Spin Currents at the Interface and Spin Hall Torque

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We compare two different approaches to compute the spin Hall torque at the interface between a spin Hall metallic layer and a ferromagnet. In one approach one attributes a spin mixing conductance $g_{\uparrow\downarrow}$ to the interface, while the other employs the thermodynamic theory for the magnetic moment currents. The main difference between the outcomes of the two approaches is the field like torque term that, in the first approach is due to the imaginary part of the spin mixing conductance Im$[g_{\uparrow\downarrow}]$, while in the second one is proportional to the effective damping α' of the ferromagnet.

***Index Terms*—spin Hall torque, spin Hall effect, Magnetization dynamic equation**

I. Introduction

The spin Hall torque is one of the most promising effects in spintronic memory devices for inducing magnetization dynamics on a thin ferromagnetic (F) layer [1], [2]. The torque is caused by the spin current electrically generated by the spin Hall effect in an adjacent normal (N) metallic layer [3] and can induce the oscillation or the switching of the magnetization of the F layer [4]–[8]. Another advantage of the spin Hall torque switching is that the detection of the state of the magnetic bit can be performed electrically thanks to the spin Hall magnetoresistance effects [9]–[11].

The description of the spin Hall torque effects on the magnetization of ferromagnet requires the introduction of two torque components in the magnetization dynamic equation: the field-like (FL) and the damping-like (DL), both proportional to intensity of the spin Hall effect [12]. By projecting all the vectors onto the plane perpendicular to the magnetization, one finds the DL in the direction of the injected magnetic moment and the FL in the perpendicular direction. The harmonic Hall method permits a detailed characterization of the two components and reveals that the FL is often much smaller than the DL and that they both depends on the thicknesses of the two layers [13], [14] .

In the literature, one finds a derivation of the two torque terms in relation to the spin mixing conductance of the interface $g_{\uparrow\downarrow}$ [15], [16]. Such a quantity, with real and imaginary parts, give rise to the two torque terms [2], [12]. However real and imaginary parts are often difficult to connect with the physical properties of the interface (quality of the interface, band structure of the metals) and requires a detailed physical interpretation case by case [17], [18]. It is therefore natural to ask if the two torque terms could be derived by some other principle not directly invoking an imaginary part for the conductance $g_{\uparrow\downarrow}$.

The alternative approach that we propose here is based on a thermodynamic treatment of the problem of the magnetic moment current [19]–[22]. In order to allow for the presence of a spin current at the interface of a ferromagnet we have to extend the boundary conditions of micromagnetism, therefore replacing the classical Brown's boundary conditions. The analysis reveals that by using the continuity of the spin current at the interface between the ferromagnet and the normal metal, one is able to derive the two torque components and to show that the, generally small, FL component is proportional to the effective damping of the ferromagnet. In the present paper we present the derivation and the properties of the DL and FL torque terms from both spin mixing conductance and thermodynamic theory and we discuss the meaning of the different outcomes.

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II. Spin Hall torque

We consider here spin orbit torque effects arising from the spin Hall effect of heavy metals such as Ta, W, Pt. The spin Hall effect corresponds to a spin dependent deviation of the transport of the electrons. It is an effect that does not require any magnetic field and is due to the spin orbit interaction of conduction electrons [23]. The spin Hall angle is a dimensionless parameter that quantifies the conversion of the electric current $\mathbf{j}_e$ into a magnetic moment current $\mathbf{j}_\mathbf{M}$. In components we have

$$j_{\mathrm{M},ij} = \theta_{\mathrm{SH}} \left(\frac{\mu_B}{e}\right) \epsilon_{ijk} j_{e,k} \tag{1}$$

