EURAMET Key Comparison EURAMET.M.D-K4.2020: Hydrometer calibration comparison from 600 kg/m3 to 2000 kg/m3

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EURAMET Key Comparison 1496
EURAMET.M.D-K4.2020

Hydrometer calibration comparison from 600 kg/m^3 to 2000 kg/m^3

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Abstract

This report presents the results of the key comparison EURAMET.M.D-K4.2020 (EURAMET 1496) that covered the calibration of high-resolution hydrometers for liquid in the density range 600 kg/m^3 to 2000 kg/m^3 at the reference temperature of 20 °C. Nine laboratories participated in this key comparison, eight European national metrology institutes and one from Asia Pacific metrology: NIM from China. The participating laboratories were initially divided into two petals linked by two density laboratories: INRIM and PTB. In each petal, four similar transfer standards, hydrometers in the density range 600 kg/m^3 to 2000 kg/m^3 are used.

The equipment used by the participants was found to have very similar characteristics. Most of the declared relative uncertainties were within the 5×10^{-5}.

The results of all participated laboratories have been linked to the CIPM key comparison CCM.D-K4 by INRIM and PTB. Discrepant results have been reported for three laboratories, which, however, do not affect their CMCs.

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1. Introduction

This comparison is the EURAMET Regional Key Comparison of the CIPM comparison CCM.D-K4. It is based on a decision at the TC-M Annual Meeting in April 2019 in Budapest (Hungary). The TC-M agreed that INRIM and PTB would organize this key comparison as pilot and co-pilot laboratories.

The CIPM key comparison CCM.D-K4, was coordinated by the Istituto Nazionale di Ricerca Metrologica (INRIM, Italy). The transfer standards involved in the CIPM comparison were hydrometers in the range between 600 kg/m^3 to 2 000 kg/m^3. The measurements were performed from January 2011 to April 2012. The Final Report was published in 2016 [1].

The main purpose of this project was to compare the experimental results and uncertainty calculations in calibrating hydrometers in the range from 600 kg/m^3 to 2 000 kg/m^3 and linking the intra-regional European results with the results obtained in the previous inter-regional CIPM key comparison CCM.D-K4. The participants of this comparison also involved in the comparison CCM.D-K4 were: INRIM, PTB and BFKH. The protocol was essentially equivalent to the protocol of CCM.D-K4 [2].

The comparison was organized in two petals, in the first took part laboratories, which needed ATA Carnet. Three laboratories INRIM, PTB and BFKH performed measurements in both petals, while INIRM performed two measurements for each hydrometer at the beginning and the end of the circulation.

Each participant had about three weeks to receive the transfer standards, make the measurements and send the transfer standards to the next participant. Due to the problems caused by the COVID-19, there was a delay in the schedule.

2. Participants and schedule

Nine laboratories took part in the comparison, eight from EURAMET and one from outside EURAMET, NIM from China (APMP), with the consent of all participating members.

The comparison was organized in two petals. In petal no. 1 took part laboratories, which needed ATA Carnet. Three laboratories INRIM, PTB and BFKH, performed measurements in both petals, while INIRM performed two measurements for each hydrometer at the beginning and the end of the circulation. The measurements were performed from March 2020 to February

2021 during the global pandemic of coronavirus disease 2019 (COVID-19). The unforeseen difficulties related to the pandemic, transportation, and customs changes delayed the original schedule for the comparison.

Only one hydrometer was broken at the end of circulation, so that each participating laboratory could measure the whole density range. Table 2.1 lists all participated laboratories and the circulation scheme.

Table 2.1. List of the participating NMIs and the circulation scheme.

Petal no. 1 (With ATA Carnet)		
2nd March	8th May	INRIM
8th May	2nd June	PTB
15th June	1st July	UME
10th July	24th July	DMDM
3rd August	5th August	INRIM (only for transfer)
25th August	29th September	NIM
21st October	13th November	BFKH
17th November	2nd February	INRIM

Petal no. 2		
2nd March	8th May	INRIM
8th May	28th May	PTB
2nd June	26th August	BEV
31st August	11th September	IPQ
14th September	12th October	BFKH
24th October	16th November	DZM
18th November	2nd December	INRIM

3. Transfer Standards

For the comparison, INRIM and PTB provide two similar sets composed of four transfer standards assigned to each petal. Details are shown in Table 3.1 and Table 3.2. Each set consisted of three hydrometers with a scale division of 0.000 1 g/cm³ in the density range of 0.600 g/cm³ to 1.500 g/cm³ and one hydrometer with a scale division of 0.000 2 g/cm³ working close to 2.000 g/cm³.

The cubic expansion coefficient for all hydrometers was assumed to be $25\cdot10^{-6}$ °C^{-1} with an uncertainty of $2\cdot10^{-6}$ °C^{-1}, rectangular distribution.

Table 3.1. Hydrometers for petal no. 1 (with ATA Carnet)

Range	Resolution	Serial Number
0.600 - 0.610 g/cm^3	0.000 1 g/cm^3	20731447
0.990 - 1.000 g/cm^3	0.000 1 g/cm^3	9343118
1.490 - 1.500 g/cm^3	0.000 1 g/cm^3	9343459
1.980 - 2.000 g/cm^3	0.000 2 g/cm^3	9346686

Table 3.2. Hydrometers for petal no. 2

Range	Resolution	Serial Number
0.600 - 0.610 g/cm^3	0.000 1 g/cm^3	20731448
0.990 - 1.000 g/cm^3	0.000 1 g/cm^3	9343119
1.490 - 1.500 g/cm^3	0.000 1 g/cm^3	9343463
1.980 - 2.000 g/cm^3	0.000 2 g/cm^3	6832

Unfortunately, hydrometer 9346686 was broken during transport between NIM and BFHK. However, the breakage did not affect the comparison as all laboratories measured at least one hydrometer in each range.

The INRIM did not detect any significant changes in calibration on any transfer standards after returning from circulation. Additionally, the stability of the transfer standard was checked and conformed for values less than 5×10^{-6}, see chapter 6.

4. Comparison Protocol

In a hydrometer calibration, the measurand was the correction *C* related to the reference liquid and reference temperature, evaluated at each calibration point

$$\mathrm{C}= \rho_\mathrm{x} - \rho_\mathrm{r}, \tag{1}$$

where ρ_x is the density of the buoyant reference liquid in which the hydrometer would freely float at the scale mark ρ_r.

In this comparison, the results had to be given at the reference temperature of 20 ºC, the cubic expansion coefficient for all hydrometers was assumed to be $25\cdot10^{-6}$ °C^{-1}, with an uncertainty of $1.2\cdot10^{-6}$ °C^{-1}.

Participating laboratories should use their routine calibration procedure based on Cuckow's method.

In Table 4.1, the calibration points and the reference surface tension of the liquid, in which each hydrometer was intended to be used, are shown.

Table 4.1. Calibration points and reference surface tensions

Range	Reading scale/ g/cm^3	Ref. surface tension/ mN/m
0.600 - 0.610 g/cm^3	0.601 0	15.1
	0.605 0	15.3
	0.609 0	15.5
0.990 - 1.000 g/cm^3	0.991 0	75.0
	0.995 0	75.0
	0.999 0	75.0
1.490 - 1500 g/cm^3	1.491 0	55.0
	1.495 0	55.0
	1.499 0	55.0
1.980 - 2.000 g/cm^3	1.981 0	75.0
	1.990 0	75.0
	1.999 0	75.0

5. Calibration method and equipment used

According to the protocol, the Cuckow's method was used by all participating NMIs. For each hydrometer, at least five weighings were performed in air and in the working fluid at each of the three scale marks. Scale readings were matched to the liquid level when the centre of the line was aligned with the horizontal liquid plane and detected with the specified visual alignment. A summary of the equipment used at the different NMIs is given in Table 5.1. The equipment was found to have very similar characteristics.

Table 5.1. Summary of the equipment's for all participating NMIs.

Laboratory	Balance Maximum load, resolution	Thermometer	Working liquid	Visualisation and alignment
INRIM	520 g, 0.01mg	0.001	n-Nonane	CCD camera, Automatic
PTB	410 g, 0.1 mg	0.001	n-Tridecane	CCD camera, Manual
BFHK	510 g, 0.1mg	0.0001	n-Nonane	Magnifier, Manual
NMI	520 g, 0.01mg	0.001	n-Nonane	CCD camera
BEV	5000 g, 0.1mg	0.001	n-Dodecane	Magnifier, Manual
DZM	510 g, 0.1 mg	0.001	Pentadecane	Magnifier, Manual
IPQ	400 g, 0.1 mg	0.001	n-Dodecane	CCD camera, Automatic
DMDM	520 g, 0.1 mg	0.001	n-Tridecane	CCD camera, Manual
UME	1200 g, 0.1 mg	0.001	n-Tridecane	Digital Microscope, Automatic

6. Results of check measurements

The stability of the transfer standard was checked by the pilot laboratory comparing the correction of hydrometer at each scale mark, before starting (C_s) and at the end (C_e) of the comparison. The results are shown in Table 6.1.

The maximum relative variation $\Delta_C/\rho = (C_s–C_e)/\rho$, observed between the first and the second measurement, was less than 5×10^{-6}. This difference can be considered negligible and is coherent with the typical uncertainty of reproducibility.

Consequently, the INRIM values for the comparison have been calculated as mean values of the two measurements and used in the following evaluation.

Table 6.1. Measurements performed by the INRIM, as pilot laboratory.

Hydrometer S/N	Value ρ / g/cm^3	C_s $\times 10^{-6}$	$U(C_s)$ $\times 10^{-6}$	C_e $\times 10^{-6}$	$U(C_e)$ $\times 10^{-6}$	$\Delta_C = C_s - C_e$ $\times 10^{-6}$	Δ_C/ρ $\times 10^{-6}$
20731447	0.601 0	53	16	53	16	0.1	0.1
	0.605 0	51	16	52	16	-0.7	-1.1
	0.609 0	52	16	50	16	1.2	2.0
9343118	0.991 0	-13	24	-12	24	-1.3	-1.3
	0.995 0	-5	24	0	24	-4.1	-4.1
	0.999 0	-4	24	-2	24	-2.3	-2.3
9343459	1.491 0	38	35	39	35	-0.1	-0.1
	1.495 0	48	35	55	35	-7.2	-4.8
	1.499 0	54	35	61	35	-6.6	-4.4
93436686	1.981 0	161	48				
	1.990 0	158	48				
	1.999 0	176	48				
20731448	0.601 0	31	16	33	16	-1.4	-2.4
	0.605 0	29	16	30	16	-0.5	-0.9
	0.609 0	26	16	28	16	-2.5	-4.1
9343119	0.991 0	-30	24	-29	24	-0.3	-0.3
	0.995 0	-32	24	-30	24	-2.8	-2.8
	0.999 0	-29	24	-27	24	-2.4	-2.4
9343463	1.491 0	73	35	74	35	-1.2	-0.8
	1.495 0	60	35	62	35	-2.4	-1.6
	1.499 0	65	35	73	35	-7.1	-4.8
6832	1.981 0	203	48	200	48	3.1	1.6
	1.990 0	276	48	277	48	-1.5	-0.8
	1.999 0	376	48	377	48	-1.2	-0.6

7. Results of participants

For each hydrometer, five weighing sequences were carried out in air and in the reference liquid at each of the three scale readings. Each participated laboratory calculated the average value of the correction C at 20 °C and the standard deviation u_c at the indicated scale reading. The measurement results presented by each participant are shown in Table 7.1.1 till 7.2.4 and Figure 7.1.1 till 7.2.4 (error bars given in term of expanded uncertainty U_{95}).

