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1	JOURNAL: Microfluidics and Nanofluidics
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3	Brief Communication
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5	Fluid flow-based description of the geometrical features in fluidic channels using the Shannon's
6	information theory: an exploratory study
7	
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17 Keywords

18 Shannon Entropy, microfluidics, information theory, thermodynamics, fluids

19

20 Abstract

21	Inspired by Nature, where storing information is an intrinsic ability of natural systems, here we
22	investigate the capability of interacting systems to transport/store the information
23	generated/exchanged in the interaction process in the form of energy or matter, preserving it
24	over time. In detail, here we test the possibility to consider a fluid as a carrier of information,
25	speculating about how to use such information. The final goal is to demonstrate that
26	information theory can be used to illuminate physical observations, even in those cases where
27	the equations describing the phenomenon under investigation are intractable, are affected by
28	a budget of uncertainty that makes their solution not affordable or may not even be known. In
29	this exploratory work an information theory-based approach is applied to microfluidic data. In
30	detail, the classical study of the fluid flow in a microchannel with obstacles of different
31	geometry is faced by integrating fluid mechanics theory with Shannon's theory of information,
32	interpreted in terms of thermodynamics. Technically, computational fluid dynamics
33	simulations at Reynolds' numbers (<i>Re</i>) equal to 1 and 50 were carried out in fluidic channels
34	presenting obstacles with rectangular and semicircular shape, and on the simulated flow fields

35	the Shannon's information theory was applied evaluating the fluid dynamics information
36	entropy content. It emerged that the Shannon Entropy (SE) evaluated at the outflow section of
37	the flow channel depends upon the geometric features (i.e. position, shape, aspect ratio) of the
38	obstacles. This suggests an interpretation of the fluid dynamics establishing in a flow channel
39	presenting obstacles in terms of information theory, that can be used to identify <i>a posteriori</i>
40	the geometric features of the obstacles the fluid interacts with. The proposed approach can be
41	applied to flow data at the boundaries of fluid domains of interest to extract information on the
42	process occurring inside a system, do not making any appeal to the governing equations of the
43	phenomenon under observation or intrusive measurements.
44	
45	1 Introduction
46	In principle, the behavior of fluids in motion can be fully described by three equations: an
47	equation describing the conservation of mass, a second equation based on the second Newton's
48	law of motion, and a third equation based on the conservation of energy (Cox 2015; Khasanov
49	2011; Merdasi et al. 2018).

50 A condition for a real fluid to be in motion is that sufficient energy is to be spent. It follows

51 that the fluid in motion is a system that modifies its internal energy, thus generating a variation

52	of entropy (S) of the system in a purely thermodynamic meaning. The concept of entropy (not
53	univocally accepted indeed), which can be explained as the level of disorder of a system
54	describing its evolution under the effect of the external environment, was taken up by Claude
55	Shannon in 1948 (Shannon 1948) and applied to the field of information theory in order to
56	describe the level of complexity of a signal in data communication systems. In general, it is still
57	uncommon to apply information theory for the purpose of the analysis of flow fields
58	characterized by different levels of complexity (Ikeda and Matsumoto 1986). Indeed, the idea
59	of considering a fluid in motion as an information carrier is not new, inspired by the fact that
60	Nature uses fluids to transport a plethora of biochemicals. Looking at a fluid in motion as an
61	information carrier (e.g. nutrients to cells, proteins through blood, etc.), a parallelism between
62	fluid flow and information theory based on the definition of the SE can be easily established.
63	Motivated by the possibility of interpreting a fluid as information carrier, in this study
64	information theory is applied to microfluidics, where its capability to discriminate the shape of
65	obstacles in a channel based on the knowledge of the motion of fluid particles upstream and
66	downstream of the obstacle is tested. In detail, the capability of the SE to discriminate the shape
67	of the obstacle based on the distortion of fluid streamlines is tested, with the final aim to use
68	the SE of the system as a <i>fingerprint</i> of the obstacle shape-specific flow perturbation. The

potency of the herein proposed approach has not been largely explored. To cite a valuable
example, Pozo et al. (Pozo et al. 2017) analyzed the flow complexity in open systems,
approaching the distortion of fluid streamlines in a channel at the level of information
transmission.