where $j_{\mathrm{M},ij}$ is a tensor with two indexes: i, giving the direction of the current and j giving the direction of the transported magnetic moment, ϵ_{ijk} is the Levi-Civita symbol, μ_B is the Bohr magneton and e is the elementary charge. The spin Hall angle for the magnetic moment θ_{SH} has opposite sign with respect to the one for the spin which is often reported in the literature $\theta_{\mathrm{SH}} = -\theta_{\mathrm{SH}}^{\mathrm{spin}}$, because the current of spin is opposite with respect to the magnetic moment one $\mathbf{j_s} = -[(\hbar/2)/\mu_B]\mathbf{j_M}$. For the magnetic moment we have a negative spin Hall angle for Pt, $\theta_{\mathrm{SH}}(\mathrm{Pt}) \simeq -0.1$, and positive for Ta, $\theta_{\mathrm{SH}}(\mathrm{Ta}) \simeq 0.07$, and W, $\theta_{\mathrm{SH}}(\mathrm{W}) \simeq 0.14$, [24]. Therefore in Pt an electric current along y will produce a positive $j_{\mathrm{M},xz}$ component: $j_{\mathrm{M},xz} = -\theta_{\mathrm{SH}}(\mathrm{Pt})(\mu_B/e)j_{e,y}$.

The spin Hall torque is often described by adding a torque term to the dynamic equation

$$\frac{d\mathbf{M}}{dt} - \alpha \mathbf{m} \times \frac{d\mathbf{M}}{dt} = -\mu_0\gamma_G\, \mathbf{M}\times\mathbf{H}_{eff} + \mathbf{T}_{\mathrm{ST}} \tag{2}$$

In the previous equation, $\mathbf{M}$ is the magnetization of constant amplitude M_s, $\mathbf{m} = \mathbf{M}/M_s$ is the magnetization versor, α is the damping constant, μ_0 is the magnetic constant and γ_G is the gyromagnetic ratio for electrons. The torque $\mathbf{T}_{\rm ST}$ has both field-like and damping-like terms. It is found that

$$\mathbf{T}_{\rm ST} = \frac{(\mu_B/e) j_e}{d_F} \left[\xi_{DL} \mathbf{m} \times (\mathbf{m} \times \mathbf{e}_p) + \xi_{FL}\, \mathbf{m} \times \mathbf{e}_p\right] \quad (3)$$

where $\mathbf{e}_p$ is the versor of the polarization of the moments induced by the spin Hall layer and d_F is the thickness of the F layer. The damping-like (ξ_{LD}) and field-like (ξ_{FL}) efficiencies are both proportional to the spin Hall angle, but also depend on the properties of the two layers.

To derive the expression for the torque one typically considers a normal metal (N) layer with the spin Hall effect of thickness d_N from $x = -d_N$ to $x = 0$ and a ferromagnetic (F) layer of thickness d_F from $x = 0$ to $x = d_F$ (see Fig.1). The method used to derive the torque is to write the constitutive equations relating the electric current $\mathbf{j}_e$ and the magnetic moment current $\mathbf{j}_\mathbf{M}$ to their potentials for both the F layer and the N layer and then joining the solutions by setting appropriate boundary conditions [12]. One approach that simplifies the notation is to measure the magnetic moment current in the same units of the electric current, i.e. to set $\mathbf{j_p} = -(e/\mu_B)\mathbf{j_M}$. In a normal metal with the spin Hall effect the constitutive equations are written as

$$\begin{aligned} j_{e,i} &= -\sigma_e \nabla_i \mu_e - \sigma_e \theta_{SH} \epsilon_{ijk} \nabla_j \mu_{p,k} & (4) \\ j_{p,ij} &= -\sigma_e \theta_{SH} \epsilon_{ijk} \nabla_k \mu_e - \sigma_e \nabla_i \mu_{p,j} & (5) \end{aligned}$$

where σ_e is the electric conductivity, μ_e is the electro-chemical potential and $\boldsymbol{\mu}_p$ is the potential associated with the magnetic moment current (in the same units of the electro-chemical one). These equations, once coupled to the continuity equation for the magnetic moment, provide the solution of the transport problem. In a ferromagnet one can write the constitutive equations

