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Metrologia

ACCEPTED MANUSCRIPT

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To cite this article before publication: Fabio Saba *et al* 2020 *Metrologia* in press https://doi.org/10.1088/1681-7575/abc410

IOP Publishing Journal Title
Journal **XX** (XXXX) XXXXXX https://doi.org/XXXX/XXXX

A dynamic gravimetric standard for liquid flow measurements

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Received xxxxxx
Accepted for publication xxxxxx
Published xxxxxx

Abstract

An improved primary method for the calibration of liquid flow meters based on dynamic weighing has been adopted at the Istituto Nazionale di Ricerca Metrologica (INRIM). Compared to the other dynamic methods currently in use, based on submerged liquid jet flows, the proposed method has the advantage of not requiring the correction for the hydrostatic buoyancy force exerted on the immersed tube. This is possible since the immersion depth of the tube is held constant during the dynamic weighing process, entailing, consequently, a constant back pressure at the outlet section of the immersed tube and a more stable liquid meniscus at the tube outer surface, which both improve the accuracy of measurement. Although at present the proposed apparatus has been tested in the range between 10 kg h^{-1} and 60 kg h^{-1}, this improved dynamic method would be particularly suitable for lower flow rate as well. The results have shown that relative uncertainties of the order of 0.01% can be obtained.

Keywords: dynamic weighing, liquid flow standard, flow meter calibration

1. Introduction

A primary calibration system for liquid flow meters allows determining the amount of a specific type of liquid, in terms of mass or volume, which passes through the flow meter in a given time interval, under specific measurement conditions, namely flow rate, temperature, and pressure.

The first classification of primary methods for liquid flow rate measurement is based on the definition of the reference standards, i.e. either mass or volume flow, which yields to the identification of gravimetric and volumetric methods, respectively. Although the two methods could be considered equivalent, provided that the density of the liquid is known, the gravimetric method is the one that normally allows obtaining more accurate measurements. In fact, typically, the volume of reference standards used in volumetric methods is derived from gravimetric measurements, rather than from direct dimensional measurements. In addition, in volumetric methods, proper corrections are required to compensate for the effects of thermal expansion and compressibility of the liquid. On the other hand, even if the gravimetric method would be more accurate and easier to carry out, also thanks to modern electronic balances, the accuracy strongly depends on a proper system to collect the liquid to be weighed and on the methods used to reduce evaporation losses.

The gravimetric methods can be classified according to the way in which the liquid is weighed, that is either statically or dynamically. In the static weighing, the most common method is based on the measurement of the mass of liquid that has passed through the flow meter, having defined the start and stop flow rate conditions during the test. The two commonly used conditions are the following: 1) the test begins and ends with zero flow rate, 2) the test begins and ends with a constant flow rate.

In the first method, which is obviously simpler than the second, it is not possible to maintain constant flow rate and pressure during the whole duration of the test. For this method, known as “standing start and stop method”, the test duration is defined by an on-off valve. Flow meter calibration carried out by such a method is affected by the initial and final flow transients, due to the fast opening and closing of the valve, whose effects can be reduced by sufficiently extending the duration of the test, with respect to the response time and possible sensitivity to pressure pulses of the flow meter under test.

With the second method, commonly known as “flying start and stop method”, tests are carried out without ever stopping the flow. Each test starts when the steady-state flow is diverted into the weightank and ends when a suitable diverting device redirects the flow outside the weightank. The diverter may be either a simple three-way valve, or a sophisticated mechanism that quickly diverts a free liquid jet. Hence, with this method, the forth and back operation of the diverter system defines the duration of the test. Although this method, in principle, is not affected by the response time of the flow sensor, it is not easy to completely avoid the influence of the flow diversion system on measurement accuracy. As the system cannot instantly divert the flow, the so-called timing error arises if the start and stop positions of the diverter are not properly identified and set in order to correctly gate the timer. Timing errors introduce both random and systematic components to the uncertainty. Several experimental methods allow providing an estimate of this error [1] (whose average amount can be corrected for), and of the associated uncertainty components. However, the accuracy of such determinations depends on many factors, such as the flow rate value, its stability, operation and repeatability of the diverter, turbulences in the jet flow, and velocity profile over the jet cross section [2]. As for the standing start and stop method, the uncertainty contribution due to transients can be made negligible by sufficiently lengthening the duration of the test. However, this approach limits the testing capability in terms of both maximum flow rate and productivity.