The uncertainty of measurement for the corrections was calculated with a given Excel sheet, as well as the effective degrees of freedom ν_{eff} of the combined standard uncertainty u_c, the t-factor $t_{95}(\nu_{eff})$ taken from the t-distribution for a 95% confidence level and the expanded uncertainty for the corrections as $U_{95} = t_{95}(\nu_{\text{eff}}) \cdot u_c$. The summary of the uncertainty budget for each participant is shown in Appendix Tables 14.6 to 14.9.

The BEV was unable to calibrate the hydrometers 9343463 and 6832 due to a temporary breakdown in its measuring system.

7.1. Petal no. 1

Table 7.1.1 Measurements results as reported by participants for petal no. 1 and hydrometer 20731447 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 20731447 0.600 – 0.610 g/cm³		INRIM	PTB	UME	DMDM	NIM	BFKH
0.601 0	x 10⁻⁶ / g/cm³	53	54	42	16	-84	84
0.605 0		52	45	41	12	-62	64
0.609 0		51	42	38	4	-33	47
u_c		8	8	13	30	20	7
U_{95}		16	16	26	58	40	15

Figure 7.1.1 Correction C reported by participants for petal no. 1 and hydrometer 20731447.

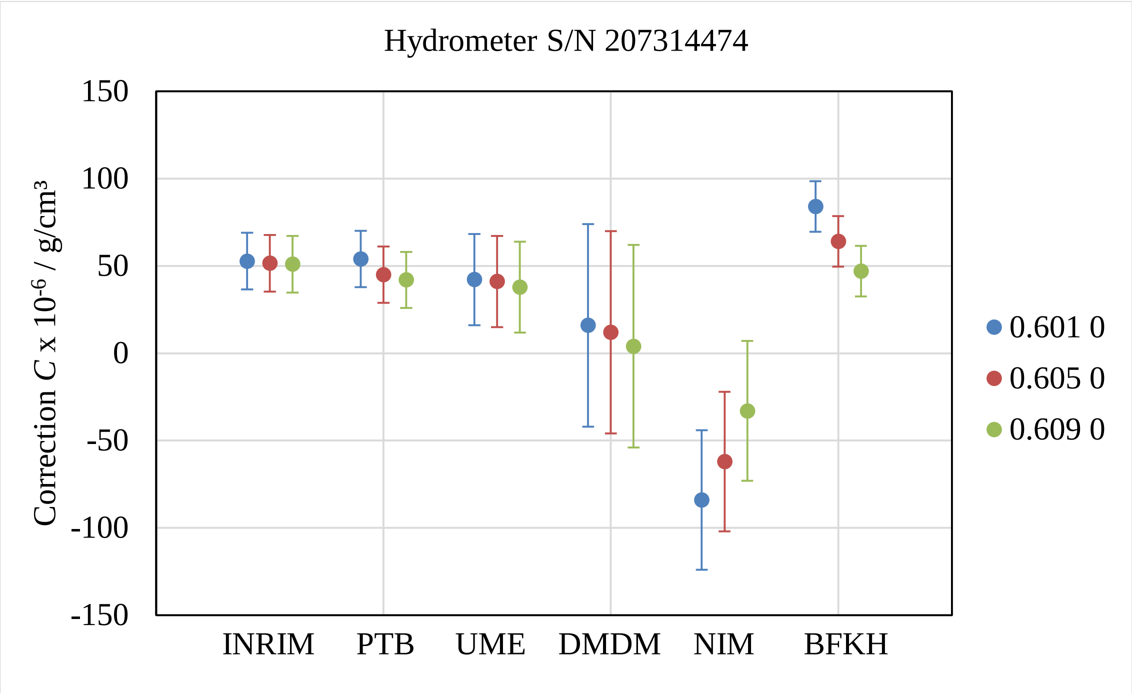


Table 7.1.2 Measurements results reported by participants for petal no. 1 and hydrometer 9343118 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 9343118 0.990 – 1.000 g/cm^3		INRIM	PTB	UME	DMDM	NIM	BFKH
0.991 0	x 10^{-6} / g/cm^3	-12	1	-36	-6	-3	32
0.995 0		-2	12	-13	-14	2	7
0.999 0		-3	3	-28	-9	11	-7
u_c		12	9	16	30	15	13
U_{95}		24	17	32	58	30	25

Figure 7.1.2 Correction C reported by participants for petal no. 1 and hydrometer 9343118.

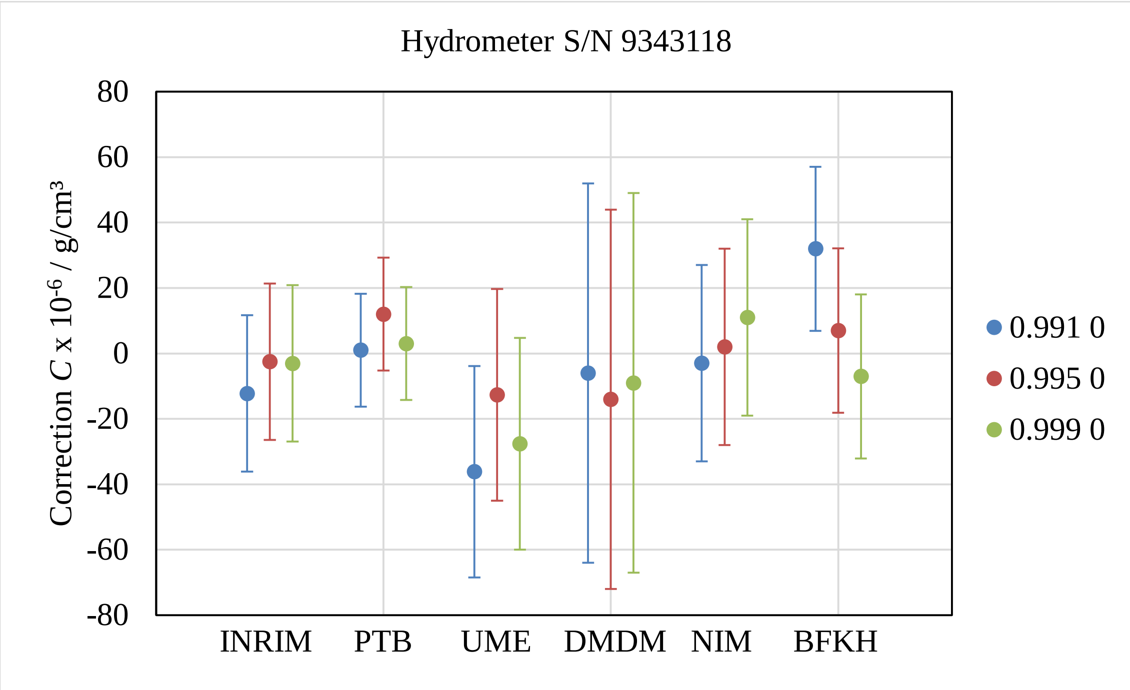


Table 7.1.3 Measurements results reported by participants for petal no. 1 and hydrometer 9343459 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 9343459 1.490 – 1.500 g/cm^3		INRIM	PTB	UME	DMDM	NIM	BFKH
1.491 0	x 10^{-6} / g/cm^3	38	52	17	27	42	69
1.495 0		52	66	62	18	57	47
1.499 0		58	54	43	23	65	30
u_c		18	12	19	30	20	15
U_{95}		35	23	38	58	40	30

Figure 7.1.3 Correction *C* reported by participants for petal no. 1 and hydrometer 9343459.

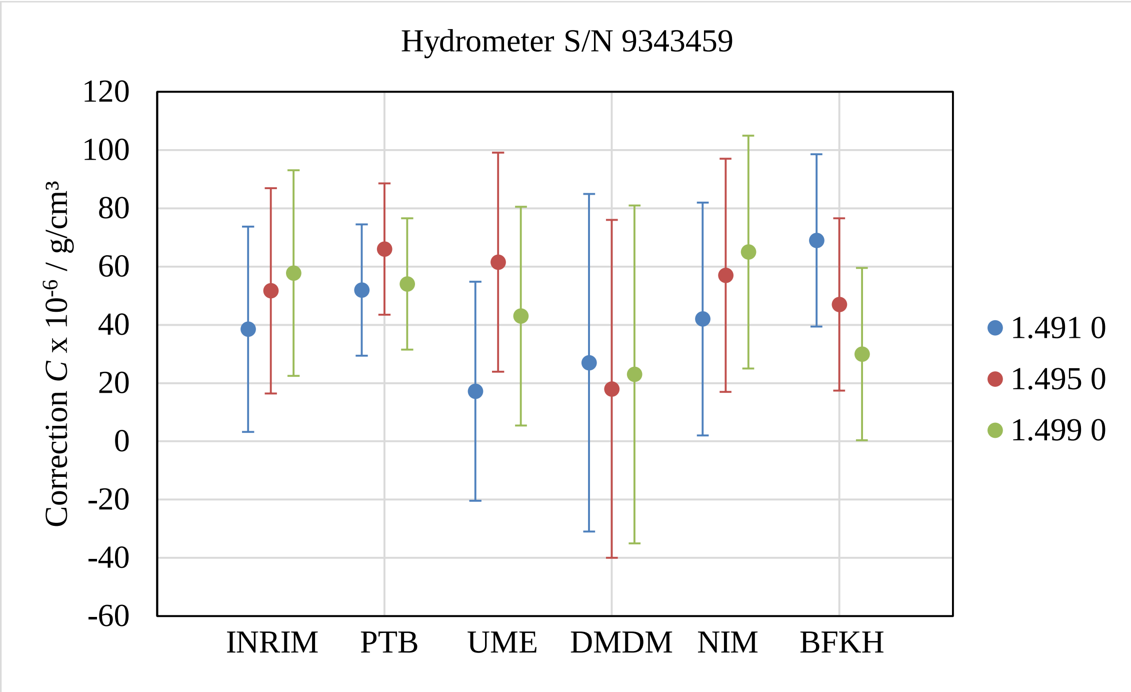
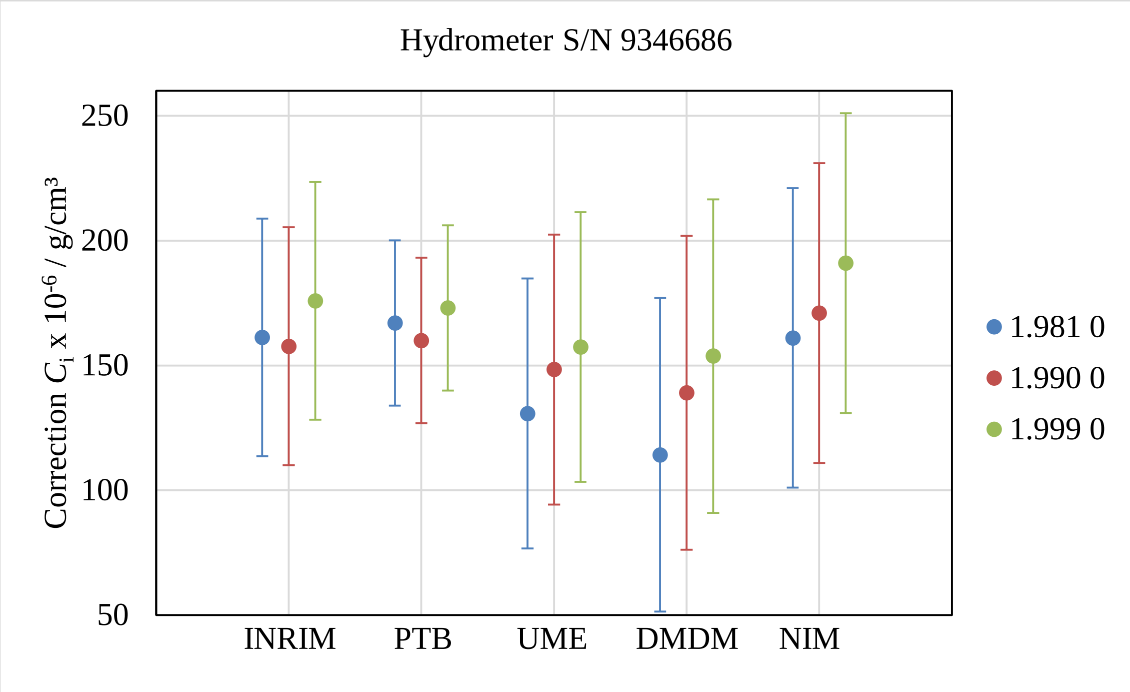


