Determination of sensitivity coefficients and their uncertainties in Rockwell hardness measurement: a Monte Carlo method for multiple linear regression

Determination of sensitivity coefficients and their uncertainties in Rockwell hardness measurement: a Monte Carlo method for Multiple Linear Regression.

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Abstract

In the last decades, many researches have been studying how hardness measurements can be affected by possible influence variables (i.e. velocity of the indenter, dwell times, temperature, etc.). This interest is particularly motivated by the newly adopted international definitions for the realization of Rockwell superficial hardness scales (HR45N, HR30N and HR15N) provided by the Consultative Committee for Mass and Related Quantities of Bureau International des Poids et Mesures, which deal with all the above-mentioned parameters.
In this paper, the effect of two of such parameters, namely the velocity of the final load application and the time interval of the force variation from the preliminary force value to the total force value, on superficial Rockwell hardness scales at different levels is studied and the related sensitivity coefficients are determined. The coefficients obtained are in the order of 10^{-3} HR s μm^{-1} and 10^{-2} HR s^{-1}, respectively, in agreement with other National Metrology Institutes (NMIs), i.e. NIST and NPL. However, the uncertainties associated by the other NMIs are usually underestimated since they are simply given as the standard deviation calculated from the Ordinary Least Squares method for the Multiple Linear Regression, or, in other cases, not reported. For this reason, we propose a methodology for calculating the uncertainties of the sensitivity coefficients via a Monte Carlo Method applied to Multiple Linear Regression in order to consider the variability of both input and output quantities: with this method, the uncertainties are given as the squared sum of the standard deviation calculated from the Ordinary Least Squares method and the uncertainty contribution due to the repeatability obtained via the proposed Monte Carlo Method. The proposed method yields uncertainties of about 10^{-2}HR, while the uncertainties reported in other related published papers are in the order of 10^{-3}HR.

Keywords: Hardness Measurement, Monte Carlo, Uncertainty, Sensitivity Coefficients

1 Introduction

In the field of Hardness measurements, in order to evaluate the measurement uncertainty, researchers have tried to understand and quantify the effect of possible influence parameters on the

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measurement itself [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Such scientific interest is greatly motivated by the new international definitions recently adopted for the realization of these hardness scales at the level of National Metrology Institutes (NMIs), which prescribe the operative measurement procedures and fix *reference values* for influence parameters (i.e. preliminary and total test force, different dwell times, mean indentation velocity, temperature, etc.). It is well understood that creep phenomena and, in general, all elasto-plastic and dynamic effects may have a non-negligible impact on the material behaviour. However, such contributions are not directly considered in the mathematical model of the different hardness scales, which rely only on geometrical factors (i.e. the depth of the indenter h for the Rockwell scale), but are examined in the related Standards [12, 13, 14].

In this paper, two influence parameters are investigated for superficial Rockwell Hardness scales: 1) the velocity of the total force application; 2) the time interval of the force variation from the preliminary force to the total force value. One major difficulty is related to the physical decoupling of the two variables (velocity and time) [6], so a careful experimental design has been carried-out beforehand. Finally, sensitivity coefficients and their uncertainties are calculated for each influence parameter via a Monte Carlo method applied to multiple linear regression. Comparisons between the results obtained by other National Metrology Institutes are also shown [15]. As a final remark, it is known that, due to the non-uniformity of the hardness block, repetition of the same '*hardness measurement*' (which are performed at different points on the same block) do not measure the same quantity [16, 15]; therefore the non-uniformity of the blocks can easily mask the effects of the influence variables (especially at the lower force scales). The authors will investigate on this topic in a future paper.

2 Methods

2.1 Rockwell hardness testing cycle

Due to the effect of the behaviour of the block material (creep, elasto-plasticity, etc.) and technical issues related to the dynamic response of the machinery, it is essential to understand the phases of a Standardized Rockwell hardness testing cycle. At first, the indenter approaches the surface of the hardness block (*approach velocity*); then, a preliminary force F_0 is applied during a time interval t_{pa}; the preliminary force F_0 is maintained for some time and an initial depth measurement is performed at time t_{pd}. After that, the force is increased from F_0 to its total value F during a time interval t_{aa}: the loading procedure is actually split into two sub-phases as the force is increased from F_0 to $0.8F_0$ to F. The final force is maintained for a time interval t_{td}. Finally the force is reduced rapidly to the preliminary force value F_0 during an interval t_{ar} and maintained at F_0 until a final depth measurement is performed at time t_{rd} and after the force is completely removed. A schematic of the process for both force and depth of the indenter over time is given in fig.1.