In dynamic methods, the weighing is performed while the liquid flow is collected into the weightank. Besides being faster than the static methods, they are also easier to be implemented, as they do not require the realization and characterization of flow diverter systems. Although dynamic methods have no limits on flow rate, they show a great effectiveness for very low flows and have been recently developed in many national metrological institutes for the calibration of medical drug delivery systems [3].

Dynamic methods are no longer affected by the measurement error associated to the flow diverter system, but on the other hand, the weighing is influenced by the dynamic response of the balance and the disturbances caused by the motion of the liquid inside the weightank. Moreover, concerning very low flows (down to the mg h^{-1} range), the variation of capillary and meniscus forces exerted by the liquid during the dynamic weighing process plays an important role on measurement accuracy, so that several solutions have been proposed to reduce their effects [4-6].

Dynamic methods can be classified into two main categories, namely the methods in which the liquid jet outflows in free atmosphere before entering the weightank, and the methods in which the liquid is collected by means of an immersed tube. The main difference is that in free jet methods, the measurement model does not require any mass correction except for the aerostatic buoyancy, as the variation of the impact force of the jet is fully compensated by the weight of the jet column intercepted between the initial and final liquid levels [7-9]. On the other hand, in immersed jet methods, it is necessary to correct the measurement for the hydrostatic buoyancy force exerted by the liquid on the immersed tube [10]. More details about these typical dynamic methods are given in the following sections.

A different approach has been adopted at INRIM, which allows performing dynamic flow measurements by means of an immersed jet method, characterized by the advantage of not requiring the correction for the variation of the hydrostatic buoyancy force on the submerged tube. This is possible since the immersion depth of the tube is held constant during the dynamic weighing process, entailing, both a constant back pressure at the outlet section of the immersed tube and a more stable liquid meniscus at the tube outer surface. This improved dynamic method is based on the approach of “overflowing an inner vessel inside a weighing vessel”, which has been adopted for micro flow rate delivery and measurement [11]. The method of the inner overflow vessel has been also used for the upgrade of the small liquid hydrocarbon facility at the National Metrology Institute of Japan (NMIJ), where flow measurements are carried out by static methods [12,13].

The structure of this paper is as follows. In section 2, we describe the metrological key concerns of the two main categories of dynamic weighing methods. In section 3, the improved dynamic calibration method is described, as well as the associated uncertainty. The analysis of the experimental results is given in section 4. Conclusions and an outlook to future work are set out in section 5.

2. Dynamic weighing methods for liquid flow rate measurement

The measurement principle of dynamic gravimetric methods for liquid flows is based on the measurement of the time variation of the weighing force associated to the mass of liquid being collected into a weightank. Unlike static gravimetric methods, where the liquid flow is first collected into the weightank for a certain time period and then weighed, in dynamic methods the liquid is weighed during flow collection, in order to evaluate the change over time in the mass of liquid poured into the weightank.

The systems used for dynamic flow measurement can be basically divided into those in which the liquid jet is freely outflowing from a nozzle before entering the weightank, and those in which the measurement is performed with the liquid jet outflowing from a tube immersed under the liquid surface. The main difference in terms of measurement is that in the immersed tube method, the additional effect associated to the hydrostatic buoyancy force exerted by the liquid on the submerged filled tube must be taken into account.

The measurement models associated with the typical dynamic methods used for liquid flow measurement, will be briefly described in sections 2.1 and 2.2.

