Sensitivity Analysis of a Certifiable Synthetic Sensor for Aerodynamic Angle Estimation

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Abstract—Nowadays, some alternative methods exist for the replacement of physical vanes (or probes) for aerodynamic angles (angle of attack and sideslip) with synthetic solutions. The results are promising and there is a growing interest for the industry in this particular solution. However, a lack of methods has been observed to estimate their performance and to compare them. The MIDAS project, funded in the Clean Sky 2 frame, will provide the aerospace community with an innovative modular digital air data system (ADS) based on synthetic sensors for aerodynamic angles. To meet the system requirement specifications given by the project leader, a method of uncertainty estimation must be implemented. This paper proposes a method of estimation of the overall uncertainty based on a consolidated metrological procedure. This method holds a certain degree of generality because it can be applied to different kinds of architecture of the synthetic sensor. In this paper, it has been applied to the preliminary design of the synthetic sensor of the MIDAS air data system and the results have been reported as example.

Index Terms—synthetic sensor, neural network, metrology, uncertainty propagation, flight safety

I. INTRODUCTION

During the last decades, technology and regulations brought to the significant reduction of the aircraft incidents and accidents caused by technical reasons. However, the flight safety still remains an important topic and recent tragedies demonstrate that the physical probes can suffer from exposure to the external agents. The main system addressed by this paper is the ADS (Air Data System), which needs to measure physical quantities that are inherently external to the aircraft. Several research groups proposed solutions to the problem of the estimation of the aerodynamic angles. Some of them are based on explicit mathematical models [1]–[5] whereas others are based on machine learning techniques [6]–[15]. Unfortunately, to the authors’ knowledge, there is a lack of metrological theory behind these estimators. One of the first questions arising on this topic concerns the possibility of ensuring the nominal functioning of the algorithm. However, a design flow that must be followed during the design of a synthetic sensor still does not exist. According to common aeronautical procedures, a set of checkpoints should be defined in order to obtain a reliable design. On the other hand, the second question usually regards the possibility to compare the output of a virtual sensor with the traditional AOA (Angle of Attack)/AOS (Angle of Sideslip) vane. This aspect highly influences the first lack of design flow. In fact, a design procedure can be properly defined only when a set of metrological procedures is identified and used as a reference. This paper shows a preliminary analysis based on consolidated metrological procedures that allow to obtain an uncertainty value that is comparable and repeatable. The general nature of this method is one of its advantages. In fact, it can be applied to different architectures of synthetic sensor, without being strictly related to the one shown in this paper.

This paper focuses on the EU project MIDAS (Modular and Integrated Digital Probe for SAT Aircraft Air Data System). The MIDAS project is funded under Clean Sky 2 to design a modular and integrated digital air data system for the SAT (Small Aircraft Transportation) segment. Sec. II provides a general description of the MIDAS ADS. The method is described in Sec. III. The sensitivity analysis reported in Sec. IV is based on the preliminary design of the synthetic sensor.
sensor showed in [16]–[19].

II. STRUCTURE OF THE MIDAS SYNTHETIC ESTIMATION

The MIDAS ADS schematics is shown in Fig. 1. It mainly consists of two protruding probes and a synthetic (or virtual) sensor. The external probes are a Pitot-Static probe and a TAT (Total Air Temperature) probe. The synthetic sensor allows to complete the so-called air triplet with the evaluation of AOA and AOS. This preliminary design has been already described in [16] and [18].

![MIDAS ADS](image)

Fig. 1. High-level schematic of the MIDAS ADS.

The synthetic sensor is currently based on the Smart-ADAHRS (Air Data, Attitude and Heading Reference System) algorithm that can be disentangled in two main steps:

1) an initial evaluation of the AOA or AOS
2) the evaluation of a correction \( \Delta \alpha \) (respectively \( \Delta \beta \)) to fill the gap between \( \hat{\alpha} \) (respectively \( \hat{\beta} \)) and the real value \( \alpha \) (respectively \( \beta \)).