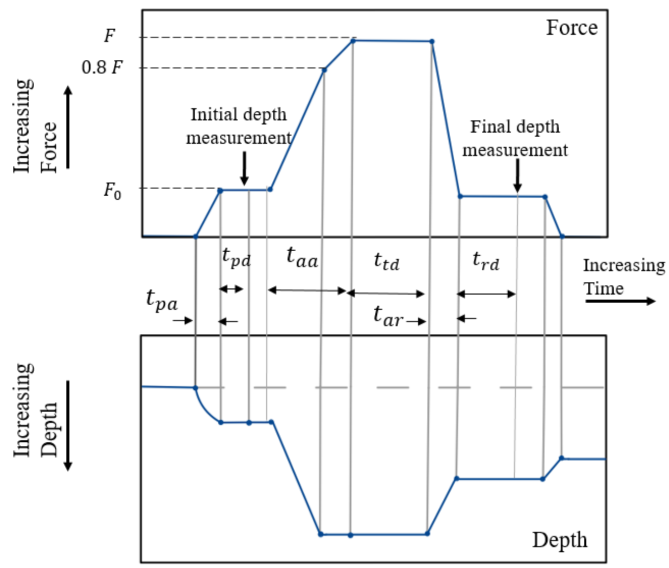

Figure 1: Rockwell testing cycle [17]

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Having described the testing cycle, it is intuitive to understand the reasoning behind each step of the measurement procedure: creep effects are particularly related to the maintenance of force over a period of time: longer time often results in a lower measured hardness, more affected by possible vibrational effects but less critical in controlling the timing of the testing cycle. On the other hand, a fast application of the loads may lead to hardening phenomena which can result in a higher measured hardness as well as more difficulties in respecting the timing of the testing cycle. Analogously, during the unload phase, a so-called elasto-plastic recovery mechanism takes place which is, as all irreversible phenomena, time-dependent. For all the above-mentioned reasons, the steps of constant load application are, in reality, characterized by noise, vibrations and/or non-linear behaviors.

2.2 Experimental Plans

In this paper, we studied the effect of the velocity of the final load application V_{fa} and the time interval of the force variation from the pre-load to its final value t_{aa} for Rockwell superficial hardness measurements, since being one of the variables more interestingly associated to creep and elasto-plastic effects (as explained in the previous sections). A long and careful set-up of the testing machine parameters has been carried-out in order to get the desired physical decoupling of the velocity-time experimental planes [15, 18]: indeed, the time of application of the additional load and its velocity are intrinsically correlated. Many experiments carried out in the past by other scientists have not highlighted this correlation, but since the velocity effect is mainly related to the final velocity of the force application (standard reports from 80% to 100% of the total force [14]), thanks to the flexible setting of the INRiM Primary Hardness Standard Machine (PHSM) (fig.2) [19, 20, 21] it was possible to carry out loading cycles that have the same application times but with different final velocities (by changing both the initial velocity and/or the velocity changing point during the additional force application phase). The PHSM records the velocity and the force parameters over time: in this way we collected the measurement for such parameters.

