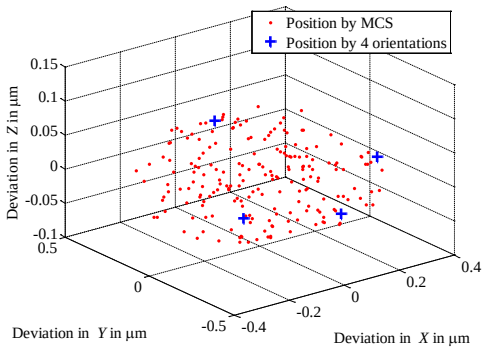


Fig. 2. Distributions of 4th vertex position of the tetrahedron in a single deformed ROI.

Figure 3 relates the expanded uncertainties in the coordinate measurements of the 4th vertex evaluated from the 200 random orientation measurements ($U_{\text{geo, MCS}}$) to those obtained from the four specified orientations ($U_{\text{geo, 15530-2}}$)[15] in the 500 corresponding ROIs. The standard uncertainties in the simulated 4th vertex coordinates, $u_{\text{geo, MCS}}$ and $u_{\text{geo, 15530-2}}$, respectively, were computed as the standard deviations of the Euclidean distances between the mean point of the repeated measurements and the corresponding measured point. Here $u_{\text{geo, MCS}}$ was the standard uncertainty in the 200 measurement results in a given ROI and $u_{\text{geo, 15530-2}}$ was that of the four measurement results in the same ROI. The coverage factors ($k = 2$ for $U_{\text{geo, MCS}}$ and $k = 3$ for $U_{\text{geo, 15530-2}}$) were determined by considering their effective degrees of freedom. In most cases, the proposed strategy overestimated the precisely estimated uncertainty, meaning that $U_{\text{geo, 15530-2}} \geq U_{\text{geo, MCS}}$. In the remaining 5% of cases in the MCS trial, the proposed strategy under-estimated the precise uncertainty.

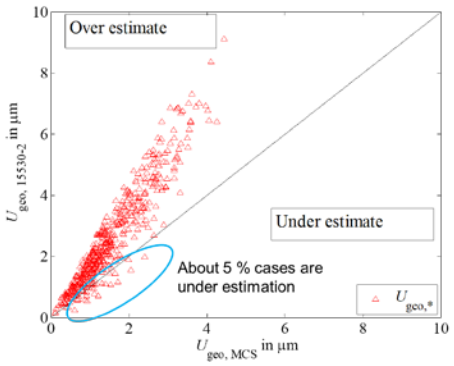


Fig. 3. Comparison of expanded uncertainties derived from the two evaluations.

5. Discussion

In industry, the measurement uncertainty is used to make decisions whether a product meets its specification requirements [3]. To avoid the risk of false acceptance or false rejection, the uncertainty approximated by the easy-to-use approach should over-estimate or under-estimate the actual uncertainty by as little as possible.

Figure 4 shows the degree of under estimation by the proposed method. The degree of under estimation under the given conditions was unacceptable in only one case. Therefore, the risk of false acceptances and rejections using the proposed method is deemed sufficiently small.

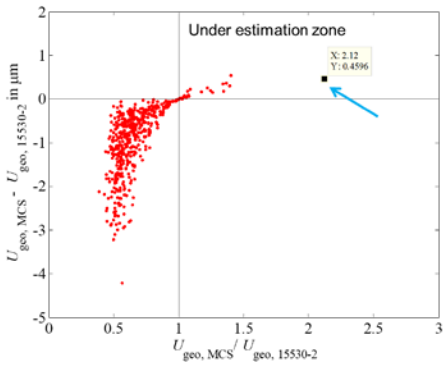


Fig. 4. Degree of under estimation in the proposed strategy.

6. Summary

Single-point coordinate measurements are essential in dimensional metrology; therefore, uncertainty evaluation of coordinate measurements is the basic process to estimate the complex task-specific uncertainties using CMMs. In this article, the authors proposed and demonstrated a simple method for estimating measurement uncertainties. After combining the measurement results in four specified orientations on a CMM, the unknown systematic errors in the CMM system are sufficiently randomized, and their contributions to the measurement results can be approximated with small risk of under estimation.

Acknowledgments

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