Comment on “Physics without determinism: Alternative interpretations of classical physics”

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Comment on “Physics without determinism: Alternative interpretations of classical physics”,

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Abstract

The paper “Physics without determinism: Alternative interpretations of classical physics” [Phys. Rev. A, 100:062107, Dec 2019] defines finite information quantities (FIQ). A FIQ expresses the available information about the value of a physical quantity. We show that a change in the measurement unit does not preserve the information carried by a FIQ, and therefore that the definition provided in the paper is not complete.

The expression of the state of knowledge about a measurand as a probability distribution (or some summary of it, such as its mean and standard deviation) is the conventional approach for expressing a measurement result [1–4]. However, it does not intuitively parallel the much more immediate concepts of “certain” and “uncertain digits” that every experimentalist feels when taking note of a measurement outcome in the lab notebook.

In [5], Del Santo and Gisin introduce the concept of finite information quantities (FIQ). A FIQ ranging in the interval $[0, 1]$ is expressed by the binary number $Q = 0.Q_1Q_2Q_3\ldots$, where the individual bits $Q_k$ are Bernoulli random variables having propensities $q_k$ for the realisation of the case $Q_k = 1$. A specific FIQ $Q$ is thus defined by the vector of propensities $q = [q_1, q_2, \ldots, q_k, \ldots, q_M, \frac{1}{2}, \frac{1}{2}, \ldots]$ of its bits $Q_k$; it is assumed that $q_k = \frac{1}{2}$ for $k > M$, i.e., all bits beyond position $M$ have a 50% propensity of being either 0 or 1 and therefore carry no information. Only a finite number $M$ of propensities are needed to specify $Q$.

The FIQ concept is very appealing and it is tempting to adopt it to express the value and uncertainty of a quantity as an alternative to probability distributions. However, for the concept of FIQ to become a practical alternative to the current way of representing the state of knowledge about a quantity, it is mandatory that calculations with them be possible and, hopefully, simple.

Consider for example the expression of the value of a quantity, traditionally written as $Q = \{Q\}[U]$, where $\{Q\}$ is the numerical value and $[U]$ is the unit. Changing the unit to $U' = U/L$, $L$ being a constant, implies $Q = \{Q'\}[U']$, with $\{Q'\} = L\{Q\}$. So, even such an elementary transformation as the change of measurement unit implies the multiplication of a FIQ by a constant.

Indeed, the FIQ definition suggests that it is possible to identify simple, practical calculation rules operating on the finite (and, intuitively, small) number of indeterminate bits and their propensities; rules suitable to be converted in efficient computation algorithms.
The arithmetic relevant to a unit change (Appendix A) shows that the transformation $Q' = LQ$ generates bits $Q'_k$ of $Q'$ which are not mutually independent even if the original $Q_k$ bits are independent. Therefore, expressing $Q'$ by providing only the propensities $q'_k$ of its individual bits deletes some of the original information.

Random variables $Q$ with independent binary digits $Q_k$ have been considered in mathematical literature [6–8]. In general, $Q$ has a ‘reasonable’ probability density function (pdf) only if the $q_k$ satisfy strict conditions, and in that case the pdf is necessarily an exponential [6]; otherwise, it becomes a fractal [7], hence difficult to associate with a physical quantity.

In conclusion, it appears that a specification of the state of knowledge about a quantity $Q$ by means of a FIQ should also include information on the dependencies among the $Q_k$, and therefore that, although the FIQ concept might be physically sound and useful, its definition as given in [5] is not complete, and deserves further development.

**Appendix A: Minimal FIQ maths**

A FIQ arithmetics can be established by generalizing operations on binary numbers. The sum $S = Q + R = 0.S_1S_2S_3\ldots$ of two FIQs, $Q = 0.Q_1Q_2Q_3\ldots$ and $R = 0.R_1R_2R_3\ldots$, is given by the full adder rule, Tab. I.

<table>
<thead>
<tr>
<th>$Q_k$</th>
<th>$R_k$</th>
<th>$C_{k+1}$</th>
<th>$S_k$</th>
<th>$C_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</table>

**TABLE I.** Binary full adder truth table. $C_k$ is the carry bit.

If $q$ is the vector of propensities associated with $Q$, and $r$ with $R$, then under the as-
assumption of independence of $q_k$ and $r_k$, the propensity $s_k$ of each sum bit $S_k$ can be written as the sum of the four propensities of the $S_k = 1$ cases in Tab. II.

\[
 s_k = (1 - q_k)(1 - r_k)c_{k+1} + (1 - q_k)r_k(1 - c_{k+1})
 + q_k(1 - r_k)(1 - c_{k+1}) + q_k r_k c_{k+1}
 = q_k + r_k + c_{k+1}
 - 2(q_k r_k + q_k c_{k+1} + r_k c_{k+1}) + 4q_k r_k c_{k+1}
\] (A1)

and similarly the propensity $c_k$ of the carry bit $C_k$ is

\[
 c_k = q_k r_k + q_k c_{k+1} + r_k c_{k+1} - 2q_k r_k c_{k+1}
\] (A2)

For example for the case $c_{k+1} = \frac{1}{2}$, we have $s_k = \frac{1}{2}$ and $c_k = \frac{1}{2}(q_k + r_k)$: the information provided by $q_k$ and $r_k$ is transferred, through the carry bit $C_k$, to bit $S_{k-1}$.

Multiplication by a deterministic constant $L$ can be performed by repeated shifting and addition. Table II gives a simple example. If $P = LQ$, where $q = [0, 0, q_3, \frac{1}{2}, \ldots]$ and $L = (11)_2 = (3)_{10}$, then

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>0.0Q_3\ldots</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.0Q_3\ldots</td>
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<tr>
<td></td>
<td>0.0</td>
<td>0.0Q_3Q_4\ldots</td>
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<tr>
<td></td>
<td>0.0</td>
<td>0.0P_1P_2P_3\ldots</td>
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</table>

**TABLE II.** Multiplication table, $P = LQ$ where $Q = 0.0Q_2Q_3\ldots$ and $L = (11)_2 = (3)_{10}$.

$L = (11)_2 = (3)_{10}$, then

\[
 p_1 = \frac{1}{2}q_3^2 + \frac{1}{4}q_3,
 p_2 = q_3 - q_3^2 + \frac{1}{4},
 p_3 = \frac{1}{2}, \ldots
\] (A3)

The propensity of occurrence of specific digit couples can also be computed. For example, denoting as $p_{12}$ the propensity of the event \{ $P_1 = 1, P_2 = 1$ \} we have $p_{12} = 0$ (to have $P_1 = 1$, it should occur that $Q_3 = 1$ and $C_3 = 1$ at the same time, hence $C_2 = 1$. However,
the case \(\{Q_3 = 1, C_3 = 1\}\) always generates \(P_2 = 0\), so \(\{P_1 = 1, P_2 = 1\}\) is never possible. Since \(p_{12} = 0 \neq p_1 p_2\), bits \(P_1\) and \(P_2\) are not independent.