$$\begin{aligned} j_{e,i} &= -\sigma_e \nabla_i \mu_e - \sigma_e \beta m_k \nabla_i \mu_{p,k} & (6) \\ j_{p,ij} &= -\sigma_e \beta m_j \nabla_i \mu_e - \sigma_e \nabla_i \mu_{p,j} & (7) \end{aligned}$$

where β is the spin polarization and m_i are the components of the magnetization versor $\mathbf{m}$. These equations are appropriate to describe the transport of magnetic moment parallel to the magnetization direction $\mathbf{m}$, however the absorption of the magnetic moment in the perpendicular direction has to be derived from the magnetization dynamic equation.

The method followed by several authors [2], [12], [17] is to assume that the spin current perpendicular to the magnetization is completely absorbed by the ferromagnet and to compute the torque as $\mathbf{T}_{\rm ST} = -\nabla \cdot \mathbf{j}_{\mathbf{M},\perp}$. The result is finally added to the dynamic equation written in the Gilbert form as in Eq.(2). However to obtain the DL and FL terms one has to attribute special properties to the interface, namely the spin mixing conductance $g_{\uparrow\downarrow}$, a quantity which is governing the passage of the perpendicular current from the N layer to the F layer with real and imaginary parts. The interpretation of the imaginary part is that the magnetic moment of the electron traversing the interface experiences an interaction at the ferromagnet side giving a precession of the moment of a finite angle around the magnetization [16]. In terms of the torque, the real part gives rise to the DL torque and the imaginary part to the FL torque. There are two critical points in this approach: first, to attribute the FL part of the torque to a precession occurring at the interface rather than at the bulk seems not appropriate; second, to simply add the torque term to the Gilbert form (Eq.(2)) may be questionable [25].

In the next two sections we both review the approach with the spin mixing conductance of Ref. [12] and propose a different approach based on the thermodynamic theory for magnetic moment currents [19].

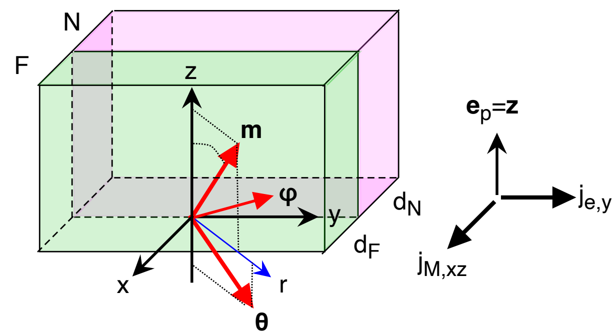

Fig. 1. Bilayer composed by a normal metal (N) with the spin Hall effect of thickness d_N and a ferromagnet (F) of thickness d_F. In the sketch the electric current flows along y and the magnetic moment current generated by the spin Hall effect is transporting moment along z in the direction x. The DL torque is along $\hat{\boldsymbol{\theta}}$ while the FL torque is along $\hat{\boldsymbol{\varphi}}$, both versors are perpendicular to the magnetization versor $\mathbf{m}$.

III. Spin mixing conductance

To derive the spin Hall torque term Haney et al. [12] (see also [2], [17]) use the boundary conditions as proposed by Brataas et. al [15], [16] in their magnetoelectronic circuit theory. The theory of Brataas et al. is based on ballistic transport, therefore the potential drop occurs only at the interfaces. The contact resistance is attributed to the mismatch between the band structures of different metals. Therefore at the contact one employs the electrical conductance g_c (i.e $j_e = -g_c \Delta\mu_e$). The constitutive equations at the contact between the normal metal and the ferromagnet are written by subdividing the potential into the component parallel to the magnetization $\mathbf{m}$ of the F layer, $\mu_{p,\parallel}$, and the component perpendicular $\boldsymbol{\mu}_{p,\perp}$. The equations are