Table 7.1.4 Measurements results reported by participants for petal no. 1 and hydrometer 9346686 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 9346686 1.980 – 2.000 g/cm³		INRIM	PTB	UME	DMDM	NIM
1.981 0	x 10^{-6} / g/cm³	161	167	131	114	161
1.990 0		158	160	148	139	171
1.999 0		176	173	157	154	191
u_c		24	17	27	32	30
U_{95}		48	33	54	63	60

Figure 7.1.4 Correction C as reported by participants for petal no. 1 and hydrometer 9346686.



7.2. Petal no. 2

Table 7.2.1 Measurements results reported by participants for petal no. 2 and hydrometer 20731448 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 20731448 0.600 – 0.610 g/cm^3		INRIM	PTB	BEV	IPQ	BFKH	DZM
0.601 0	x 10^{-6} / g/cm^3	32	26	17	10	55	-39
0.605 0		29	22	35	13	36	-35
0.609 0		27	17	40	8	26	-32
u_c		8	8	20	30	7	31
U_{95}		16	16	40	59	15	61

Figure 7.2.1 Correction C reported by participants for petal no. 2 and hydrometer 20731448.

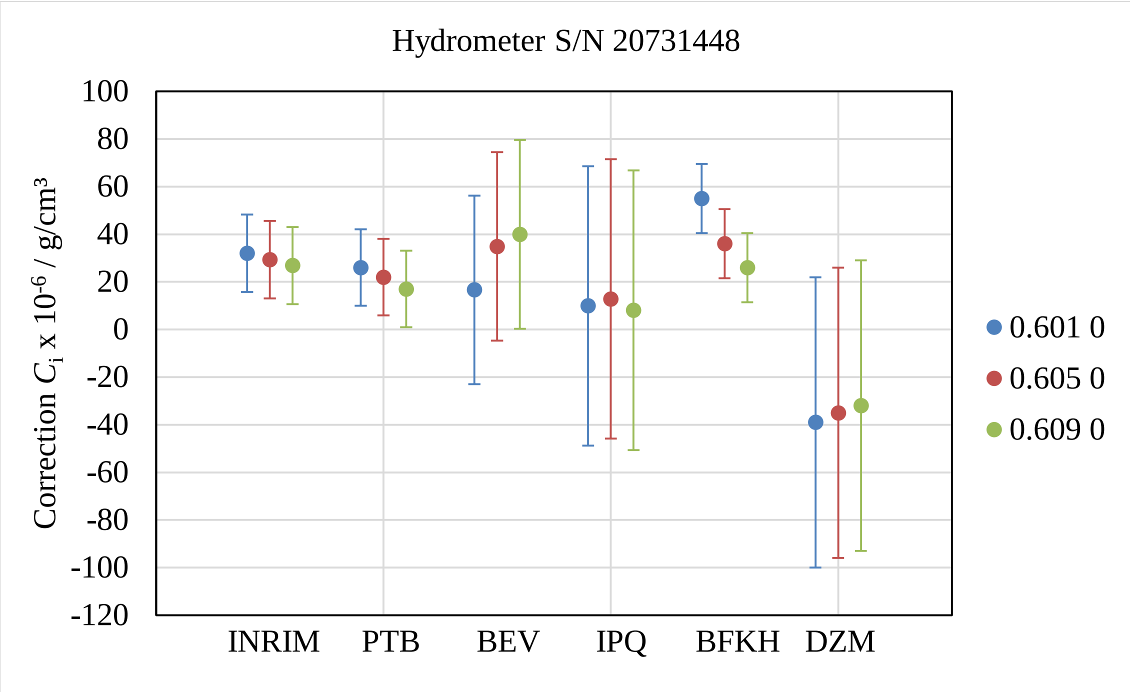


Table 7.2.2 Measurements results reported by participants for petal no. 2 and hydrometer 9343119 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 9343119 0.990 – 1.000 g/cm^3		INRIM	PTB	BEV	IPQ	BFKH	DZM
0.991 0	x 10^{-6} / g/cm^3	-29	-20	-36	-74	16	-104
0.995 0		-31	-24	-17	-59	-14	-104
0.999 0		-28	-21	-17	-41	-24	-100
u_c		12	8	42	68	13	32
U_{95}		24	17	84	139	25	65

Figure 7.2.2 Correction C reported by participants for petal no. 2 and hydrometer 9343119.

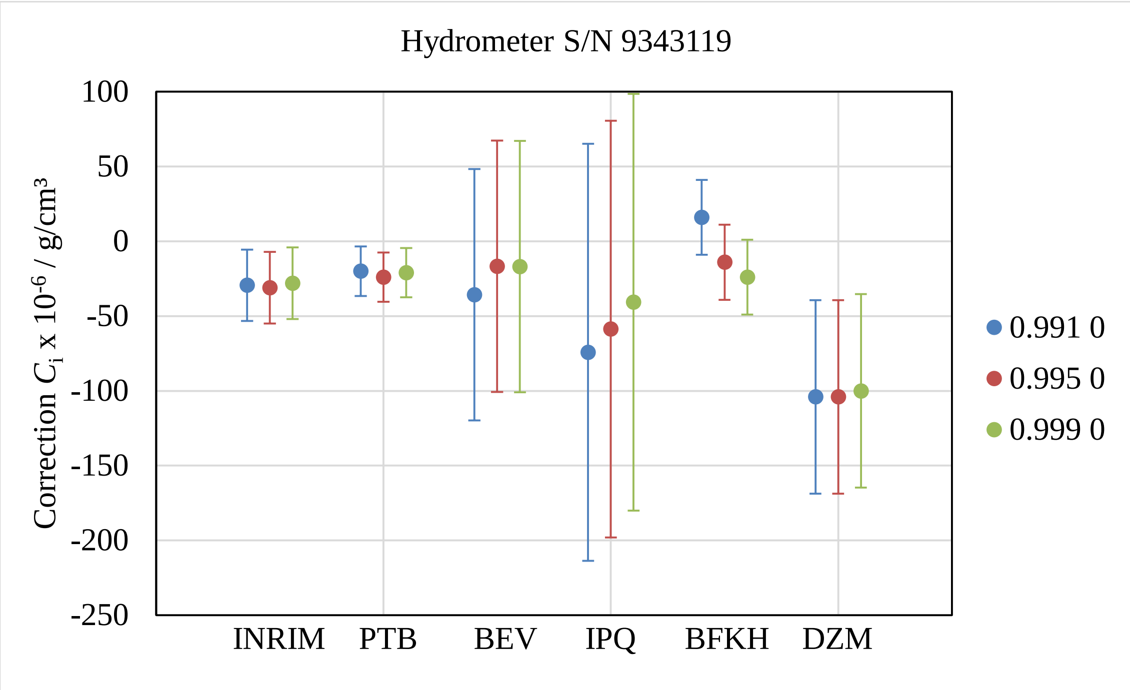


Table 7.2.3 Measurements results reported by participants for petal no. 2 and hydrometer 9343463 standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 9343463 1.490 – 1.500 g/cm^3		INRIM	PTB	IPQ	BFKH	DZM
1.491 0	x 10^{-6} / g/cm^3	73	90	13	101	-30
1.495 0		61	68	17	57	-47
1.499 0		69	66	45	58	-46
u_c		18	11	62	15	36
U_{95}		35	22	122	30	72

Figure 7.2.3 Correction *C* reported by participants for petal no. 2 and hydrometer 9343463.