We chose three reference blocks at nominal low, medium and high hardness values, using three

superficial Rockwell hardness scales HR45N, HR30N, HR15N. Due to the material inhomogeneity, since the hardness measurements of the blocks vary according to the location of the indentation, the blocks were divided into 9 sectors, as in fig.3: the measurements were performed along the radial and circumferential directions.
In summary, the experimental plans, related to the hardness scales, the nominal and actual experimental conditions tested for the velocity of the final load application V_{fa} and the time interval of the force variation from the preliminary force value to the total force value t_{aa}, are depicted in fig.4. For each scale and for each block a total of 27 measurements have been performed, using 3 blocks and 3 hardness scales for a total of 243 measurements. The 27 measurements are given by $3t_{aa}$ values $\times$ $3V_{fa}$ values $\times$ 3 repetitions at the same nominal velocity of the total force application V_{fa} and same nominal time interval of the force variation from the preliminary force to its total force value t_{aa}. According to the experimental planes obtained (fig.4), it was possible to study how the measurement is affected by the change in one parameter among the two (V_{fa}, t_{aa}) at a time, while keeping the other one as fixed as possible. The central values of the parameters correspond to the reference values prescribed in the current new definition of superficial Rockwell Hardness [17]. In order to obtain the sensitivity coefficients and their uncertainty, a Monte Carlo Method is applied to MLR.

Figure 2: Primary Hardness Standard Machine at INRiM

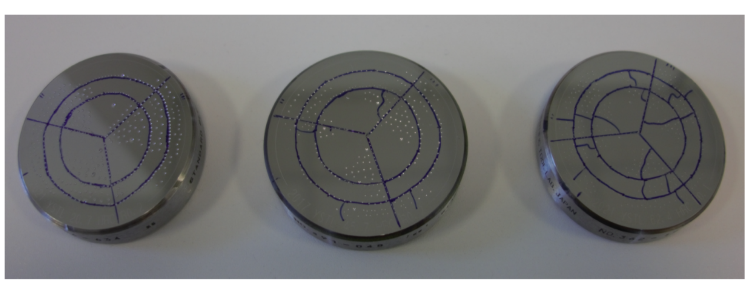

Figure 3: The three hardness blocks used in the measurements.

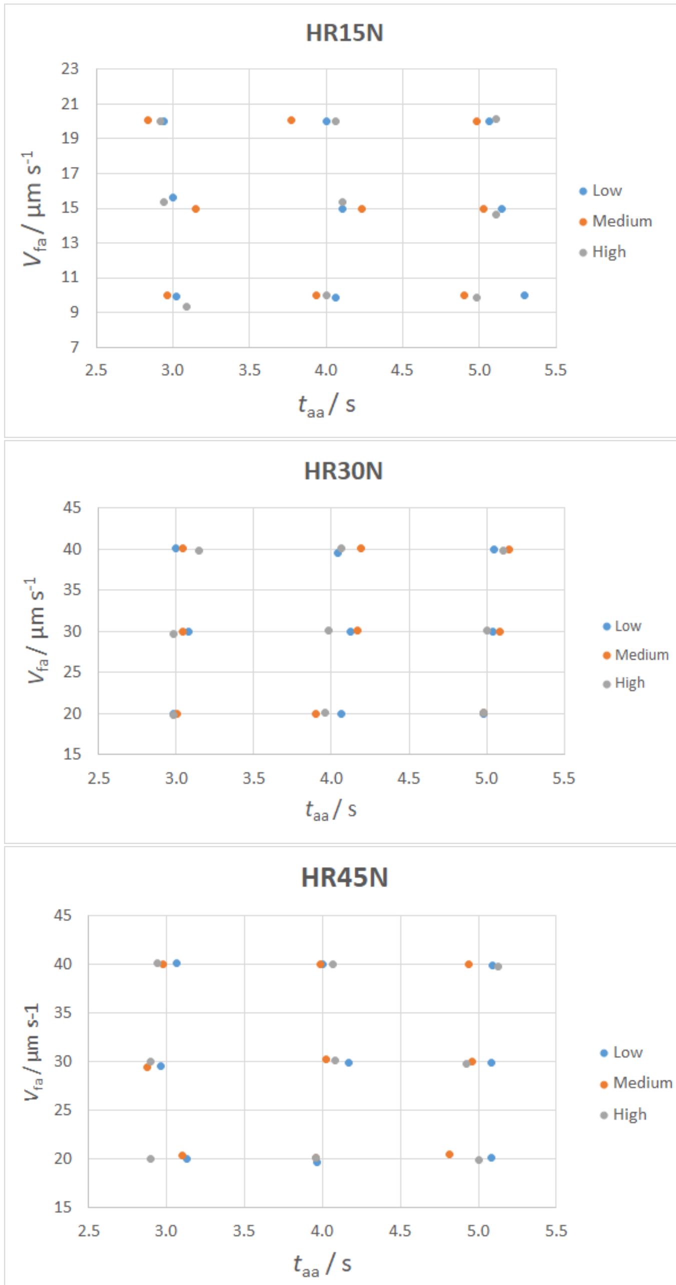

Figure 4: Experimental plans, for the three superficial Rockwell hardness scales HR45N, HR30N and HR15N.