$$\begin{pmatrix} \mathbf{j}_e \\ \mathbf{j}_{p,\parallel} \\ \mathbf{j}_{p,\perp} \end{pmatrix} = -g_c \begin{pmatrix} 1 & P & 0 \\ P & 1 & 0 \\ 0 & 0 & \eta \end{pmatrix} \begin{pmatrix} \Delta\mu_e \\ \Delta\mu_{p,\parallel} \\ \Delta\boldsymbol{\mu}_{p,\perp} \end{pmatrix} \quad (8)$$

where the conductance $g_c = g_\uparrow + g_\downarrow$ is the sum of $g_\uparrow$ and $g_\downarrow$, the conductances for the majority and minority spins, respectively, $P = (g_\uparrow - g_\downarrow)/(g_\uparrow + g_\downarrow)$ is the polarization at the contact and

$$\eta = \frac{2 g_{\uparrow\downarrow}}{g_\uparrow + g_\downarrow} \quad (9)$$

is the relative mixing conductance for the perpendicular current which is related to the spin mixing conductance $g_{\uparrow\downarrow}$. The previous approach is not limited to metals and can also be applied to the contact between a metal and a ferromagnetic insulator. In that case the conductance quantifies the amount of spin transmitted from the metal to the insulator. By allowing both real and imaginary parts to the spin mixing conductance one obtains

$$\mathbf{j}_{p,\perp} = -2\mathrm{Re}[g_{\uparrow\downarrow}]\Delta\boldsymbol{\mu}_{p,\perp} + 2\mathrm{Im}[g_{\uparrow\downarrow}](\mathbf{m}\times\Delta\boldsymbol{\mu}_{p,\perp}) \quad (10)$$

i.e. the potential difference $\Delta\boldsymbol{\mu}_{p,\perp}$ produces not only a current in the same direction of the potential, proportional to $\mathrm{Re}[g_{\uparrow\downarrow}]$, but also a component in the perpendicular direction, proportional to $\mathrm{Im}[g_{\uparrow\downarrow}]$. The torque is computed as

$$\mathbf{T}_{\mathrm{ST}} = \left(\frac{\mu_B}{e}\right)\frac{j_{\mathbf{p},\perp}(0)}{d_F} \quad (11)$$

bacause one assumes $j_{\mathbf{p}}(d_F) = 0$. With a few passages (reported in appendix A) one can compute $j_{\mathbf{p},\perp}(0)$ and the torque results Eq.(3) with efficiencies

$$\xi_{DL} = \theta_{SH}\frac{2\mathrm{Re}[g_{\uparrow\downarrow}]g_N\tanh(\hat{d}_N) + |2g_{\uparrow\downarrow}|^2}{|2g_{\uparrow\downarrow} + g_N\tanh(\hat{d}_N)|^2} \quad (12)$$

$$\xi_{FL} = \theta_{SH}\frac{2\mathrm{Im}[g_{\uparrow\downarrow}]g_N\tanh(\hat{d}_N)}{|2g_{\uparrow\downarrow} + g_N\tanh(\hat{d}_N)|^2} \quad (13)$$

where $\hat{d}_N = d_N/l_M$, $g_N = \sigma_e/l_M$ is the conductance and l_M is the diffusion length of the N layer. With the approach using the spin mixing conductance, the FL term is associated to the imaginary part of the spin mixing conductance $\mathrm{Im}[g_{\uparrow\downarrow}]$.