The correction is evaluated by a NN (Neural Network) properly trained to conduct a sort of calibration of the initial evaluation. The architecture selected is a fully-connected feed-forward MLP (Multilayer Perceptron) corresponding to the map in (1)

\[
[\Delta \alpha, \Delta \beta]^T = f_{VS} (\text{TAS}, \hat{\alpha}, n_x, n_y, n_z, \theta, \phi, p, q, r, \\
\delta_c, \delta_a, \delta_r, \delta_{th}, \Delta_{tk}, \delta_{hs})
\]

where TAS is the true airspeed, \( n_x, n_y, n_z \) are the accelerations measured by the accelerometers respectively in \( X_B \), \( Y_B \) and \( Z_B \) axes, \( \theta, \phi \) are the pitch angle and the roll angle respectively, \( p, q, r \) are the body angular rates, \( \hat{\alpha} \) is the initial estimation for the AOA, \( \delta_c \) is the elevator deflection, \( \delta_a \) is the aileron deflection, \( \delta_r \) is the rudder deflection, \( \delta_{th} \) is the throttle command, \( \Delta_{tk} \) is the difference between the torque on the left and right propellers and \( \delta_{hs} \) is the horizontal stabilizer angle.

Loosely speaking, the MLP contains a set of values called weights used to conduct a series of nonlinear combinations of the input signals. The training operation is mainly related to the optimization of the weights such that an overall metric, evaluated on the entire training set, is minimized. A huge literature exist on this subject and more details on the applied architecture can be found in previous research [20].

In details, in this work a single hidden layer with 24 neurons is trained with the Levenberg-Marquardt rule. As a result of this optimization process based on a single error value, the local estimation error can be unacceptable. For this reason, the sensitivity analysis is important because it can provide indications on the local capability of the synthetic sensor to estimate the desired flight parameter.

III. NONLINEAR UNCERTAINTY PROPAGATION

Mathematically speaking, in this case the MLP is a function of several variables whose elements of the codomain can be both scalar numbers or vectors. Thanks to its structure, if the training is conducted properly, it can represent any function in a given hypercube of definition and hence it can become strongly nonlinear. For this reason, the analysis of the sensitivity of the function cannot be conducted truncating the Taylor series to the first order and linearizing the function.

In metrology there is a common procedure in these cases. The function is tested with a Monte Carlo simulation using a Gaussian distribution on the input variables and analysing the distribution of the output around the nominal values. From a metrological standpoint, uncertainties are often given in terms of coverage factor \( k = 2 \), equivalent to \( 2 \sigma \) for non-Gaussian distributions, which expresses a 95 % confidence interval in the measurement once all relevant sources of uncertainty are considered. For this reason, a coverage factor \( k \) equal to 2 has been considered in this work, assigning the expanded uncertainty value to half of the interval between \( F(x) = 0.97725 \) and \( F(x) = 0.02275 \), where \( F \) stands for the cumulative density function of the error around each nominal point. Piaggio Aerospace, in quality of project leader, required that the uncertainties of the final estimation on AOA/AOS would depend on the value of the aerodynamic angles itself. As can be seen from Fig. 2, the AOA/AOS envelope has been divided in 2 regions, a primary zone with

\[
E_1 = \{ \alpha, \beta \in \mathbb{R} | 0^\circ < \alpha < 15^\circ, -5^\circ < \beta < 5^\circ \}
\]

and a secondary zone with

\[
E_2 = \left\{ (\alpha, \beta) \in \mathbb{R}^2 | A \left( \begin{array}{c} \alpha \\ \beta \end{array} \right) \leq b \right\} \setminus E_1
\]

where

\[
A = \begin{bmatrix}
-1 & 0 \\
1 & 0 \\
0 & -1 \\
1 & -1 \\
1 & 1 \\
-1 & -6 \\
-1 & 2 \\
-1 & 3 \\
\end{bmatrix}
\quad \text{and} \quad
b = \begin{bmatrix}
6 \\
20 \\
15 \\
25 \\
15 \\
9 \\
9 \\
\end{bmatrix}
\]

(4)
However, any pair \((\alpha, \beta)\) can represent several flight conditions. For sake of clarity, it is not specified, for instance, if the flight is stationary or there is a linear or angular acceleration. For this reason, the resultant distribution of uncertainty values will be analyzed statistically, if there are enough points (here corresponding to different flight instants) to define a distribution. The dispersion of the uncertainty value is also of great importance. In fact, if several flight conditions are grouped inside the same \(\alpha, \beta\) bin, it is interesting to study if the flight condition has an effect on the final uncertainty. Statistically speaking, it must be checked if the expanded uncertainty at a given pair \((\alpha, \beta)\) is biased from the flight mechanics point of view. Finally, the obtained charts are compared with the estimation error, to try to understand if the classical approach gives at least an estimation of the uncertainty.