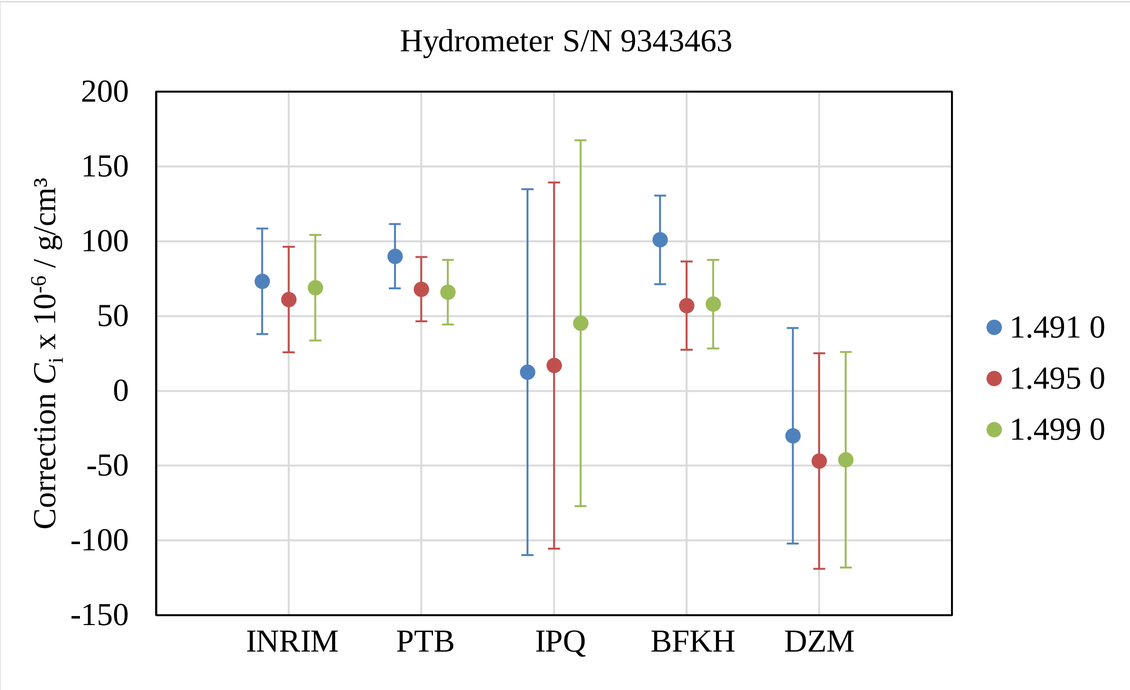
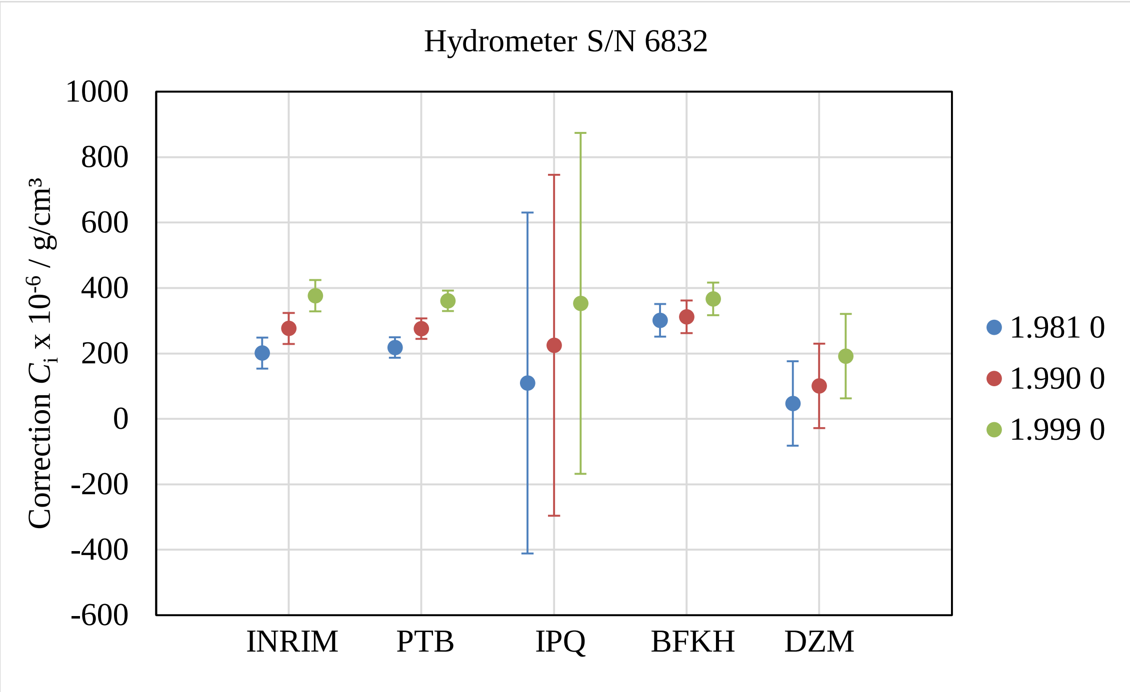


Table 7.2.4 Measurements results reported by participants for petal no. 2 and hydrometer 6832 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 6832 1.980 – 2.000 g/cm^3		INRIM	PTB	IPQ	BFKH	DZM
1.981 0	x 10^{-6} / g/cm^3	201	218	110	301	47
1.990 0		277	276	225	312	101
1.999 0		376	361	353	367	192
u_c		24	16	249	26	65
U_{95}		48	31	521	50	129

Figure 7.2.4 Correction C reported by participants for petal no. 2 and hydrometer 6832.



8. Link to CCM.D-K4 and Degree of Equivalence

According to the EURAMET Guide No. 4 [3], in this EURAMET key comparison, no reference value was determined, the degrees of equivalence were calculated by linking to the *KCRV* of the CIPM key comparison CCM.D-K4 [1] in which similar hydrometers were used as travelling standards.

The link has been provided by the differences between the mean values measured by the linking laboratories that took part in the comparison CCM.D-K4 and in in both petals of this comparison. These are INRIM, PTB and BFHK.

In order to assess the consistency of the measurements carried out by the three linking laboratories, the degree of equivalence with respect to the weighted mean value calculated from their measurements was evaluated.

The results show that some measurements performed by the BFKH were not consistent with the value of the weighted mean. For this reason, the link to the comparison was calculated from the measurements of INRIM and PTB. The BFKH discrepant values are revealed in chapter 8.

Hence, the link between measured artefacts in the CIPM and EURAMET comparisons was made by mean of difference of the two linking laboratories. Each linking laboratory i yield a value for the difference $\Delta\rho_\mathrm{i}$ between the measurand of the CIPM hydrometer $x_\mathrm{i}(CCM)$ and the measurand of the EURAMET hydrometer $x_\mathrm{i}(EUR)$:

$$\Delta\rho_\mathrm{i} = x_\mathrm{i}(CCM) - x_\mathrm{i}(EUR). \qquad (2)$$

Which can be evaluated as

$$\Delta\rho_\mathrm{i} = x(KCRV) - x(WME) + \Delta_\mathrm{i} \qquad (3)$$

Where

- $\Delta_\mathrm{i} = -\Delta_{CIPM_\mathrm{i}} + \Delta_{WME_\mathrm{i}}$.
- $x(KCRV)$ is the reference value (*KCRV*) for the CIPM comparison and $\Delta_{CIPM_\mathrm{i}}$is the difference between the measurement of the laboratory with respect to the *KCRV*, that is the difference of the degree of equivalence (*DoE*).
- $x(WME)$ is the weighted mean obtained from the linking laboratories in the EURAMET comparison and Δ_{WME_i} is the difference of the *DoE*.

As the link correction $\Delta\rho$ is evaluated by the mean value of the two linking laboratories, INRIM and PTB, the value is

$$\Delta\rho = x(KCRV) - x(WME) + \Delta. \tag{4}$$

Δ is difference of the mean values, $\Delta = -\Delta_{CIPM} + \Delta_{WME}$, where

$$\Delta_{CIPM} = \frac{\Delta_{CIPM_{\mathrm{INRIM}}} + \Delta_{CIPM_{\mathrm{PTB}}}}{2}, \text{ and } \Delta_{WME} = \frac{\Delta_{WME_{\mathrm{INRIM}}} + \Delta_{WME_{\mathrm{PTB}}}}{2}. \tag{5}$$

The measurements of each participant $\rho_{\mathrm{i\ EUR}}$ are therefore correct as:

$$\rho_{\mathrm{i\ CIPM}} = \rho_{\mathrm{i\ EUR}} + \Delta\rho. \tag{6}$$

The uncertainty of the difference $\Delta\rho$ is evaluated considering that $x_{\mathrm{i}}(CIPM)$ and $x_{\mathrm{i}}(EUR)$ are correlated.
As the measuring range of each hydrometer was very small (0.010 g/cm^3), a nominal density range ρ_{j} (0.6 – 1.0 – 1.5 – 2.0 g/cm^3) has been defined in both comparisons. In each comparison, both INRIM and PTB calibrated two hydrometers which belong to the two petals of the comparison with the same measurement range at three scale marks. Thus, in both comparisons the mean value $\Delta_{CIPM_{\mathrm{i}}}$ and $\Delta_{WME_{\mathrm{i}}}$ of the six measurements could be determined for each range. The uncertainties $u(\Delta_{CIPM_{\mathrm{i}}})$ and $u(\Delta_{WME_{\mathrm{i}}})$ have been evaluated by considering the uncertainty of reproducibility of the measurements of the two laboratories, which can be evaluated by the standard deviation of the six differences of the Degrees of Equivalence (*DoE*s) of $\Delta_{CIPM_{\mathrm{i}}}$ and $\Delta_{WME_{\mathrm{i}}}$ for each density range.

The degree of equivalence D_{i} is calculated as

$$D_{\mathrm{i}} = \rho_{\mathrm{i\ CIPM}} - KCRV = \rho_{\mathrm{i\ EUR}} + \Delta\rho - KCRV \tag{7}$$

and

$$U(D_{\mathrm{i}}) = 2\, u(D_{\mathrm{i}}), \tag{8}$$

where

$$u(D_{\mathrm{i}}) = \sqrt{u^2(\rho_{\mathrm{i\ EUR}}) + u^2(\Delta\rho) + u^2(KCRV)}. \tag{9}$$

For the linking laboratories, due to the correlation [4]

$$u(D_{\mathrm{i}}) = \sqrt{u^2(\rho_{\mathrm{i\ EUR}}) + u^2(\Delta\rho) - u^2(KCRV)}. \tag{10}$$

Table 8.1 shows the values of the weighted means *WME* and the associated uncertainties *u*(*WME*) for the EURAMET comparison estimated by the INRIM and PTB measurements.

Table 8.1. Weighted means *WME* and the associated uncertainties *u*(*WME*) for the EURAMET comparison estimated by the INRIM and PTB measurements.

Range 0.600 - 0.610 g/cm³		Petal no. 1, 20731447		Petal no. 2, 20731448	
		WME	$u(WME)$	WME	$u(WME)$
0.6010	x 10⁻⁶ / g/cm³	53	6	29	6
0.6050		48	6	26	6
0.6090		46	6	22	6
Range 0.990 - 1.000 g/cm³		Petal no. 1, 9343118		Petal no. 2, 9343119	
		WME	$u(WME)$	WME	$u(WME)$
0.9910	x 10⁻⁶ / g/cm³	-4	7	-23	7
0.9950		7	7	-26	7
0.9990		1	7	-23	7
Range 1.490 - 1500 g/cm³		Petal no. 1, 9343459		Petal no. 2, 9343463	
		WME	$u(WME)$	WME	$u(WME)$
1.4910	x 10⁻⁶ / g/cm³	48	10	85	9
1.4950		62	10	66	9
1.4990		55	10	67	9
Range 1.980 - 2.000 g/cm³		Petal no. 1, 9346686		Petal no. 2, 6832	
		WME	$u(WME)$	WME	$u(WME)$
1.9810	x 10⁻⁶ / g/cm³	165	14	213	13
1.9900		159	14	276	13
1.9990		174	14	366	13

Table 8.2 and 8.3 show the mean degrees of equivalence of the measurements for the two linking laboratories, and the correspondent mean differences for the CIPM and EURAMET comparisons, respectively.

Table 8.2. Degrees of equivalence for the linking laboratories in CIPM comparison.