Technically, here computational fluid dynamics (CFD) solutions of laminar flow in channels 73 74 presenting obstacles were compared to an "ideal communication" channel where the 75 transmitted and received messages are identical and SE was applied to CFD data. The characterization of microflows as information carriers finds several applications, e.g. to assess 76 77 mixing efficiency in micromixers (Camesasca et al. 2006; Pennella et al. 2010; Pennella et al. 78 2012) or in microreactors and in scaffolds and bioreactors for tissue engineering applications 79 (Eijkel and Van den Berg 2005; Bilen and Yapici 2002; Yojina and Ngamsaad 2010; Tariq et al. 80 2020) and in many other technological fields like solar energy (Ali 2020), nanofluids for manufacturing processes (Wang et al. 2020), heat and mass transfer (Chen et al. 2020; Sajjad 81 82 et al. 2020; Sözen et al. 2021).

The principal aim of this study, and the reason of its novelty, was to investigate the behavior of a fluid that flows in a channel in presence of obstacle/s characterized by different shape, multiplicity and disposition and employing the information theory. This exploratory study

- 86 highlights that at defined conditions a simplified approach based on the information theory can
- 87 be applied to extract information on the processes occurring within closed systems, and that
- this can be done by measuring only the "information" at systems' boundaries.
- 89

90 2 Methods

In this study a microchannel geometry without and with obstacles of different shape and position was investigated. Overall, as depicted in Fig. 1, nine configurations were considered, i.e. one without obstacles and 8 with obstacles. The ratio between the height of the channel h_{ch} and the characteristic obstacle dimension h₁ was set to h₁ = h_{ch}/6. In microchannel configurations where more than one obstacle was considered, inter-distance was set equal to h_{ch}/2 (Fig. 1).



Fig. 1 Schematic illustration of the investigated channel geometries with obstacles ($h_1 = h_{ch}/6$). The analysis on models a, d, e and h is presented in the Results Section. Coordinates (*i*, *j*) correspond to the logical coordinates useful for the computational analysis

The analytical solution of the fluid motion in the microchannel without obstacles can be easily
obtained from Navier-Stokes equations. In detail, for an incompressible, homogeneous,
Newtonian fluid in steady-state laminar condition, the law of motion in the microchannel can
be expressed as:

105
$$u_x = -\frac{1}{2\mu} \cdot \frac{dp}{dx} \cdot (y_0^2 - y^2) (1)$$

106 where u_x is the velocity in axial (x, see Fig. 1) direction, μ the dynamic viscosity, p the pressure, 107 y the general position in vertical direction (Fig. 1) and y_0 the coordinate of the microchannel 108 wall, where velocity is equal to 0 (i.e. no-slip conditions).

109 The finite volume-based CFD commercial code Fluent (ANSYS Inc., USA) was adopted to solve the discretized governing equations of fluid motion in microchannels with obstacles. In detail, 110 the Navier-Stokes equations in their discretized form and under steady-state laminar 111 conditions were solved using a second order pressure discretization and a second order 112 upwind momentum discretization scheme. The fluid was assumed to be isotropic, 113 114 incompressible and Newtonian with a density value ρ equal to 998.2 [kg·m⁻³] and a dynamic viscosity μ equal to 10⁻² [kg·s·m⁻²]. To ensure grid-independence of the solution, based on a 115 mesh sensitivity analysis, quad-mesh with elements size of $1 \cdot 10^{-4}$ m was adopted. On average, 116 the resulting computational grids consisted of 24000 elements and 24381 nodes. Two different 117

118	flow regimes, characterized by <i>Re</i> equal to 1 and 50, were simulated by applying Dirichlet
119	conditions at the inflow section of the microchannel geometry (in terms of flat velocity profile),
120	while the reference pressure Neumann boundary condition was imposed at the outflow section.
121	Walls were assumed to be rigid, and the no-slip condition was imposed.
122	The concept that a confined fluid in motion is an information carrier, and the parallel with a
123	communication system, was translated into a scheme where: (1) the fluid in the microchannel
124	is the carrier; (2) the inflow section (or more in general a section upstream of the channel
125	segment presenting obstacles) is the transmitter; (3) the outflow section (or more in general a
126	section downstream of the channel segment presenting obstacles) is the receiver; (4) the
127	difference in flow features between the two sections is the information transmitted. In this
128	regard, flow perturbations induced by the presence of obstacles can be regarded as the "noise"
129	affecting the system (Fig. 2). The channel without obstacles was considered as the reference
130	case, i.e. the case where the carrier (fluid) is not disturbed and the transmitted information is
131	not modified.