Due to the structure of the estimator, this analysis is repeated twice. In fact, although the expanded uncertainty is metrologically important, the effect of the NN on the initial estimation can greatly help the design of the virtual sensor.

A total number of 6 charts per estimated parameter are obtained, as reported in TABLE 1.

### TABLE I  
**ANALYSIS RESUME**

<table>
<thead>
<tr>
<th>(\Delta \alpha - (\Delta \alpha)_{nom})</th>
<th>Expectation Expanded uncertainty value</th>
<th>Dispersion of the expanded uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \beta - (\Delta \beta)_{nom})</td>
<td>(k = 2)</td>
<td>(k = 2)</td>
</tr>
<tr>
<td>(\alpha - (\alpha)<em>{nom},\beta - (\beta)</em>{nom})</td>
<td>Expectation Expanded uncertainty value</td>
<td>Dispersion of the expanded uncertainty</td>
</tr>
<tr>
<td></td>
<td>(k = 2)</td>
<td>(k = 2)</td>
</tr>
</tbody>
</table>

IV. RESULTS

This section shows some preliminary results of the analysis. The data has been provided by the project leader based on the Consortium requirements. The origin of the data is an high-fidelity flight simulator, which considers delays and noises of the system. Following the methodology described in Section III, a synthetic sensor for AOA has been analysed. Fig. 3 and 4 allow a series of important observations. First of all, the expanded uncertainty is not compatible with the project specification in every zone of the AOA/AOS. In particular, the region with \(\alpha < 5^\circ\) and \(\beta < -5^\circ\) is characterized by higher uncertainty. The same reasoning can be conducted for \(\beta \approx 0^\circ\) and \(5^\circ < \alpha < 8^\circ\). Second, the expected value of the distributions around each nominal point is close to \(0^\circ\) for AOA, which represents a certain degree of radial symmetry of the nonlinear function. The same value is slightly higher for the AOS estimator. Third, the dispersion of uncertainty due to different flight condition is generally lower than \(1^\circ\) for both estimators with peak at \(3^\circ\) for AOA and \(4^\circ\) for AOS. This means that the obtained uncertainty could not be considered valid for any flight condition at the given \((\alpha, \beta)\) pair. Actually, further research must be conducted on this aspect. In fact, this dispersion might be solved using a bigger data-set. This consideration is also supported by the concentration of this behaviour in some particular regions of the envelope. Last, the architecture of the virtual sensor as sum of two terms does not affect the final results and the major source of uncertainty comes from the NN, as expected. In fact, there are very few differences between the columns of Fig. 3 and Fig. 4.

V. CONCLUSIONS

Nowadays, a growing interest in the synthetic sensors for AOA/AOS is observed. Unfortunately, in literature a very few example of them considers a metrologically valid sensitivity analysis. To lead to the definition of a reliable design process for synthetic sensors, a set of metrological procedures must be adopted. This paper proposes the application of a classical method for the estimation of the expanded uncertainty of a nonlinear estimator. The method has been applied to the estimators for both AOA and AOS designed for the certifiable MIDAS probe, resulting from a Clean Sky 2 project. As expected, the obtained uncertainty values demonstrate the fact that the simple estimation error is not enough for the complete definition of the performance of the sensor. Some regions on the AOA/AOS showed an higher uncertainty, suggesting a redefinition of the training set. Moreover, the standard deviation of the expected uncertainty has been given, showing a certain bias of the results. This effect comes from the project specifications, which required to join every flight condition corresponding to a given pair \((\alpha, \beta)\) in the same point of the chart. Further research must be conducted to understand if this bias can be reduced using bigger data-sets or not.

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Fig. 3. Results for the AOA estimator on the AOA/AOS envelope

REFERENCES


Fig. 4. Results for the AOS estimator on the AOA/AOS envelope.