Range g/cm³	$\Delta_{CIPM_{\mathrm{INRIM}}}$ × 10^{-6}	$u(\Delta_{CIPM_{\mathrm{INRIM}}})$ × 10^{-6}	$\Delta_{CIPM_{\mathrm{PTB}}}$ × 10^{-6}	$u(\Delta_{CIPM_{\mathrm{PTB}}})$ × 10^{-6}	Δ_{CIPM} × 10^{-6}	$u(\Delta_{CIPM})$ × 10^{-6}
0.600	3.5	4.0	-0.2	2.5	-1.8	2.3
1.000	-3.7	8.5	5.7	3.7	1.0	4.6
1.500	-2.2	9.1	6.2	7.6	2.0	5.9
2.000	-9.5	12.6	15.2	13.8	2.8	9.3

Table 8.3. Degrees of equivalence for the linking laboratories in EURAMET comparison.

Range g/cm³	$\Delta_{WME_{\mathrm{INRIM}}}$ × 10^{-6}	$u(\Delta_{WME_{\mathrm{INRIM}}})$ × 10^{-6}	$\Delta_{WME_{\mathrm{PTB}}}$ × 10^{-6}	$u(\Delta_{WME_{\mathrm{PTB}}})$ × 10^{-6}	Δ_{WME} × 10^{-6}	$u(\Delta_{WME})$ × 10^{-6}
0.600	3.1	2.0	-3.1	2.0	0.0	1.4
1.000	-6.4	2.3	3.2	1.3	-1.6	1.3
1.500	-5.4	6.5	2.1	2.5	-1.6	3.5
2.000	-0.7	7.4	0.3	3.2	-0.2	4.0

Table 8.4 shows, for each range, the difference of the mean values Δ of the EURAMET and CIPM comparison, the associated uncertainty $u(\Delta)$, and the relative associated uncertainty $u(\Delta)/\rho$.

Table 8.4. Difference of the mean values Δ, uncertainty $u(\Delta)$ and the relative uncertainty $u(\Delta)/\rho$.

Range g/cm³	x 10^{-6} / g/cm³	Δ	$u(\Delta)$	$u(\Delta)/\rho$
0.600		1.9	2.7	4.5
1.000		-2.6	4.8	4.9
1.500		-3.6	6.9	4.6
2.000		-3.0	10.2	5.1

The link was calculated with respect to petal A of the CIPM comparison CCM.D-K4, the reference value, the associated uncertainties, and the correction $\Delta\rho$ and uncertainty $u(\Delta\rho)$ for each calibration point and for the two petals are shown in Table 8.5. As the relative uncertainty $u(\Delta)/\rho$ is similar enough for the whole range, it was decided to use the maximum value of $u(\Delta)/\rho = 5.1\times10^{-6}$ for all points.

The degree of equivalence D_i, the associated uncertainty $U(D_i)$ and normalized error $E_n = D_i / U(D_i)$, obtained by each participant, are shown in Table 8.6 and Figure 8.1 till 8.4, where the error bars are shown in terms of expanded uncertainty.

Table 8.5. Linking values of the CIPM comparison CCM.D-K4 petal A by the reference value *KCRV*, the associated uncertainties $u(KCRV)$ to this EURAMET comparison with the correction $\Delta\rho$ and uncertainty $u(\Delta\rho)$ for each calibration point and petals.

Value g/cm³	*KCRV* × 10^{-6} g/cm³	$u(KCRV)$ (k = 1) × 10^{-6} g/cm³	Petal no. 1 $\Delta\rho$ × 10^{-6} g/cm³	Petal no. 2 $\Delta\rho$ × 10^{-6} g/cm³	$u(\Delta\rho)$ (k = 1) × 10^{-6} g/cm³
0.601	-95	3.5	-147	-122	3.1
0.605	-98	3.5	-144	-122	3.1
0.609	-90	3.5	-135	-110	3.1
0.98592	-29	6.5	-28	-9	5.1
0.99106	-41	6.5	-51	-17	5.1
0.9967	-22	6.5	-26	-1	5.1
1.491	75	7.5	23	-14	7.6
1.495	85	7.5	20	15	7.6
1.499	73	7.5	14	3	7.6
1.981	108	10.5	-60	-108	10.1
1.990	91	10.5	-71	-188	10.1
1.999	148	10.5	-29	-220	10.2

Table 8.6. Degree of equivalence D_i for all NMIs participated in this EURAMET comparison linked to the CIPM comparison CCM.D-K4 petal A, the associated uncertainties $U(D_{i,j})$ and the normalized error E_n.

Range	INRIM			PTB			BFKH			UME			DMDM			NIM			BEV			IPQ			DZM		
	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n
g/cm^3	$\times 10^{-6}$ g/cm^3			$\times 10^{-6}$ g/cm^3			$\times 10^{-6}$ g/cm^3			$\times 10^{-6}$ g/cm^3			$\times 10^{-6}$ g/cm^3			$\times 10^{-6}$ g/cm^3			$\times 10^{-6}$ g/cm^3			$\times 10^{-6}$ g/cm^3			$\times 10^{-6}$ g/cm^3		
0.601	3	16	0.2	1	16	0.0	30	17	1.7	-9	28	-0.3	-36	60	-0.6	-136	41	-3.3	-10	41	-0.3	-17	61	-0.3	-66	62	-1.1
0.605	5	16	0.3	-2	16	-0.1	15	17	0.9	-5	28	-0.2	-34	60	-0.6	-108	41	-2.6	11	41	0.3	-11	61	-0.2	-59	62	-1.0
0.609	7	16	0.4	-3	16	-0.2	4	18	0.2	-7	28	-0.2	-41	60	-0.7	-78	41	-1.9	20	41	0.5	-12	61	-0.2	-52	62	-0.8
0.985	-10	23	-0.4	1	15	0.1	35	30	1.1	-35	36	-1.0	-5	61	-0.1	-2	34	-0.1	-15	86	-0.2	-54	136	-0.4	-84	67	-1.3
0.991	-10	23	-0.4	1	15	0.1	4	30	0.1	-22	36	-0.6	-24	61	-0.4	-8	34	-0.2	7	86	0.1	-35	136	-0.3	-80	67	-1.2
0.996	-7	23	-0.3	0	15	0.0	-7	30	-0.2	-31	36	-0.9	-13	61	-0.2	7	34	0.2	4	86	0.0	-20	136	-0.1	-79	67	-1.2
1.491	-15	36	-0.4	1	23	0.0	15	37	0.4	-35	43	-0.8	-25	63	-0.4	-10	45	-0.2				-77	125	-0.6	-119	75	-1.6
1.495	-11	36	-0.3	-1	23	0.0	-16	37	-0.4	-4	43	-0.1	-47	63	-0.8	-8	45	-0.2				-53	125	-0.4	-117	75	-1.6
1.499	-1	36	0.0	-5	23	-0.2	-21	37	-0.6	-16	43	-0.4	-36	63	-0.6	6	45	0.1				-25	125	-0.2	-116	75	-1.5
1.981	-11	48	-0.2	0	32	0.0	85	59	1.4	-37	61	-0.6	-54	70	-0.8	-7	56	-0.1				-106	499	-0.2	-169	132	-1.3
1.990	-4	48	-0.1	-3	32	-0.1	33	59	0.6	-14	61	-0.2	-23	70	-0.3	9	56	0.2				-54	499	-0.1	-178	132	-1.3
1.999	3	48	0.1	-6	32	-0.2	-2	59	0.0	-20	61	-0.3	-23	70	-0.3	14	56	0.3				-15	499	0.0	-177	132	-1.3

Figure 8.1. D_i of each particacapated NMI in the density range of 0.600 to 0.610 g/cm³.

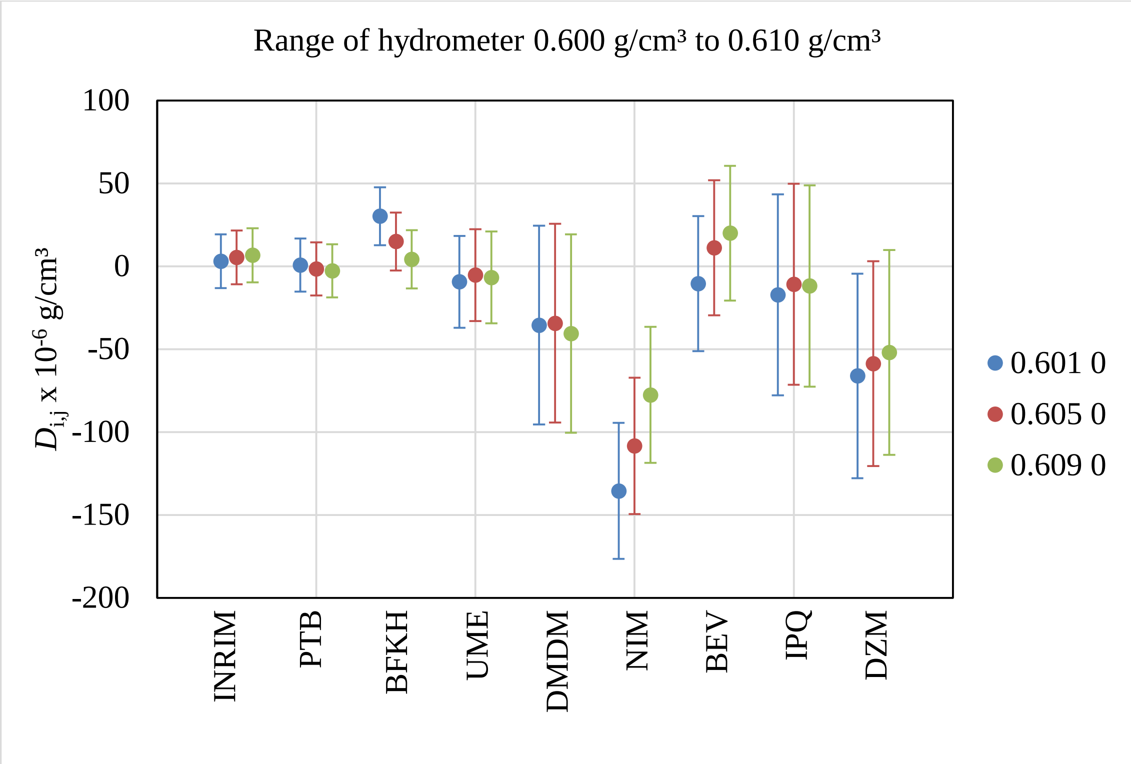


Figure 8.2. D_i of each particacapated NMI in the density range of 0.985 to 0.996 g/cm³.