IV. Thermodynamic theory

An alternative way to derive the torque is to use the thermodynamic approach to the magnetic moment currents. This approach is inspired to the thermodynamic theory of Johnson and Silsbee [19] and has been developed by several groups [20], [22], [26]. In non equilibrium thermodynamics each current has an associated potential and in the linear system approximation the current is proportional to the gradient of the potential $\mathbf{H}^*$ with units of a magnetic field. The conversion between the units of the previous section is $\mu_0\mu_B\mathbf{H}^* = e\boldsymbol{\mu}_p$. The magnetic moment is a non conserved quantity therefore the continuity equation must include the presence of sources and sinks. One finds that the potential $\mathbf{H}^*$ is the quantity describing the generation and absorption of the magnetic moment

$$\frac{\partial\mathbf{M}}{\partial t} + \nabla\cdot\mathbf{j}_{\mathbf{M}} = \frac{\mathbf{H}^*}{\tau_M} \quad (14)$$

and τ_M is a constant describing the rate of magnetic moment generation [21]. Ferromagnets are a special case of magnetic materials in which the magnetization has constant amplitude and one has also to take into account the phenomenon of precession. Therefore the continuity equation is not simply Eq.(14), but rather a dynamic equation that in the Gilbert form reads

$$\frac{d\mathbf{M}}{dt} - \alpha\mathbf{m}\times\frac{d\mathbf{M}}{dt} = -\mu_0\gamma_G\,\mathbf{M}\times\mathbf{H}_{eff} \quad (15)$$

where the effective field

$$\mathbf{H}_{eff} = l_{EX}^2 M_s\nabla^2\mathbf{m} + \mathbf{H}_{AN} + \mathbf{H}_M + \mathbf{H}_a \quad (16)$$

includes contributions from exchange, anisotropy, magnetostatic field and applied field and l_{EX} is the exchange length. Eq.(15) contains both damping and precessional terms, but the magnetic moment current is not explicit. Our task is therefore to write the dynamic equation for ferromagnets as a continuity equation. To do so, we first rewrite Eq.(15) in the equivalent Landau-Lifshitz form

$$\frac{d\mathbf{M}}{dt} = -\mu_0\gamma_L\mathbf{M}\times\mathbf{H}_{eff} + \mu_0\gamma_L M_s\alpha\mathbf{H}_{eff,\perp} \quad (17)$$

where $\gamma_L = \gamma_G/(1+\alpha^2)$, and then we notice that in the effective field the exchange term can be written as a divergence. Therefore by setting $\mathbf{H}^* = \mathbf{H}_{AN} + \mathbf{H}_M + \mathbf{H}_a$ from Eq.(17) written as a continuity-type equation, like Eq.(14), we get

$$\frac{\partial\mathbf{M}}{\partial t} + (\nabla\cdot\mathbf{j}_{\mathbf{M},\perp})_\perp = -\mu_0\gamma_L\mathbf{M}\times\mathbf{H}_\perp^* + \mu_0\gamma_L M_s\alpha\mathbf{H}_\perp^* \quad (18)$$

where the magnetic moment current is defined as

$$\mathbf{j}_{\mathbf{M},\perp} = \mu_0\gamma_L M_s^2 l_{EX}^2(\mathbf{m}\times\nabla\mathbf{m} - \alpha\nabla\mathbf{m}) \quad (19)$$

This is the component of the magnetic moment current perpendicular to $\mathbf{m}$. Even if the continuity type equation (18), in which the current is explicit, is relevant only in presence of interfaces with other layers, it is worth to make two general comments on its specific form. First, as a dynamic equation, it has the form of Landau-Lifshitz rather than the Gilbert one [25]. Second, as a continuity equation, it has two source terms. It contains the same source term of Eq.(14) (set $\tau_M^{-1} = \mu_0\gamma_L M_s\alpha$ to verify it), but it also has an additional source term resulting from the precession of the magnetization. Indeed, from the point of view of a continuity equation, the precessional term is a source/sink term which mixing the components of the magnetization. If, for example, we choose the precession around $\mathbf{z}$ (i.e. set $\mathbf{H}_\perp^* = H\mathbf{z}$), in Eq.(18) we can see that the first term at the right hand side becomes $\mu_0\gamma_L M_s H(-m_y\mathbf{x} + m_x\mathbf{y})$ i.e. (with $m_x > 0$ and $m_y > 0$) it is a sink of moments along $\mathbf{x}$ and a source along $\mathbf{y}$.