laminar conditions, will perturb the flow with the consequence that streamlines will be deflected losing their parallelism. This behavior was translated building up a binary matrix, representative of the phenomena. To build up the representative binary matrix, here fluid velocity data from CFD simulations were considered (i.e. for each cell of the quad-mesh a value of velocity was extrapolated). Technically, a $N \times M$ matrix $B_{i,j}^{x}$ was built:

150
$$B_{i,j}^{x} = \begin{bmatrix} b_{1,1} & \dots & b_{1,M-1} \\ \vdots & \ddots & \vdots \\ b_{N,1} & \dots & b_{N,M-1} \end{bmatrix}$$
 (2)

151 with $i = \{1, ..., N\}$, where N is the number of grid cells in y direction, and $j = \{1, ..., M - 1\}$, where

152 M is the number of grid cells in x direction (Fig. 1). The binary elements $b_{i,j}$ of the matrix were

153 calculated as follows:

154
$$b_{i,j} = \begin{cases} 1, & \frac{\overline{u}_{i,j}^{x}}{\overline{u}_{i,j-1}^{x}} \leq \zeta_{i,j} \\ 0, & otherwise \end{cases}$$
(3)

where $\bar{u}_{i,j}^x$ and $\bar{u}_{i,j-1}^x$ are the values of the *x*-component of the velocity at grid cell location (*i*, *j*) and (*i*, *j*-1), respectively. The threshold values $\zeta_{i,j}$ for matrix $B_{i,j}^x$ binarization were set according

157 to:

158
$$\zeta_{i,j} = \frac{\overline{u}_{i,j}^{num.}}{\overline{u}_{i,j-1}^{num.}} - \frac{\overline{u}_{i,j}^{ana.}}{\overline{u}_{i,j-1}^{ana.}} = 0$$
 (4),

and depend on the results of CFD analysis and Eq. (1). From this equation, $\bar{u}_{i,j}^{num.}$ and $\bar{u}_{i,j}^{ana.}$ are the CFD and the analytical velocity values in *x* direction at grid cell location (*i*, *j*), and $\bar{u}_{i,j-1}^{num.}$ and $\bar{u}_{i,j-1}^{ana.}$ are the CFD and the analytical velocity values in *x* direction at grid cell location (*i*, *j*-1), respectively.

Here the SE was employed to quantify the level of interaction between fluid flow and the obstacles in the microchannel, intended as the level of streamlines deflection with respect to the microchannel without obstacles, binarized according to matrix $B_{i,j}^{x}$. By definition, the formulation of the SE is given by:

168
$$SE(X_i) = -P(X_i) \log_2 P(X_i)$$
 (5),

169 where X_i is a discrete random variable with possible values { X_1 , ..., X_n } and $P(X_i)$ is the 170 probability distribution of X_i . Theoretically, an increase of entropy corresponds to a loss in 171 information content. In this study, the SE was evaluated according to:

$$172 \quad SE = -P_i \log_2 P_i \tag{6}$$

173 where P_i is given by:

174
$$P_i = \frac{\sum_{j=1}^{J} b_{i,j}}{N}$$
 (7)

Based on Eqs. (6) and (7), the SE ranges between zero (i.e. no information lost or, for thespecific application, no distortion of fluid streamlines) and one (i.e. all the information carried

by the fluid lost due to the disruption of the flow field as a consequence of its interaction withobstacles in the microchannel).

179

180 3 Results

The analysis was extended to all the microchannels with different obstacles geometry and 181 182 number summarized in Fig. 1. As first explanatory example, the SE values computed along two cross-sections (proximal and distal to the obstacle, respectively) of the two microchannels with 183 single obstacles of rectangular and semicircular shape, at two different *Re* numbers (1 and 50), 184 185 are presented in Fig. 3. At *Re* = 1, as expected, moderate differences are highlighted by the SE 186 both between the upstream and downstream sections (flow field distortion consequence of the presence of the obstacle) and between the two microchannels (consequence of the different 187 188 shape of the obstacles). Marked differences in SE distribution (and in absolute values as well) along the cross-sections emerge at Re = 50 (Fig. 3): as expected, the presence of the rounded 189 (semicircular) obstacle, which is expected to distort fluid streamlines less than the obstacle 190 with rectangular shape, modifies trans-obstacle SE values markedly less than the rectangular 191 single obstacle (the semicircular shape maintains almost unaltered the SE of the system even 192 increasing the inertial effects by one order of magnitude, as stated by the *Re*). The results of Fig. 193

