Evaluation of the uncertainty of coordinate measurements based on the formula for EL,MPE

Evaluation of the uncertainty of coordinate measurements based on the formula for $E_{L,MPE}$

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Abstract

The methodology of evaluating the uncertainty of coordinate measurements, developed at ATH (University of Bielsko-Biała) and verified within the EUCoM project, is presented, in which the only information about the accuracy of the CMM used is the formula for $E_{L,MPE}$. The measurement models used for individual characteristics are derived from distance point-point, point-straight line, point-plane and straight line-straight line formulae. Information about the shape and dimensions of the measured workpiece is in the form of the minimum number of appropriately distributed points needed to define individual characteristics. The measurement model is a formula expressing the measured characteristics as a function of the differences in the coordinates of the characteristic points. The GUM uncertainty framework is used to propagate individual uncertainty components. Two models and example uncertainty budget is presented.

1 Introduction

Ideas for easy and understandable ways to determine the uncertainty of coordinate measurements have been sought for many years. Currently, the "understandable" condition is met by the technique described in ISO 15530-3 [1]. However, this technique requires a time-consuming experiment (the main component of uncertainty is determined by the B method) and accurate artefacts. The ATH has developed a method that does not require experimentation – all components of measurement uncertainty are determined using the type A method [2, 3, 4]. The method was verified within the EUCoM project [5] by comparing the results of evaluations with the experimental method [6]. A calibrated cylindrical square was used for validation. 17 circles' diameters and

84 different combinations of datum length and distance of the toleranced element from the datum for measuring coaxiality were adopted as validated characteristics.

2 Measurement models

The measurement models used for the individual characteristics are derived from known distance formulas (see ISO 17450-1, Table B.7):

- point-point ($\boldsymbol{PT}_1$, $\boldsymbol{PT}_2$ – coordinates of points)

$$d(\boldsymbol{PT_1}, \boldsymbol{PT}_2) = |\boldsymbol{PT}_1, -\boldsymbol{PT}_2| \tag{1}$$

- point-straight line ($\boldsymbol{PT}_1$ – point coordinates, $\boldsymbol{A}_2$ – coordinates of a point on the straight line, $\boldsymbol{u}_2$ – unit director vector of the straight line)

$$d(\boldsymbol{PT}_1, \boldsymbol{SL_2}) = |(\boldsymbol{A}_2, -\boldsymbol{PT}_1) \times \boldsymbol{u_2}| \tag{2}$$

- point-plane ($\boldsymbol{PT}_1$ – point coordinates, $\boldsymbol{A}_2$ – coordinates of a point on the straight line, $\boldsymbol{u}_2$ – unit normal vector of the plane)

$$d(\boldsymbol{PT}_1, \boldsymbol{PL_2}) = |(\boldsymbol{A_2} - \boldsymbol{PT}_1) \cdot \boldsymbol{u_2}| \tag{3}$$

- straight line-straight line ($\boldsymbol{A}_1$, $\boldsymbol{A}_2$ – coordinates of points on the straight lines, $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ – director vectors of the straight lines)

$$d(\boldsymbol{SL}_1, \boldsymbol{SL_2}) = \left|(\boldsymbol{A}_2 - \boldsymbol{A}_1) \cdot \frac{\boldsymbol{u}_1 \times \boldsymbol{u}_2}{|\boldsymbol{u}_1 \times \boldsymbol{u}_2|}\right| \tag{4}$$

The measurement model is a formula expressing the measured characteristic (output quantity) y as a function of coordinate differences (input quantities) x_i of the so-called "characteristic points".

3 Shape and dimensions

Information about the shape and dimensions of the measured workpiece are the coordinates (in any coordinate system) of the minimum number of characteristic points needed to define individual characteristics. These points can be points of integral or derived features (e.g. axis points) and should be distributed analogously to the sampling strategy used.

4 Uncertainties of input quantities

The measurement uncertainties of the individual input quantities are calculated by the formula (see ISO 14253-2, 8.4.5) [7]:

$$u_i = E_{L,MPE} \cdot b \tag{5}$$

Assuming that the distribution of CMM indication errors is normal and 95% of these errors fall within the range $\pm E_{L,MPE}$ the coefficient b = 0,577.

If the results of acceptance tests or reverification tests are known, the following formula is used to calculate *b* (usually *N* = 105):

$$b = \sqrt{\frac{1}{N}\sum_{i=1}^{N} E_i} \tag{6}$$

where

$$E_i = \frac{E_{L,i}}{E_{L,MPE,i}} \tag{7}$$

5 Propagation of uncertainty components

The GUM uncertainty framework (according to ISO/TS 15530-1 – sensitivity analysis) is used to propagate individual uncertainty components:

$$u_c = \sqrt{\left(\sum \frac{\partial y}{\partial x_i} u_i\right)^2} \tag{8}$$

The values of partial derivatives needed for the budget can be easily calculated analytically or numerically using a program written in a language that uses vector notation (Matlab, Maple, Python).