The solution of Eq.(18) is obtained by imposing the boundary conditions with other layers. Indeed we cannot impose anymore the Brown's natural boundary conditions $\partial\mathbf{m}/\partial\mathbf{n} = 0$, where $\mathbf{n}$ is the normal to the surface of the F layer, but rather we have to impose the continuity of the current, $\mathbf{j}_{\mathbf{M},\perp}$, and of the potential, $\mathbf{H}_\perp^*$, with the other layers. To do so we have to solve the problem of the transport of magnetic moment layer by layer and finally joining the solutions at the boundaries. For a thin F layer we approximate

$$(\nabla\cdot\mathbf{j}_{\mathbf{M},\perp})_\perp = \left(\frac{\mu_B}{e}\right)\frac{j_{\mathbf{p},\perp}(0)}{d_F} \quad (20)$$

where again we have set $\mathbf{j}_{\mathbf{p},\perp}(d_F)=0$. With a few passages (reported in appendix B) we can compute $j_{\mathbf{p},\perp}(0)$. To understand the difference with respect to the derivation of the previous section we take a real conductance for the contact ($\eta=1$ in Eq.(8)). The result is

$$j_{\mathbf{p},\perp}(0)=j_{p,S,eq}\mathbf{e}_{p,\perp}-g_{N,eq}\left(\frac{\mu_B}{e}\right)\mu_0\mathbf{H}^*_\perp(0) \quad (21)$$

where $\mathbf{e}_{p,\perp}=-\mathbf{m}\times(\mathbf{m}\times\mathbf{e}_p)$,

$$j_{p,S,eq}=\theta_{SH}\frac{g_c\tanh(\hat{d}_N)\tanh(\hat{d}_N/2)}{g_c+g_N\tanh(\hat{d}_N)}j_{e,y} \quad (22)$$

is an equivalent current source and

$$g_{N,eq}=\frac{g_c g_N\tanh(\hat{d}_N)}{g_c+g_N\tanh(\hat{d}_N)} \quad (23)$$

is an equivalent conductance. As the F layer is thin we also approximate $\mathbf{H}^*_\perp(d_F)\simeq\mathbf{H}^*_\perp(0)$ and then simply call it $\mathbf{H}_\perp$. Therefore Eq.(18) becomes

$$\frac{d\mathbf{M}}{dt}=-\mu_0\gamma_L\,\mathbf{M}\times\mathbf{H}_\perp+\mu_0\gamma_L M_s\alpha'\mathbf{H}_\perp-\left(\frac{\mu_B}{e}\right)\frac{j_{p,S,eq}}{d_F}\mathbf{e}_{p,\perp} \quad (24)$$

where

$$\alpha'=\alpha+\left(\frac{\mu_B}{e}\right)^2\frac{g_{N,eq}}{\gamma_L M_s d_F} \quad (25)$$

is an effective damping. With the use of the thermodynamic theory we have derived Eq.(24) which contains two results. First, the effective damping is enhanced with respect to the pure ferromagnet by the presence of the side metallic layer, a well known effect which is independent of the spin Hall activity of the metal [27]. Second, the torque exerted at the interface appears as a term in the Landau-Lifshitz form and not in the Gilbert one [25]. Therefore if we rewrite it in the Gilbert form we get Eq.(2) with α' instead of α and we get the two terms for the torque of Eq.(3) with

$$\xi_{DL}=\theta_{SH}\frac{g_c\tanh(\hat{d}_N)\tanh(\hat{d}_N/2)}{g_c+g_N\tanh(\hat{d}_N)} \quad (26)$$

and $\xi_{FL}=\alpha'\xi_{DL}$. With the thermodynamic theory the FL term is associated to the effective damping α'.