194 3 confirm the capability of the SE of discriminating between obstacle shapes, properly







200 Figure 3 also highlights an expected lack of symmetry (more pronounced at Re = 50) in the SE 201 cross-sectional distribution, reflecting the absence of geometrical and fluid dynamical 202 symmetry in the two microchannels with single obstacles. In this regard, the second explanatory example of Fig. 4 reports the SE values computed along two cross-sections 203 (proximal and distal to the obstacle, respectively) of the two microchannels presenting 204 205 obstacles with rectangular and semicircular shape, symmetrically located with respect to the microchannel axis. In this case, the cross-sectional SE distributions: (1) confirm the capability 206 of capturing the different influence of the shape of the obstacle in the flow field, with marked 207 208 differences related to the obstacle shape clearly evident also at Re = 1; (2) adequately reflect 209 the presence of a geometrical (and fluid dynamical) symmetry of the microchannel system. 210 Summarizing, the cross-sectional distributions of Figs. 3 and 4 clearly demonstrate that SE 211 is an indirect measure of the impact that obstacles with different shape and configuration have on the microchannel fluid dynamics, suggesting that SE can be adopted to a posteriori 212 discriminate the shape of obstacles (e.g. cultured cells in biomicrofluidic applications) in 213 systems without optical access. This is like to say that SE can be used as a sort of *fingerprint* 214 215 that specific obstacles leave on fluid streamlines, depending upon their shape, configuration and fluid dynamics conditions (as defined by the Reynolds' number). 216



Fig. 4 SE computed on two cross-sections (proximal to the obstacle, upper panel; distal to the obstacle,
lower panel) of two microchannels presenting obstacles with rectangular and semicircular shape, at *Re*= 1 and 50. Obstacles are symmetrically located with respect to the axis of the microchannels. h_{ch} is the
general height of the channel.

223	In order to check for the robustness of SE with respect to the CFD grid cardinality (i.e. the
224	number of nodes considered for the SE calculation), a sensitivity analysis was carried out where
225	nodes were decimated. Technically, the average value of the SE was calculated considering the
226	entire microchannel fluid domain as follows: (1) considering the 100% of the mesh grid
227	elements; (2) considering only 50% of the mesh grid elements; (3) averaging the velocity
228	values of two adjacent mesh grid elements. The percentage differences among the average SE
229	values, summarized in Fig. 5 for microchannels with four different semicircular shape obstacle
230	configurations, clearly show that a substantial reduction (50%) in the number of mesh grid
231	elements weakly influences SE evaluation (with differences lower than 6%).



Fig. 5 Impact of the number of mesh grid elements on microchannel average SE values. The analysis was

carried out considering the 100% and the 50% of the mesh grid elements, as well as averaging the

velocity values of two adjacent mesh grid elements. The results refer to four microchannels withdifferent configurations of semicircular obstacles

237

238 4 Conclusions

239 In this study the fluid dynamics in microchannels with obstacles was investigated using the 240 concept of entropy, here approached in a different way compared to the present state-of-the-241 art (Camesasca et al. 2006, Pozo et al. 2017, Rocha et al. 2008). The analysis suggests that a quantity linked to entropy, SE, is capable to discriminate among different shapes and 242 243 configurations of obstacles within a microchannel. In the microfluidic field the capability to infer presence and configuration of obstacles in microsystems from SE differences between 244 inflow and outflow sections could allow to monitor e.g. cells shape and growth in 245 microbioreactors, as well as mixing of species in the microsystem itself. Applications other than 246 247 biomicrofluidics, where fluid streamlines entropy can be employed to describe the physics within the microsystem just looking at SE input and output variations are manifold, ranging 248 from microelectronics to chemistry (Ghaneifar et al. 2021, Shahsavar et al. 2020, Khalid et 249 250 al. 2021). In this sense, the here presented results represent a starting point for future

251	dedicated applications where the extraction of information on the processes occurring inside			
252	not accessible system is critical.			
253	Integrating statistical mechanics with information theory, in the long run will allow			
254	establish a clearly distinguishable link between thermodynamics perturbation of a system a			
255	the level of interaction of the individual elements of which a system is made.			
256				
257	Conflicts of interest			
258	The authors declare that they have no competing interests.			
259				
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