3 Monte Carlo method for MLR

One of the major problems regarding the calculation of sensitivity coefficient is due to an handful of factors: the first one is due to a poor experimental design, which leads to the difficulty in decoupling the parameters and therefore the inability to clearly attribute the cause of a certain change in the measurement to a specific variable; the second one is that, even in the case of a successful experimental design, many doubts arise on how to calculate the uncertainty of the sensitivity coefficients. For instance, in the field of Hardness measurement, it often happens that the uncertainty of the hardness measurement is associated directly to the sensitivity coefficients [3, 2], while in other cases, the uncertainty contribution of the sensitivity coefficients is simply given by the standard error from the Multiple Linear Regression. However, the authors point out that due to the variability of the hardness measurements and the variability of the input influence variables (V_{fa} and t_{aa}), the usual MLR cannot be used to evaluate the sensitivity coefficients. In general, the case where both input and output variables are associated with uncertainties is managed using the Weighted Total Least Squares (WTLS), which is based on a minimization process to be implemented numerically [22]. Instead the author propose a new simple methodology based on a Monte Carlo Method applied to Multiple Linear Regression [23]. The mathematical measurement model we are dealing with in this paper is in the form:

$$HR = f(h, X, Y) \tag{1}$$

where the scalar output HR is function of the indentation depth h and two additional input variables $X = V_{\mathrm{fa}}$ and $Y = t_{\mathrm{aa}}$. Each variable is obtained experimentally, therefore it must be treated as a random variable with associated uncertainty.

3.1 Algorithmic set up

Suppose that from the experimental analysis a set of N_{exp} experimental points $\{a, b, c, \dots\}$ was established. Then, experiments are carried out performing M repetitions for each experimental point: for example, for point a we measure the *input* $X_{a,1}, X_{a,2}, \dots, X_{a,M}$, $Y_{a,1}, Y_{a,2}, \dots, Y_{a,M}$ and the output $HR_{a,1}, HR_{a,2}, \dots, HR_{a,M}$. From these experimental data, it is assumed we can assign to each variable (input and output) a certain Probability Density Function (i.e. a Normal distribution or a Student's t-distribution). Via a Monte Carlo (MC) method, it is possible to sample $X_a, X_b, \dots$, $Y_a, Y_b, \dots$ and $HR_a, HR_b, \dots$ from their realted Probability Density Functions (PDFs) and perform a Multiple Linear Regression (MLR). For a first order model, from each sampling a *regression plane* is generated corresponding to a function in the form:

$$HR = \underbrace{N - \frac{h}{S}}_{=A} + c_{\mathrm{t}}^{i} t_{\mathrm{aa}} + c_{\mathrm{v}}^{i} V_{\mathrm{fa}} \tag{2}$$

where A is the intercept and the c^i are the *sensitivity coefficients* at the i-th MC iteration (sampling). Finally, after the last sampling, we have N_{MC} regression planes spanning over a *cuboid* which is used to define the uncertainties of the sensitivity coefficients (fig.5). For a bi-linear model, one can simply calculate the mean $c_{V_{\mathrm{fa}}}, c_{t_{\mathrm{aa}}}$ and their standard uncertainties (given as standard deviation) u_{MC} of the sensitivity coefficients obtained during each MC iteration.

3.2 Evaluation of the standard uncertainties of the sensitivity coefficients

Assuming that the sensitivity coefficients are independent random variables, we evaluate the standard uncertainties for each of the sensitivity coefficients individually: in the following, we denote either $c_{V_{\mathrm{fa}}}$ or $c_{t_{\mathrm{aa}}}$ by c_i.