V. Discussion and conclusion

We have compared two different approaches to compute the spin Hall torque on a ferromagnet. In the first approach one attributes real and imaginary parts to the spin mixing conductance $g_{\uparrow\downarrow}$ and finds that the FL terms is directly associated to the imaginary part $\mathrm{Im}[g_{\uparrow\downarrow}]$. In the second approach, based on the thermodynamic theory [19], the FL term is associated to an effective damping, α', including the intrinsic α of the ferromagnet and the contributions of the interface and of the metallic layer. It is interesting to observe that the FL term is the natural consequence of the presence of the damping term in the dynamic equation of the ferromagnet. To understand the role played by the bulk and by the interface one can take the limit in which the bulk conductance is very large ($g_N\gg g_c$). Both expressions are greatly simplified and it is interesting to compare the ratios ξ_{FL}/ξ_{DL}. For the theory with the spin mixing conductance the ratio is $\mathrm{Im}[g_{\uparrow\downarrow}]/\mathrm{Re}[g_{\uparrow\downarrow}]$ while for the thermodynamic theory is $\alpha'=\alpha+(\mu_B/e)^2 g_c/(\gamma_L M_s d_F)$. In the second case we have an explicit dependence on the conductance of the contact g_c and on the thickness of the F layer d_F. This second type of behavior seems to be the one observed in experiments. In W/CoFeB bilayers with different thicknesses d_F of the F layer, the measured DL and FL efficiencies actually show that the FL one decreases strongly as the thickness is increased, while the DL is practically insensitive to d_F [18]. Future work will be devoted to compare the two different outcomes with more detailed experiments with both metallic and insulating ferromagnets.

Appendix A
Torque with spin mixing conductance

To derive the torque we have to compute $j_{\mathbf{p},\perp}(0)$. We take $\mathbf{e}_p=\hat{\mathbf{z}}$ and define the reference system $(\mathbf{m},\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\varphi}})$ as in Fig.1, where $\hat{\boldsymbol{\varphi}}=\hat{\mathbf{z}}\times\mathbf{m}/|\hat{\mathbf{z}}\times\mathbf{m}|$ and $\hat{\boldsymbol{\theta}}=\hat{\boldsymbol{\varphi}}\times\mathbf{m}$. At the interface we have $j_{e,x}=0$ then from Eqs.(8) we get

$$j_{p,\parallel}=-\frac{4g_\uparrow g_\downarrow}{g_\uparrow+g_\downarrow}\Delta\mu_{p,\parallel} \quad (27)$$

Next we assume that the parallel part of the current is not absorbed (i.e $j_{p,\parallel}=0$ and therefore $\mu_{p,\parallel}(F)=\mu_{p,\parallel}(N)$) and that the perpendicular part is completely absorbed (i.e. $\mu_{p,\theta}(F)=0$ and $\mu_{p,\varphi}(F)=0$). Therefore in Eq.(10) we remain with the potential at the metal side only. By dropping the (N) label and as using $(\boldsymbol{\mu}_p)_\perp=\mu_{p,\theta}\hat{\boldsymbol{\theta}}+\mu_{p,\varphi}\hat{\boldsymbol{\varphi}}$ we get Eq.(10) as

$$j_{p,\theta} = 2\mathrm{Re}[g_{\uparrow\downarrow}]\mu_{p,\theta}+2\mathrm{Im}[g_{\uparrow\downarrow}]\mu_{p,\varphi} \quad (28)$$

$$j_{p,\varphi} = -2\mathrm{Im}[g_{\uparrow\downarrow}]\mu_{p,\theta}+2\mathrm{Re}[g_{\uparrow\downarrow}]\mu_{p,\varphi} \quad (29)$$

