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1. Introduction

Low-cost sensors and in particular MEMS (Micro-Electro-Mechanical Systems) devices are widely used in many applications, including consumer electronics, healthcare, automotive, and industrial automation. Their production, which is typically in the order of millions per week in a single factory, involves the calibration of these devices on a large scale which can be costly and time-consuming. This is a significant challenge for manufacturers who need to guarantee the required traceability to SI. To address these challenges, as also requested by the Consultative Committee for Acoustics, Ultrasound, and Vibration of BIPM [1], a solution can be found in the use of statistical process control techniques [2]. Based on probabilistic models that take into account prior knowledge and uncertainties, Bayesian approaches can be very useful for "statistically calibrating" large batches of sensors. In this paper, a Bayesian method is investigated and proposed in this respect: it implies experimentally calibrating a sample of sensors from an unknown batch of larger dimension and using them to estimate the true number of reliable sensors in the whole batch, process that can be considered as a "statistical calibration" of the batch. The advantage of this statistical approach is that it allows for the incorporation of the prior knowledge coming from the calibration of a golden batch representative of the whole production process. This method allows for the calibration of large batch sensors by reducing the number of actual experimental calibrations.

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2. The proposed method in brief

The first step of the proposed method consists in the usual calibration in the laboratory of a golden batch of N sensors, representative of the production process: each sensor is provided with an estimate of its sensitivity S and the associated uncertainty. Using a mixture distribution model to account for the calibration uncertainties of the individual sensors, the overall distribution of the sensitivities of the golden batch is modelled. By setting a bearable probability (p_{def}) of finding out-of-tolerance sensors in the golden batch, the mixture distribution is used to find the minimum and maximum sensitivity limits, S_{min} and S_{max} respectively, encompassing the 1-p_{def} fraction of acceptable sensors. Hence, the expected number of out-of-tolerance sensors in the golden batch is $C_{gold} = p_{def}\, N$. For other/future unknown batches of the same kind of sensors, only a sample of $n < N$ devices from each batch is required to be experimentally calibrated, hence reducing time and cost efforts. The calibrated devices are checked whether their sensitivity is within or outside the out-of-tolerance limits: k indicates that number. The statistical calibration of the whole batch of N sensors is then based on the following Bayesian model. Assuming that the expected number of out-of-tolerance sensors in the unknown batch is equal to that in the golden batch, a binomial prior distribution $P_{prior}(C_{unknown}; N, p_{def})$ is used to model the number of $C_{unknown}$ defective sensors in the unknown batch. This prior (assuming that experimental calibration and manufacturing process do not change from batch to batch) models the state of knowledge of a typical batch from that production. Multiplying the prior by a likelihood function $P_{likelihood}(k; n, C_{unknown}, N)$ defined as a hypergeometric distribution with n sensors, k of which are defective, drawn from the unknown batch of N sensors, $C_{unknown}$ of which are defective, yields an un-normalized posterior $P_{post,un}(C_{unknown}; k, n, N, p_{def})$ for $C_{unknown}$ out-of-tolerance sensors in the unknown batch. Normalizing this distribution leads to a probability mass function for the number $C_{unknown}$ of out-of-tolerance pieces in the unknown batch, provided that k out-of-tolerance devices were found in the small calibrated sub-batch. At this point, it is possible to define different metrics based on the posterior cumulative probability function that can provide an assessment of the reliability and quality of the whole unknown batch.

References

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