6 Examples

Example 1. Axis straightness
The points containing the information needed to build the measurement model are the two points (*A* and *B*) from the ends of the axis and one point from its centre (*S*) (Fig. 1).

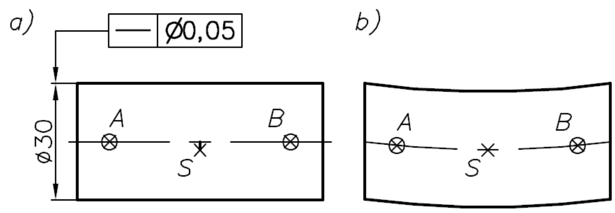

Fig. 1. Characteristic points for the axis straightness measurement model: a) drawing with points, b) deviation visualization

The measurement model results directly from the formula for the distance of point *S* from the straight line *AB*:

$$STR(x_{AS}, y_{AS}, z_{AS}, x_{AB}, y_{AB}, z_{AB}) = l(\boldsymbol{AS}, \boldsymbol{AB}) = \left|\boldsymbol{AS} \times \frac{\boldsymbol{AB}}{|\boldsymbol{AB}|}\right| \tag{9}$$

The uncertainty budget is presented in Table 1.

Table 1. Uncertainty budget for or the distance between point *S* and straight line *AB* $E_{L,MPE} = 2 + L/250$ and $b = 0.333$ and the workpiece length 100 mm

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	u_{xi} , µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
x_{AS}	50	0	0.73	0
y_{AS}	0	0	0.67	0
z_{AS}	0.01	1	0.67	0.67
x_{AB}	100	0	0.80	0
x_{AB}	0	0	0.67	0
x_{AB}	0	-0.50	0.67	0.33
			u_c =	0.75

The standard uncertainty of axis straightness measurement is 0.75 µm. The largest uncertainty component (with sensitivity coefficient 1) is connected with measurement of a small distance (0.1 mm in the example) between the point *S* and the straight line *AB*. The second significant component (with sensitivity coefficient 0.5) depends on the length of the workpiece.

Example 2. Coaxiality

The points containing the information needed to build the measurement model are two points (*A* and *B*) defining the datum and one point (*S*) defining the toleranced feature (Fig. 2)

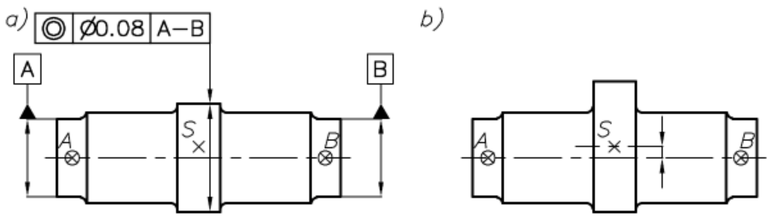

Fig. 2. Characteristic points for the measurement model of coaxiality: a) drawing with characteristic points, b) visualization of the deviation

The measurement model results directly from the formula for the distance of point *S* from the straight line *AB*:

$$COAX(x_{AS}, y_{AS}, z_{AS}, x_{AB}, y_{AB}, z_{AB}) = 2 \cdot l(\boldsymbol{AS}, \boldsymbol{AB}) = \\ = 2\left|\boldsymbol{AS} \times \frac{\boldsymbol{AB}}{|\boldsymbol{AB}|}\right| \tag{10}$$

For the same CMM and workpiece length, the uncertainty budget for the distance of point *S* from line *AB* will be the same as for the straightness deviation, but the uncertainty value will be twice as large (u_c = 1.5 µm).

Example 3. Coaxiality (the toleranced feature aside of the datum)
The points containing the information needed to build the measurement model are two points (*A* and *B*) defining the datum and one point (*S*) defining the toleranced feature (Fig. 3)

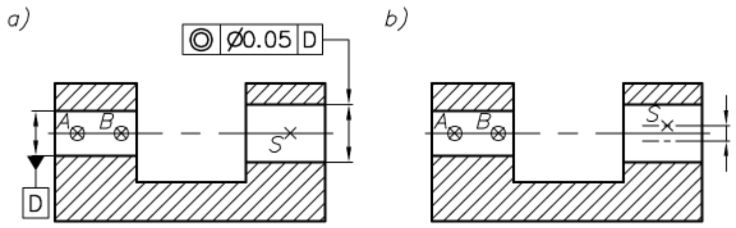

Fig. 3. Characteristic points for the measurement model of coaxiality: a) drawing with characteristic points, b) visualization of the deviation

The measurement model is the same as in the example 2. The uncertainty budget is presented in Table 2.