The potentials, $\mu_{p,\theta}$ and $\mu_{p,\varphi}$ at the metal side are given by the solution of the diffusion equation for the spin Hall layer (see appendix B). Along the $\hat{\boldsymbol{\varphi}}$ axis we have $j_{p,\varphi}=-g_{N,eff}\mu_{p,\varphi}$ then we get the equation

$$-g_{N,eff}\mu_{p,\varphi}=-2\mathrm{Im}[g_{\uparrow\downarrow}]\mu_{p,\theta}+2\mathrm{Re}[g_{\uparrow\downarrow}]\mu_{p,\varphi} \quad (30)$$

giving

$$j_{p,\theta} = \frac{2\mathrm{Re}[g_{\uparrow\downarrow}]g_{N,eff}+4|g_{\uparrow\downarrow}|^2}{2\mathrm{Re}[g_{\uparrow\downarrow}]+g_{N,eff}}\mu_{p,\theta} \quad (31)$$

$$j_{p,\varphi} = \frac{-2\mathrm{Im}[g_{\uparrow\downarrow}]g_{N,eff}}{2\mathrm{Re}[g_{\uparrow\downarrow}]+g_{N,eff}}\mu_{p,\theta} \quad (32)$$

Along the $\hat{\boldsymbol{\theta}}$ axis we have $j_{p,\theta}=-g_{N,eff}\mu_{p,\theta}-\sin\theta j_{p,S,eff}$ therefore

$$\mu_{p,\theta}=-\sin\theta\frac{2\mathrm{Re}[g_{\uparrow\downarrow}]+g_{N,eff}}{|2g_{\uparrow\downarrow}+g_{N,eff}|^2}j_{p,S,eff} \quad (33)$$

The current $j_{\mathbf{p},\perp}(0)$ is given by Eqs.(31) and (32). By writing them in vector form, using $\sin\theta\hat{\boldsymbol{\theta}}=\mathbf{m}\times(\mathbf{m}\times\mathbf{e}_p)$ and

$-\sin\theta\hat{\boldsymbol{\varphi}} = \mathbf{m} \times \mathbf{e}_p$, we get Eq.(3) with the efficiencies of Eqs. (12) and (13).

APPENDIX B
SPIN HALL EFFECT LAYER

To find the solution of the magnetic moment transport in the N layer we use Eqs.(4) and (5) with electric gradients along y and controlled electric current $j_{e,y}$ and magnetic moment current along x and we have the constitutive equations

$$j_{p,xx} = -\sigma_e \nabla_x \mu_{p,x} \quad (34)$$
$$j_{p,xy} = -\sigma_e \nabla_x \mu_{p,y} \quad (35)$$
$$j_{p,xz} = j_{p,S} - \sigma_{e,\parallel} \nabla_x \mu_{p,z} \quad (36)$$

with a current source $j_{p,S} = \theta_{SH} j_{e,y}$ and $\sigma_{e,\parallel} = \left(1 + \theta_{SH}^2\right)\sigma_e$. As the spin Hall angle is small we can approximate $\sigma_{e,\parallel} \simeq \sigma_e$. To solve equation (39) for the spin Hall layer we have to add the continuity equation (14) that in stationary conditions provides the diffusion equation $l_M^2 \nabla_x^2 \mu_{p,j} = \mu_{p,j}$ characterized by a diffusion length $l_M = (\mu_B/e)\sqrt{\mu_0 \sigma_e \tau_M}$. With a spin Hall layer of finite thickness from $x = -d_N$ and $x = 0$ in which the left side is left open, i.e. $j_{p,j}(-d_N) = 0$, we have at $x = 0$

$$j_{p,xx}(0) = -g_{N,eff} \mu_{p,x}(0) \quad (37)$$
$$j_{p,xy}(0) = -g_{N,eff} \mu_{p,y}(0) \quad (38)$$
$$j_{p,xz}(0) = j_{p,S,eff} - g_{N,eff} \mu_{p,z}(0) \quad (39)$$

with $j_{p,S,eff} = j_{p,S} \tanh(\hat{d}_N) \tanh(\hat{d}_N/2)$, $g_{N,eff} = g_N \tanh(\hat{d}_N)$ and $g_N = \sigma_e / l_M$ is the conductance for the magnetic moment in the bulk. By adding the contact with $\eta = 1$ and writing the equation in vector form we get Eq.(21).

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