Once the N_{MC} sensitivity coefficients c_i have been evaluated from the procedure presented in the previous section, the sample standard deviation of c_i can be evaluated as:

$$u_{MC}^2 = \frac{1}{N_{\mathrm{MC}} - 1} \sum_{i=1}^{N_{\mathrm{MC}}} \left(c^i - \overline{c} \right)^2 \tag{3}$$

where $\overline{c}$ is the mean of the sensitivity coefficient $\overline{c} = \sum_{i=1}^{N_{\mathrm{MC}}} c_i / N_{\mathrm{MC}}$.

The MLR used in each of the N_{MC} iteration is itself characterized by an uncertainty contribution, that represents how much the hyperplanes obtained from the MLR fail to meet each experimental point: this contribution arises from the Ordinary Least Squares method (OLS) on which the MLR is based. In order to follow a more conservative approach, we take the maximum among all the OLS uncertainties from all the MC iterations, denoted by u_{OLS}^2. The total uncertainty of the sensitivity coefficient is thus given as the sum of the squares of the two contributions u_{MC} and u_{OLS}:

$$u_{HR} = \sqrt{u_{\mathrm{MC}}^2 + u_{\mathrm{OLS}}^2} \tag{4}$$

A similar analysis has been applied for a simple linear model in [1], which the reader is referred to as far as the implications of considering the uncertainty of each sensitivity coefficient in the measurand model are concerned. Indeed, in [1], the authors propose a *modified* mathematical measurement model (which resembles the one in (2)) via linearly introducing the additional influence parameters each with the related sensitivity coefficients. Since obtained experimentally, the sensitivity coefficients have to be treated as random variables and, therefore, their uncertainty contributions must be propagated through the law of propagation of uncertainty [1, 24].

4 Results and discussion

In the proposed case, the MC method was applied sampling from a normal and t-student distribution (due to the few number of repetitions available for each measurement condition). The results are given for the ladder PDF using $10^6 - 10^7$ iterations as suggested in [25]. The usual MLR and the MC methods applied to MLR have been compared and yield very close results as far as the evaluation of the sensitivity coefficients is concerned (this due to the linear character of the modified model). The bilinear-model is the following:

$$HR = A + B_1 \cdot V_{\mathrm{fa}} + B_2 \cdot t_{\mathrm{aa}} \tag{5}$$

It is important to remark that the repetitions of the hardness measurements for the same parameter combination (V_{fa} and t_{aa}) are performed at different points on the same hardness block. Therefore, a natural question arises: is the change in the hardness measurement caused by the variation of the parameters V_{fa} and t_{aa} or the result of the material inhomogeneity of the hardness block? Indeed, citing [16]: '*when we speak of repeated hardness measurement (e.g.* $n = 5$ *measurements of the same object) we have not measured the same quantity*'. In order to overcome this problem, once the estimate of each sensitivity coefficients is determined, corrections are performed on the hardness