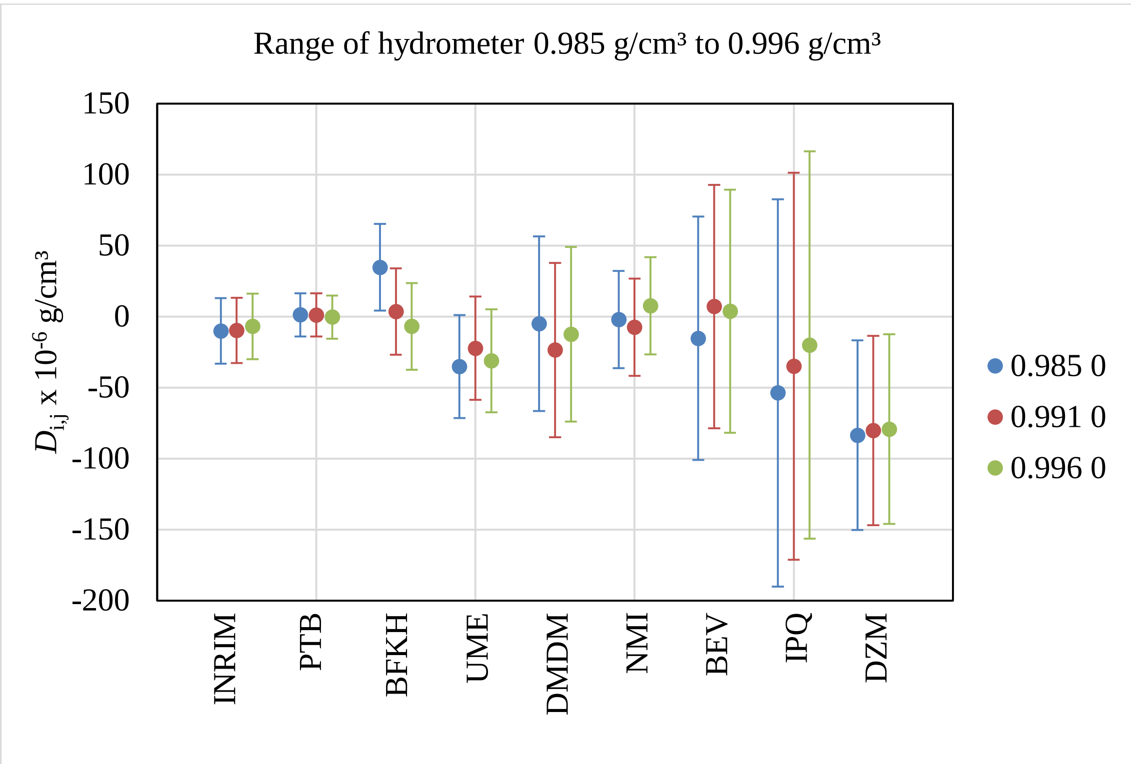


Figure 8.3. D_i of each particapated NMI in the density range of 1.490 to 1.500 g/cm³.

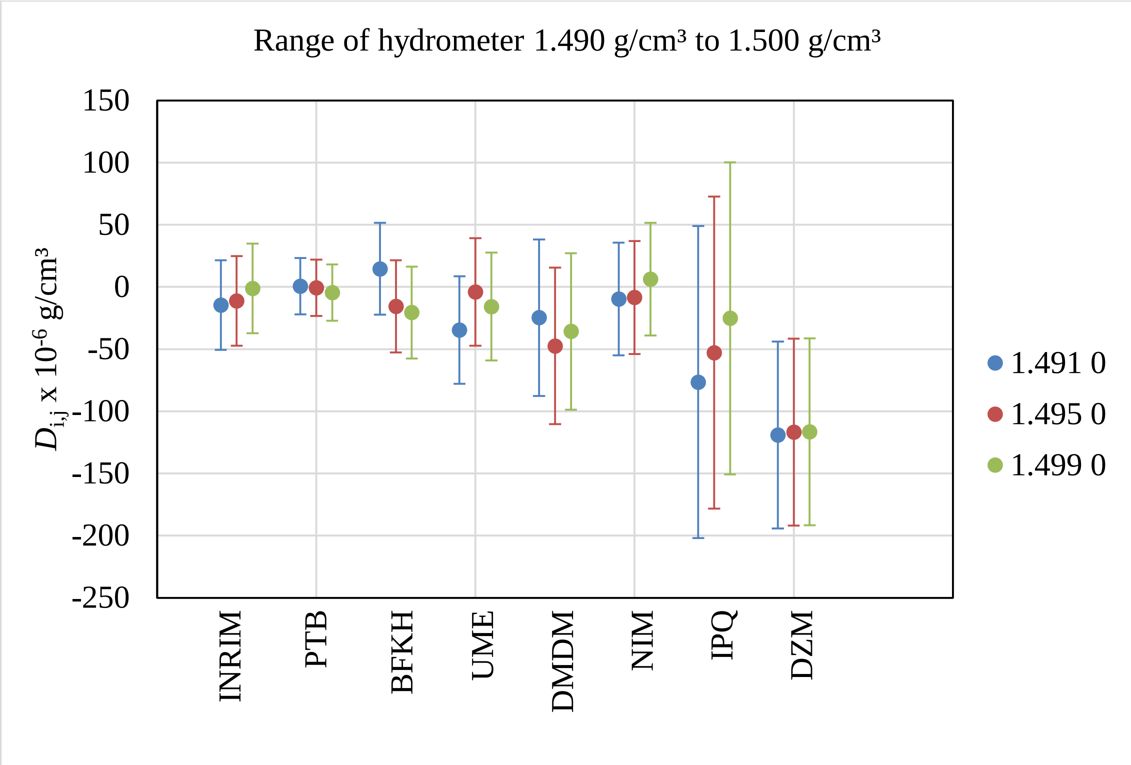
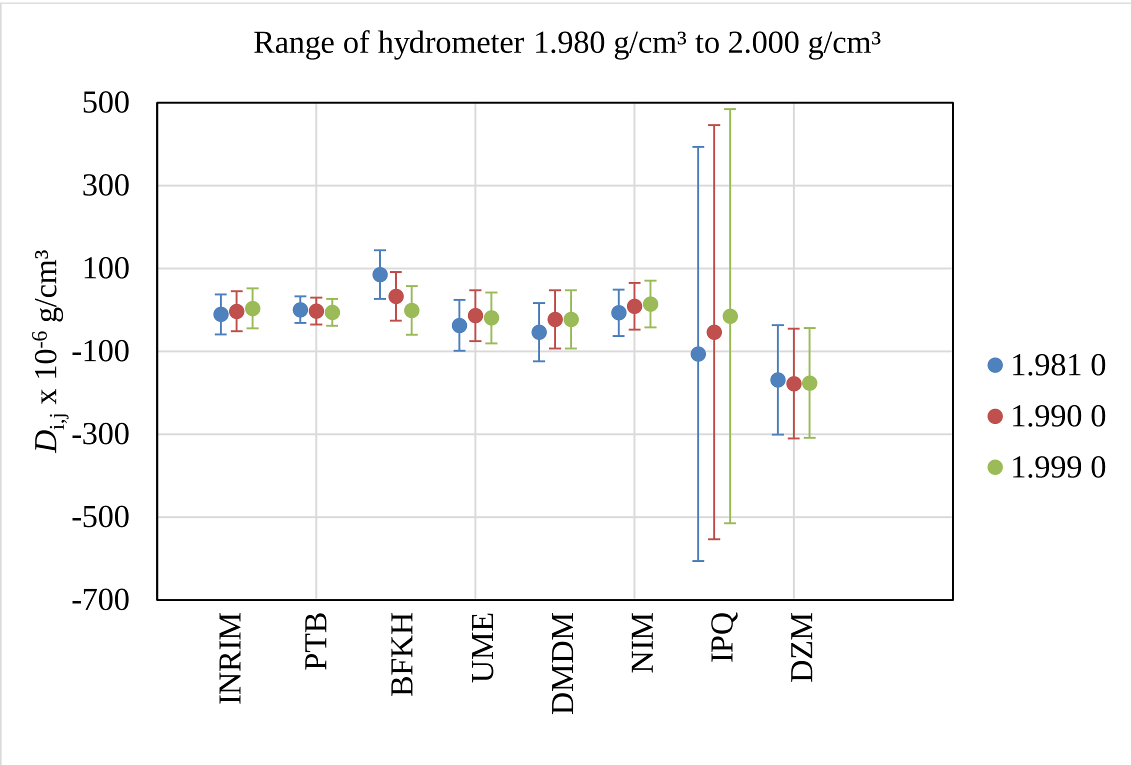


Figure 8.4. D_i of each particapated NMI in the density range of 1.980 to 2.000 g/cm³.



9. Degrees of Equivalence between laboratories

The degrees of equivalence between laboratories has been evaluated, considering the values $\rho_{\mathrm{i\ CIPM}}$ (of equation 8). For two laboratories *i* and *j*

$$D_{\mathrm{i,j}} = \rho_{\mathrm{i\ CIPM}} - \rho_{\mathrm{j\ CIPM}}. \tag{11}$$

The uncertainty depends on whether the laboratories belong to the same petal or to the different petal. For the same petal

$$U(D_{\mathrm{i,j}}) = 2\sqrt{u^2(x_{\mathrm{i}}) + u^2(x_{\mathrm{j}})}. \tag{12}$$

For the different petal

$$U(D_{\mathrm{i,j}}) = 2\sqrt{u^2(x_{\mathrm{i}}) + u^2(x_{\mathrm{j}}) + u^2(\Delta\rho)}. \tag{14}$$

In Appendix, the results are shown in Tables 14.2 to 14.5.

10. Conclusions

The comparison EURAMET Key Comparison 1496 (EURAMET.M.D-K4.2020) is the EURAMET Regional Key Comparison of the CIPM comparison CCM.D-K4, on high-resolution hydrometer calibrations. The density range was between 0.600 g/cm^3 to 2.000 g/cm^3 using three hydrometers with the resolution of 0.000 1 g/cm^3 and one hydrometer with 0.000 2 g/cm^3, similar to those used in the CCM.D-K4.

The measurements of this comparison were conducted during March 2020 and February 2021. The degrees of equivalence of all participated laboratories have been evaluated by linking to the *KCRV* values of the CCM.D-K4. In order to assess the support of CMCs entries provided by this comparison, the obtained degrees of equivalence and the declared uncertainties have been compared against the uncertainty values of the CMCs-tables *U*(CMC). It is expected that the differences of D_{i} are smaller than *U*(CMC) for supporting purposes, the results are in Appendix Table 14.1. For NMIs without present CMC entries on hydrometers, the label n/a is shown. It is to be noted that IPQ at 1.0 g/cm^3 declared an expanded uncertainty bigger than the CMC.

Three participants: BFHK, DZM and NMI, produced anomalous results, however:

- BFHK at the range of 0.6 g/cm^3, 1.0 g/cm^3 and 2.0 g/cm^3 is only one point out of three calibrated scale lines deviating.
- DZM at present does not have CMCs on hydrometers.
- NMI at the range of 0.6 g/cm^3 deviating considering the declared uncertainty, but consistent with CMCs.

Although the instrumentation used by the laboratories is very similar, the reported relative uncertainties range from 1.5×10^{-5} to 2.6×10^{-4}, but for most results (65%), the uncertainties were less than 5×10^{-5}.