Table 2. Uncertainty budget for the distance between point *S* and straight line *AB* for $E_{L,MPE}$ = 2 + *L*/250 and *b* = 0.333; the datum length is 20 mm and the toleranced feature is 90 mm aside of the datum.

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	u_{xi} , µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
x_{AB}	20	0	0.69	0
y_{AB}	0	0	0.67	0
z_{AB}	0	-4.5	0.67	-3
x_{BS}	90	0	0.79	0
y_{BS}	0	0	0.67	0
z_{BS}	0.01	1	0.68	0.68
			u_c =	3.07

The standard uncertainty of the coaxiality is twice as large as the uncertainty of straightness presented in the Table 2 and equals 6.14 µm. The sensitivity coefficient 4.5, assigned to the largest component of the uncertainty budget, is connected with significant distance of the point *S* from the datum *AB*. The second component (with the sensitivity component 1) relates to the measurement of small distance (in the example 0.1 mm) between point *S* and straight line *AB*.

Example 4. Parallelism of planes

The points containing the information needed to build the measurement model are two points (*A* and *B*) defining the datum and one point (*S*) defining the toleranced feature (Fig. 2)

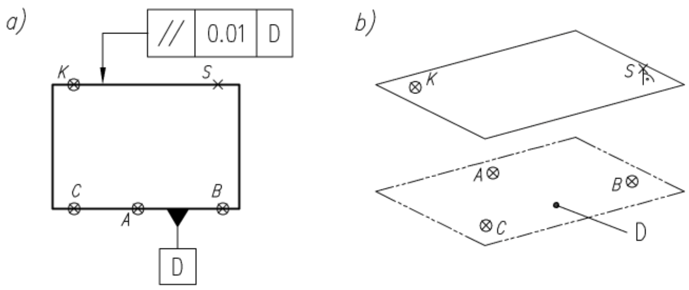

Fig. 4. Characteristic points for the measurement model of parallelism: a) drawing with characteristic points, b) visualization of the deviation

The measurement model results directly from the formula for the distance of point *S* from the plane containing point *K* and parallel to *ABC*:

$$PAR(x_{BA}, y_{BA}, z_{BA}, x_{BC}, y_{BC}, z_{BC}, x_{KS}, y_{KS}, z_{KS}) = \\ = l(\boldsymbol{BA}, \boldsymbol{BC}, \boldsymbol{KS}) = \left| \boldsymbol{KS} \cdot \frac{\boldsymbol{BA} \times \boldsymbol{BC}}{|\boldsymbol{BA} \times \boldsymbol{BC}|} \right| \tag{11}$$

The uncertainty budget is presented in Table 3.

Table 3. Uncertainty budget for $E_{L,MPE}$ = 2 + *L*/250 and *b* = 0.333 and the workpiece with dimensions (300 × 300 × 200) mm

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	u_{xi} , µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
x_{BA}	-300	0	1.07	0
y_{BA}	-150	0	0.87	0
z_{BA}	0	1	0.67	0.67
x_{BC}	0	0	0.67	0
y_{BC}	-300	0	1.07	0
z_{BC}	0	0.5	0.67	0.33
x_{KS}	300	0	1.07	0
y_{KS}	300	0	1.07	0
z_{KS}	0.01	1	0.68	0.68
			u_c =	1.00

The standard measurement uncertainty of parallelism planes is 1.00 µm. The two largest uncertainty components (both with a sensitivity coefficient of 1) are related to short distance measurements (0 mm and 0.01 mm in the example). The first concerns the accuracy of measuring the points defining the plane *ABC*, the second the distance of point *S* from the plane containing point *K* and parallel to the *ABC* plane. The third component (with a sensitivity coefficient of 0.5), similarly to the first, concerns the accuracy of measuring points defining the plane. The measurement uncertainty is not affected by the dimensions of the feature.

7 Summary

The use of the minimum number of points made it possible to "discover" the laws applicable to coordinate measurements of geometric deviations. Some deviations are small distances of a point from a straight line or a plane, others are doubled values of these distances, and still others are geometric sums of two such distances.
If the points of derived features are used as characteristic points (in the examples, these are points of the axis), then they replace the sampling points used to determine them.
Assuming coordinate differences as the input value allows for consistency with the formula for $E_{L,MPE}$, which concerns distance measurements.

References

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