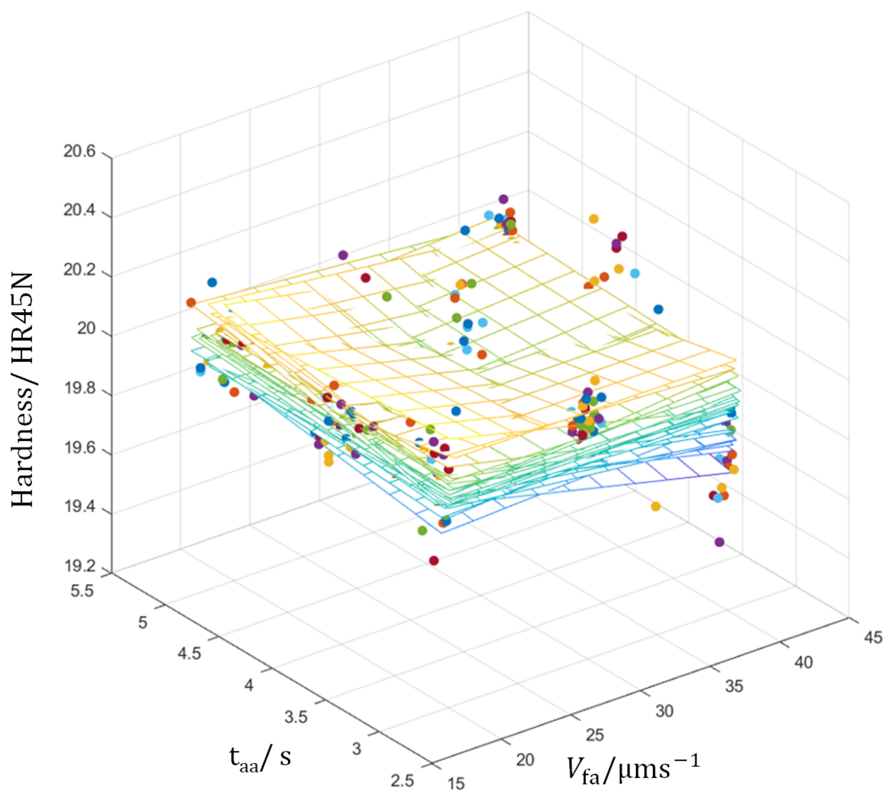

Figure 5: Example of possible Regression planes obtained from 15 MC samplings.

measurements. After that, the previous MC method for MLR is performed with a `MATLAB` script. The sensitivity coefficients and their expanded uncertainties are given in table 1. Thanks to data available in literature, it was possible to compare the data gathered in this study with the other obtained by NPL and NIST (fig. 6).
Two important remarks should be made from the analysis of fig.6 and the related literature available:

- when the uncertainties of the sensitivity coefficients are not given (as happened in most cases during the study of the literature for the writing of this paper), we could only make observations based on the expected values of the sensitivity coefficients, which generally agree among different NMIs.

- even when the uncertainties of the sensitivity coefficients are given, the authors of this paper have not found a unifying methodology that explained how such uncertainty contributions were calculated (i.e. are those uncertainties based only on the standard uncertainty given from the OLS for the Linear Regression as in [3], or are they given the same uncertainty of the hardness measurements as in [2]?). Additionally, even when given, the uncertainty contributions of the sensitivity coefficients are generally not propagated through the *law of propagation of uncertainty* to calculate the combined standard uncertainty of the measurement model. The authors believe that, when non-negligible, the uncertainty contributions must be considered in the combined standard uncertainty of the measurement model: such additional contributions take into account how much the measurements fail to be performed at exactly the reference values prescribed in the related standards. A discussion and different case studies concerning this problem have been presented in [1].

It should be noted that this first experimental plan was limited due to availability of the machine time: we hope to gain more insights carrying out again the experiments investigating the same parameter and increasing the number of experiments (both repetitions and additional points in the experimental plans) to get more realistic PDFs for the Monte Carlo MLR methodology proposed above and possibly reduce the uncertainty contributions for each sensitivity coefficient. As a final remark, when only the standard uncertainties given by the OLS method for MLR (without using the Monte Carlo procedure introduced previously) were associated to the sensitivity coefficients resulting by our experiments, we obtained comparable uncertainty budgets with respect to the ones published by the other NMIs (when such information was found!).

	Nominal	A/HR	B_1/HR s μm^{-1}	B_2/HR s^{-1}
$HR45N$	Low	19.98 ± 1.404	-0.0027 ± 0.0338	0.0098 ± 0.264
	Medium	49.30 ± 1.206	-0.0061 ± 0.0208	-0.0143 ± 0.244
	High	68.63 ± 1.092	-0.0052 ± 0.0138	0.0091 ± 0.215
$HR30N$	low	41.14 ± 0.397	-0.0039 ± 0.0101	0.0438 ± 0.271
	Medium	64.28 ± 1.67	-0.0149 ± 0.0454	-0.0013 ± 0.270
	High	78.91 ± 1.4128	-0.0050 ± 0.0252	0.0044 ± 0.213
$HR15N$	Low	68.64 ± 0.7731	-0.0062 ± 0.0316	0.000 ± 0.1398
	Medium	82.64 ± 0.801	-0.0287 ± 0.0466	0.0217 ± 0.2112
	High	91.10 ± 0.2069	-0.0216 ± 0.0116	0.0094 ± 0.0483

Table 1: Sensitivity coefficients and their expanded uncertainties $U_{95\%}$ obtained via a Monte Carlo Multiple Linear regression. For lightness of notation, HR stands for HR45N,HR30N and HR15N according to the related scale.