11. Comments

BFKH

BFKH has provided the following explanation regarding the inconsistent results. The reason was due to the type of rubber used to hang the hydrometers.

Unfortunately, the type of rubber used has absorbed the vapours of the n-Nonane reference liquid during the measurements, increasing the mass, depending on the immersion depth of the hydrometer.

This increase in mass is negligible for the first calibration mark, 1.2 mg for the second, and 4.1 mg for the third.

This effect caused a systematic error for all the measurements, particularly for the third mark in calibration, which caused inconsistent results.

With correction of the measurements for this effect, all measurements would have been consistent.

IPQ

IPQ looking at the uncertainty budget realized that there was a mistake in the air density uncertainty calculation that overestimated the final uncertainty presented in this report.

12. Acknowledgements

The authors would like to acknowledge the kind assistance of all the colleagues in the participating laboratories for helping this comparison to run so smoothly. We thank A. Leka and D. Torchio of INRIM, for their collaboration in the organisation of the comparison, and S. Gerdesmann of PTB for her great competence to measure both petals of travelling standards used in this comparison.

13. References

[1] S. Lorefice et al "CCM key comparison CCM.D-K4 'Hydrometer'", Metrologia, 2016, **53**, 07003.

[2] Malengo. A. and Eppers. D., Technical Protocol for EURAMET.M.D-K4 "Hydrometers calibration comparison from 600 kg/m³", 2020, https://www.bipm.org/kcdb/comparison?id=1656.

[3] EURAMET Guide No. 4 Version 1.0 (05/2016).

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14. Appendix

Table 14.1. Comparison of the results with the CMCs.

NMI	Range g/cm^3	U(CMC) $\times 10^{-6}$ g/cm^3	U $\times 10^{-6}$ g/cm^3	$U \leq$ CMC	Passed with U	Passed with CMC
INRIM	0.6	16	16	yes	yes	yes
	1.0	24	24	yes	yes	yes
	1.5	36	35	yes	yes	yes
	2.0	48	48	yes	yes	yes
PTB	0.6	16	16	yes	yes	yes
	1.0	22	17	yes	yes	yes
	1.5	28	23	yes	yes	yes
	2.0	35	33	yes	yes	yes
IPQ	0.6	n/a	59	-	yes	yes
	1.0	100	139	no	yes	yes
	1.5	n/a	122	-	yes	yes
	2.0	n/a	521	-	yes	yes
BFKH	0.6	15	15	yes	no for one point	no for one point
	1.0	25	25	yes	no for one point	no for one point
	1.5	50	30	yes	yes	yes
	2.0	50	50	yes	no for one point	no for one point
NIM	0.6	200	40	yes	no	yes
	1.0	80	30	yes	yes	yes
	1.5	80	40	yes	yes	yes
	2.0	80	60	yes	yes	yes
UME	0.6	n/a	26		yes	
	1.0	n/a	32		yes	
	1.5	n/a	38		yes	
	2.0	n/a	54		yes	
DMDM	0.6	n/a	58		yes	
	1.0	n/a	58		yes	
	1.5	n/a	58		yes	
	2.0	n/a	63		yes	
BEV	0.6	n/a	40		yes	
	1.0	n/a	84		yes	
	1.5	n/a	-		-	
	2.0	n/a	-		-	
DZM	0.6	n/a	61		no for one point	
	1.0	n/a	65		no	
	1.5	n/a	72		no	
	2.0	n/a	129		no	

Table 14.2. Degrees of equivalence between laboratories, range 0.6 g/cm^3.

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		BEV		IPQ		DZM	
0.601 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	2	23	-27	22	12	31	39	61	139	43	14	43	20	62	69	63
PTB	-2	23	-	-	-30	22	10	31	36	61	136	43	11	43	18	62	67	63
BFKH	27	22	30	22	-	-	40	66	66	85	166	72	41	72	47	85	96	86
UME	-12	31	-10	31	-40	66	-	-	26	65	126	48	1	48	8	66	57	67
DMDM	-39	61	-36	61	-66	85	-26	65	-	-	100	71	-25	71	-18	84	31	85
NIM	-139	43	-136	43	-166	72	-126	48	-100	71	-	-	-125	57	-118	72	-69	73
BEV	-14	43	-11	43	-41	72	-1	48	25	71	125	57	-	-	7	72	56	73
IPQ	-20	62	-18	62	-47	85	-8	66	18	84	118	72	-7	72	-	-	49	86
DZM	-69	63	-67	63	-96	86	-57	67	-31	85	69	73	-56	73	-49	86	-	-

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		BEV		IPQ		DZM	
0.605 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	7	23	-10	22	11	31	40	61	114	43	-6	43	16	62	64	63
PTB	-7	23	-	-	-17	22	4	31	33	61	107	43	-13	43	9	62	57	63
BFKH	10	22	17	22	-	-	20	40	49	66	123	50	4	50	26	67	74	68
UME	-11	31	-4	31	-20	40	-	-	29	65	103	48	-16	48	6	66	53	67
DMDM	-40	61	-33	61	-49	66	-29	65	-	-	74	71	-46	71	-23	84	24	85
NIM	-114	43	-107	43	-123	50	-103	48	-74	71	-	-	-120	57	-97	72	-50	73
BEV	6	43	13	43	-4	50	16	48	46	71	120	57	-	-	22	72	70	73
IPQ	-16	62	-9	62	-26	67	-6	66	23	84	97	72	-22	72	-	-	48	86
DZM	-64	63	-57	63	-74	68	-53	67	-24	85	50	73	-70	73	-48	86	-	-

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		BEV		IPQ		DZM	
0.609 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	9	23	2	22	13	31	47	61	84	43	-13	43	19	62	59	63
PTB	-9	23	-	-	-7	22	4	31	38	61	75	43	-23	43	9	62	49	63
BFKH	-2	22	7	22	-	-	11	27	45	60	82	41	-16	40	16	61	56	62
UME	-13	31	-4	31	-11	27	-	-	34	65	71	48	-27	48	5	66	45	67
DMDM	-47	61	-38	61	-45	60	-34	65	-	-	37	71	-61	71	-29	84	11	85
NIM	-84	43	-75	43	-82	41	-71	48	-37	71	-	-	-98	57	-66	72	-26	73
BEV	13	43	23	43	16	40	27	48	61	71	98	57	-	-	32	72	72	73
IPQ	-19	62	-9	62	-16	61	-5	66	29	84	66	72	-32	72	-	-	40	86
DZM	-59	63	-49	63	-56	62	-45	67	-11	85	26	73	-72	73	-40	86	-	-

Table 14.3. Degrees of equivalence between laboratories, range 1.0 g/cm^3.

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		BEV		IPQ		DZM	
0.991 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	2	35	-32	63	38	74	8	124	5	70	18	172	57	273	87	134
PTB	-2	35	-	-	-34	63	36	74	6	124	3	70	16	172	54	273	84	134
BFKH	32	63	34	63	-	-	65	94	35	137	32	91	45	182	84	279	114	146
UME	-38	74	-36	74	-65	94	-	-	-4	126	-7	73	6	173	44	274	74	136
DMDM	-8	124	-6	124	-35	137	4	126	-	-	-33	90	-20	181	18	279	48	146
NIM	-5	70	-3	70	-32	91	7	73	33	90	-	-	-120	176	-82	276	-52	139
BEV	-18	172	-16	172	-45	182	-6	173	20	181	120	176	-	-	43	276	73	139
IPQ	-57	273	-54	273	-84	279	-44	274	-18	279	82	276	-43	276	-	-	30	146
DZM	-87	134	-84	134	-114	146	-74	136	-48	146	52	139	-73	139	-30	146	-	-

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		BEV		IPQ		DZM	
0.995 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	4	35	2	63	28	74	29	124	13	70	-2	172	40	273	86	134
PTB	-4	35	-	-	-5	63	21	74	22	124	6	70	-9	172	33	273	79	134
BFKH	-2	63	5	63	-	-	37	78	39	126	23	75	8	174	50	274	95	137
UME	-28	74	-21	74	-37	78	-	-	18	126	2	73	-12	173	30	274	75	136
DMDM	-29	124	-22	124	-39	126	-18	126	-	-	-27	91	-41	181	1	279	46	146
NIM	-13	70	-6	70	-23	75	-2	73	27	91	-	-	-115	176	-73	276	-28	139
BEV	2	172	9	172	-8	174	12	173	41	181	115	176	-	-	46	276	91	139
IPQ	-40	273	-33	273	-50	274	-30	274	-1	279	73	276	-46	276	-	-	45	146
DZM	-86	134	-79	134	-95	137	-75	136	-46	146	28	139	-91	139	-45	146	-	-

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		BEV		IPQ		DZM	
0.999 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	7	35	14	63	38	74	19	124	-1	70	3	172	27	273	86	134
PTB	-7	35	-	-	4	63	28	74	10	124	-10	70	-7	172	17	273	77	134
BFKH	-14	63	-4	63	-	-	35	73	17	123	-3	69	0	171	24	273	84	134
UME	-38	74	-28	74	-35	73	-	-	6	126	-14	73	-10	173	13	274	73	136
DMDM	-19	124	-10	124	-17	123	-6	126	-	-	-48	91	-44	181	-21	279	39	146
NIM	1	70	10	70	3	69	14	73	48	91	-	-	-81	176	-58	276	2	139
BEV	-3	172	7	172	0	171	10	173	44	181	81	176	-	-	40	276	99	139
IPQ	-27	273	-17	273	-24	273	-13	274	21	279	58	276	-40	276	-	-	59	146
DZM	-86	134	-77	134	-84	134	-73	136	-39	146	-2	139	-99	139	-59	146	-	-

Table 14.4. Degrees of equivalence between laboratories, range 1.5 g/cm^3.

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		IPQ		DZM	
1.491 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	2	48	-12	76	38	88	28	127	13	92	80	251	122	151
PTB	-2	48	-	-	-14	76	35	88	25	127	10	92	77	251	120	151
BFKH	12	76	14	76	-	-	65	105	55	140	40	109	107	258	149	162
UME	-38	88	-35	88	-65	105	-	-	15	129	0	94	67	252	110	153
DMDM	-28	127	-25	127	-55	140	-15	129	-	-	-26	108	41	258	84	162
NIM	-13	92	-10	92	-40	109	0	94	26	108	-	-	-59	254	-16	156
IPQ	-80	251	-77	251	-107	258	-67	252	-41	258	59	254	-	-	43	162
DZM	-122	151	-120	151	-149	162	-110	153	-84	162	16	156	-43	162	-	-

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		IPQ		DZM	
1.495 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	6	48	21	76	9	88	53	127	14	92	58	251	122	151
PTB	-6	48	-	-	14	76	2	88	46	127	7	92	51	251	115	151
BFKH	-21	76	-14	76	-	-	19	92	62	129	23	96	68	253	132	153
UME	-9	88	-2	88	-19	92	-	-	42	129	3	94	48	252	111	153
DMDM	-53	127	-46	127	-62	129	-42	129	-	-	-26	108	18	258	82	162
NIM	-14	92	-7	92	-23	96	-3	94	26	108	-	-	-56	254	8	156
IPQ	-58	251	-51	251	-68	253	-48	252	-18	258	56	254	-	-	64	162
DZM	-122	151	-115	151	-132	153	-111	153	-82	162	-8	156	-64	162	-	-

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		IPQ		DZM	
1.499 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	11	48	27	76	22	88	42	127	0	92	32	251	123	151
PTB	-11	48	-	-	18	76	13	88	33	127	-9	92	22	251	114	151
BFKH	-27	76	-18	76	-	-	20	87	40	126	-2	91	29	251	121	151
UME	-22	88	-13	88	-20	87	-	-	29	129	-13	94	18	252	110	153
DMDM	-42	127	-33	127	-40	126	-29	129	-	-	-47	108	-15	258	76	162
NIM	0	92	9	92	2	91	13	94	47	108	-	-	-52	254	39	156
IPQ	-32	251	-22	251	-29	251	-18	252	15	258	52	254	-	-	91	162
DZM	-123	151	-114	151	-121	151	-110	153	-76	162	-39	156	-91	162	-	-

Table 14.5. Degrees of equivalence between laboratories, range 2.0 g/cm^3.

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		IPQ		DZM	
1.981 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	3	67	-82	119	40	124	57	142	10	114	110	999	172	265
PTB	-3	67	-	-	-84	119	38	124	55	142	8	113	107	999	170	265
BFKH	82	119	84	119	-	-	68	137	84	153	37	127	137	1001	199	271
UME	-40	124	-38	124	-68	137	-	-	45	143	-2	115	97	999	160	266
DMDM	-57	142	-55	142	-84	153	-45	143	-	-	-28	127	71	1001	133	271
NIM	-10	114	-8	113	-37	127	2	115	28	127	-	-	-29	1000	33	268
IPQ	-110	999	-107	999	-137	1001	-97	999	-71	1001	29	1000	-	-	63	271
DZM	-172	265	-170	265	-199	271	-160	266	-133	271	-33	268	-63	271	-	-

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		IPQ		DZM	
1.990 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	8	67	-27	119	19	124	29	142	-3	114	59	999	184	265
PTB	-8	67	-	-	-34	119	12	124	22	142	-10	114	52	999	177	265
BFKH	27	119	34	119	-	-	29	126	38	144	6	116	69	999	193	266
UME	-19	124	-12	124	-29	126	-	-	18	143	-14	115	49	999	173	266
DMDM	-29	142	-22	142	-38	144	-18	143	-	-	-43	127	20	1001	144	271
NIM	3	114	10	114	-6	116	14	115	43	127	-	-	-54	1000	70	268
IPQ	-59	999	-52	999	-69	999	-49	999	-20	1001	54	1000	-	-	124	271
DZM	-184	265	-177	265	-193	266	-173	266	-144	271	-70	268	-124	271	-	-

x_j	INRIM		PTB		BFKH		UME		DMDM		NIM		IPQ		DZM	
1.999 g/cm^3	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	12	67	8	119	26	124	30	142	-7	114	22	999	183	265
PTB	-12	67	-	-	-1	119	17	124	20	142	-17	114	12	999	174	265
BFKH	-8	119	1	119	-	-	24	123	27	141	-10	113	19	999	181	265
UME	-26	124	-17	124	-24	123	-	-	16	143	-21	115	9	999	170	266
DMDM	-30	142	-20	142	-27	141	-16	143	-	-	-55	127	-25	1001	136	271
NIM	7	114	17	114	10	113	21	115	55	127	-	-	-62	1000	99	268
IPQ	-22	999	-12	999	-19	999	-9	999	25	1001	62	1000	-	-	161	271
DZM	-183	265	-174	265	-181	265	-170	266	-136	271	-99	268	-161	271	-	-

Table 14.6. Uncertainty budget for the range 0.6 g/cm^3 (in density g/cm^3×10^{-6}).

Influence quantity	INRIM	PTB	UME	BEV	DMDM	IPQ	BFHK	DZM	NIM
	g/cm^3×10^{-6}								
Weighing in air	0.07	0.04	0.11	1.89	0.37	0.58	0.34	0.15	0.70
Weighing in liquid	0.57	0.31	0.47	13.12	2.52	10.40	1.88	0.52	6.00
Additional weights	0.57	1.70	7.30	0.00	0.00	0.00	0.00	0.00	0.00
Cubic thermal expansion	0.01	0.01	0.05	0.08	0.03	1.27	0.01	0.02	0.00
Diameter of stem	0.17	0.16	0.66	4.10	3.55	0.23	1.30	0.02	0.70
Air density	0.30	0.04	0.43	3.87	0.30	10.20	1.00	0.60	0.90
Liquid density	4.66	3.10	4.02	8.13	15.0	4.08	2.12	7.81	5.00
Temperature of liquid	5.70	2.10	4.20	0.15	4.02	0.12	3.20	2.25	3.00
Thermal expansion coefficient of liquid	0.01	0.42	0.93	0.00	0.40	0.76	0.21	0.00	0.00
Compressibility of liquid	0.00	0.00	0.00	0.00	0.02	0.00	0.18	0.00	0.00
Surface tension of liquid	1.16	3.80	2.76	2.90	7.15	0.86	3.40	3.17	0.00
Gravitation acceleration	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
Gradient of gravitational	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Height difference of weights and hydrometer	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
Readings error	3.00	5.40	5.77	0.00	8.00	25.15	3.00	28.90	5.00
Standard deviation of the mean of corrections	1.00	2.40	6.35	0.00	0.00	5.29	2.10	0.61	6.00
Influence of the temperature distribution in the bath		0.44							
Contact angle buoyant liquid				7.44					
Contact angle liquid				7.44					

Table 14.7. Uncertainty budget for the range 1.0 g/cm^3 (in density g/cm^3×10^{-6}).

Influence quantity	INRIM	PTB	UME	BEV	DMDM	IPQ	BFHK	DZM	NIM
	g/cm^3×10^{-6}								
Weighing in air	0.13	0.49	0.07	1.90	0.37	0.95	0.88	0.20	1.00
Weighing in liquid	0.96	0.35	0.44	24.30	2.52	58.10	3.15	0.87	9.00
Additional weights	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cubic thermal expansion	0.02	0.01	0.07	0.19	0.03	2.10	0.02	0.01	0.00
Diameter of stem	1.90	0.54	5.18	18.00	3.55	1.79	7.90	0.20	0.70
Air density	0.78	0.67	6.61	5.00	0.30	22.0	1.00	0.75	1.50
Liquid density	6.96	4.00	0.71	16.10	15.00	6.70	3.47	12.90	8.00
Temperature of liquid	9.14	2.80	6.90	0.19	4.02	0.19	5.30	3.71	4.00
Thermal expansion coefficient of liquid	0.02	0.55	1.64	0.00	0.40	1.25	0.21	0.00	0.00
Compressibility of liquid	0.00	0.05	0.00	0.00	0.02	0.00	0.18	0.00	0.00
Surface tension of liquid	1.52	3.10	3.62	3.65	7.15	1.10	2.80	2.51	0.00
Gravitation acceleration	0.00	0.00	0.00	0.00	0.00	0.03	0.01	0.00	0.00
Gradient of gravitational	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Height difference of weights and hydrometer	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
Readings error	3.00	5.70	5.77	0.00	8.00	24.60	3.00	28.90	5.00
Standard deviation of the mean of corrections	1.43	3.20	9.24	0.00	0.00	7.77	2.24	0.63	6.00
Influence of the temperature distribution in the bath		0.45							
Contact angle buoyant liquid				9.31					
Contact angle liquid				20.90					

Table 14.8. Uncertainty budget for the range 1.5 g/cm^3 (in density g/cm^3×10^{-6}).

Influence quantity	INRIM	PTB	UME	DMDM	IPQ	BFHK	DZM	NIM
	g/cm^3×10^{-6}							
Weighing in air	0.69	1.12	0.34	0.37	2.23	1.86	0.50	2.50
Weighing in liquid	1.08	0.65	0.98	2.52	24.8	3.57	1.03	10.00
Additional weights		0.00	0.00	0.00		0.00	0.00	0.00
Cubic thermal expansion	0.03	0.02	0.12	0.03	3.12	0.01	0.01	0.00
Diameter of stem	0.24	0.06	0.35	3.55	0.15	2.40	0.01	0.05
Air density	2.18	1.90	2.17	0.30	48.0	1.00	2.26	4.00
Liquid density	10.40	5.90	9.92	15.00	10.1	5.22	19.40	12.00
Temperature of liquid	13.70	4.10	10.36	4.02	0.28	8.00	5.59	7.00
Thermal expansion coefficient of liquid	0.03	0.82	2.62	0.40	1.86	0.21	0.00	0.00
Compressibility of liquid	0.00	0.06	0.00	0.02	0.00	0.18	0.00	0.00
Surface tension of liquid	1.71	3.50	4.06	7.15	1.30	2.60	1.95	0.00
Gravitation acceleration	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
Gradient of gravitational	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Height difference of weights and hydrometer	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
Readings error	3.00	6.40	5.77	8.00	26.30	3.00	28.9	5.00
Standard deviation of the mean of corrections	1.63	4.40	9.24	0.00	7.00	6.62	0.68	6.00
Influence of the temperature distribution in the bath		0.51						

Table 14.9. Uncertainty budget for the range 2.0 g/cm^3 (in density g/cm^3×10^{-6}).

Influence quantity	INRIM	PTB	UME	DMDM	IPQ	BFHK	DZM	NIM
	g/cm^3×10^{-6}							
Weighing in air	1.13	2.51	0.54	0.37	4.09	3.58	0.95	6.00
Weighing in liquid	2.16	0.82	1.04	2.52	228	5.59	1.57	18.00
Additional weights	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cubic thermal expansion	0.03	0.02	0.18	0.03	4.26	0.00	0.09	0.00
Diameter of stem	0.44	0.12	0.82	3.55	0.30	0.33	0.01	0.10
Density air	3.58	3.00	3.64	0.30	86.20	1.00	3.47	6.00
Density liquid	13.90	7.90	13.20	15.00	13.40	6.96	25.8	16.00
Temperature of liquid	18.30	5.50	13.80	4.02	0.38	10.00	7.42	9.00
Thermal expansion coefficient of liquid	0.03	1.10	3.28	0.40	2.54	0.21	0.00	0.00
Compressibility of liquid	0.00	0.02	0.00	0.02	0.00	0.18	0.00	0.00
Surface tension of liquid	2.70	6.00	7.01	7.15	1.98	18.00	2.25	0.00
Gravitation acceleration	0.00	0.01	0.00	0.00	0.01	0.01	0.00	0.00
Gradient of gravitational	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Height difference of weights and hydrometer	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
Readings error	5.00	10.00	11.60	15.00	45.10	3.0	57.70	10.00
Standard deviation of the mean of corrections	3.40	5.40	12.70	0.00	16.40	5.27	1.25	9.00
Influence of the temperature distribution in the bath		0.90						