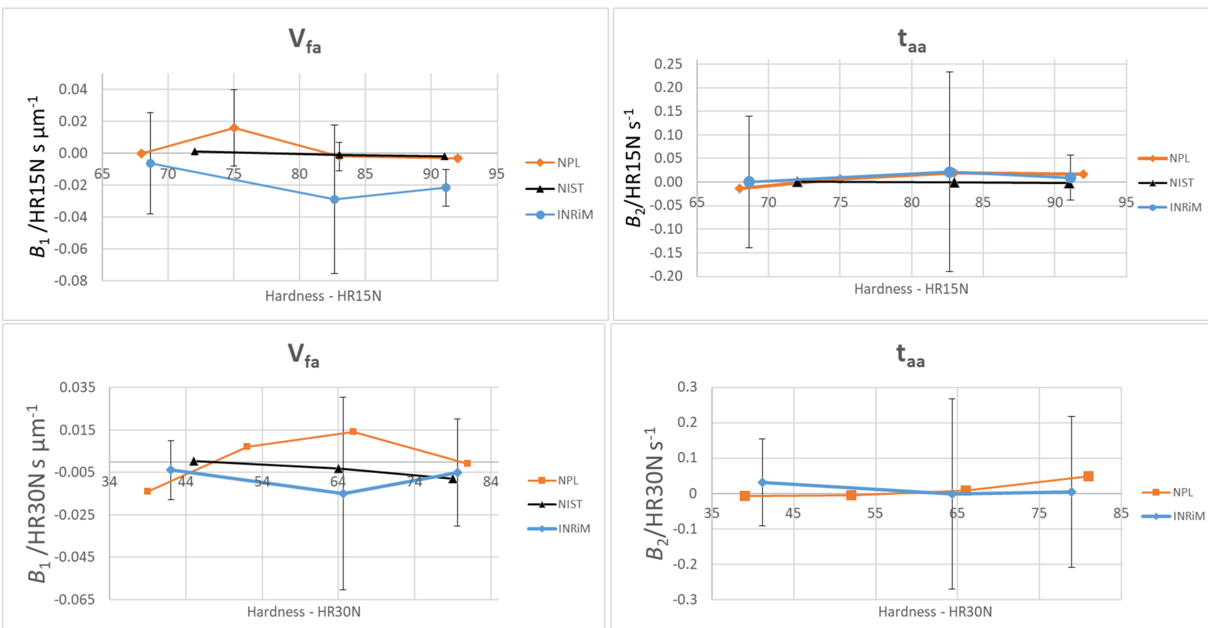

Figure 6: Comparisons for HR15N and Hr30 scale of the results obtained by INRiM (with related expanded uncertainty) with other NMIs [15].

5 Conclusion

In this paper, the influence on superficial Rockwell Hardness measurement of the velocity of the final load application V_{fa} and the time interval of the force variation from pre-load to its final value were investigated. The estimation of the related sensitivity coefficients seems to agree in most cases with the ones offered by literature (other NMIs). However an important question is raised on how to estimate the uncertainties of the sensitivity coefficients. In this paper, it is proposed that the uncertainty contribution of each sensitivity coefficient may be obtained from its repeatability via a Monte Carlo Method for Multiple Linear Regression, while the other contribution comes from the standard error from the usual Ordinary Least Squares method associated with the regression method. As a final remark on this paper, we would like to draw more attention on the urgent need to establish a unifying methodology to be applied in the determination of the uncertainty of the sensitivity coefficient: such uncertainties can be used to establish whether a given sensitivity coefficient is indeed of influence (*significant*) and must be propagated, if non-negligible, in the combined standard uncertainty in the hardness measurement model where additional variables (such as in our case V_{fa} and t_{aa}) are used in the measurement mathematical model as in [1].

Conflict of interest

The Authors declare that they do not have any conflict of interest.

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