2.1. The free jet method

The dynamic methods based on the filling up of the weightank by means of a liquid jet freely outflowing from a nozzle placed above the liquid surface, are characterized by the variation of the impact force of the jet impinging on the liquid surface. However, such a variation of the vertical momentum of the jet during the dynamic weighing process is fully compensated by the additional weight of the jet column intercepted between the initial and final level inside the weightank. In fact, in flow meter calibration by free jet methods, the additional mass of the jet between the two levels is eventually weighed by the balance, but it is not measured by the upstream meter under test during the test itself [9], as the liquid level inside the weightank rises along the jet boundary surface. Figure 1 shows the scheme of a dynamic weighing system adopting the free jet method, where the additional mass of the jet between the initial and final liquid levels is highlighted.

Assuming a constant liquid mass flow rate $\dot{m}$, the measurement model for the determination of the mass of liquid Δm collected into the weightank over the dynamic weighing period Δt, where $\Delta m = \dot{m}\Delta t$, can be expressed as follows

$$\Delta m = \Delta m_{\mathrm{w}} \frac{1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{c}}}}{1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}}} - \frac{\dot{m}\Delta v}{g\left(1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}}\right)} - \frac{\dot{m}\Delta\tau}{1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}}} + \delta m_{\mathrm{t}} + \delta m_{\mathrm{ev}} \ . \quad (1)$$

Looking at the RHS of equation (1):

- the first term is the difference between the final and the initial mass readings provided by the balance Δm_{w}, multiplied by the air buoyancy correction factor, which is calculated from the densities of air ρ_{a}, liquid ρ_{w}, and mass standards used to calibrate the balance ρ_{c}.

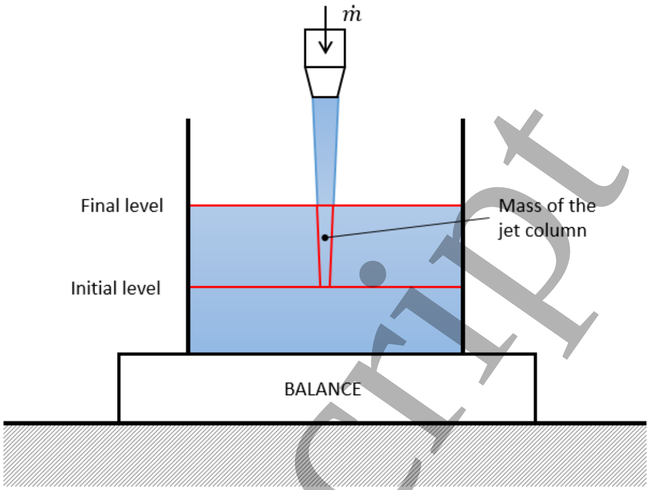

Figure 1. Scheme of a dynamic weighing system adopting the free jet method.

- the second term is the mass correction for the variation of the impact force of the jet, where Δv is the difference between the impact velocities of the jet normal to the liquid surface at the final and initial levels inside the weightank. Since the liquid jet falls from the nozzle outlet section into the weightank, the impact velocity is higher at the beginning of weighing, and decreases during the collection interval as the liquid level increases. The variation of the impact velocity depends on the distance between initial and final liquid levels inside the weightank, the outflow velocity of the jet from the nozzle, and the air friction at the jet boundary.
- the third term represents the mass correction for the additional weighed amount of liquid corresponding to the mass of the jet column between the initial and final levels inside the weightank. The mass of the jet between the two levels is calculated as the product between the constant mass flow rate $\dot{m}$ and the time $\Delta\tau$ spent by the jet to travel from the final to the initial level. Such a mass correction must be considered in the model because the mass of the jet column is weighed by the balance, but it is not measured by the upstream meter under test during the test itself.
- the fourth term δm_{t} accounts for possible net change of vertical components of momentum over the dynamic weighing period, caused by the motion of the liquid inside the weightank. This is mainly due to air bubbles entrained by the jet, which cause a net vertical motion of the center of mass of the liquid during the collection interval [9]. A rigorous analysis of this effect is difficult to be considered in the model, and the mass correction δm_{t} is usually evaluated according to approximated models.

- finally, the last term δm_{ev} represents the mass of liquid evaporated during the dynamic weighing period, which is usually reduced by ensuring almost saturated air conditions around the weightank.

As noticed by Shafer and Ruegg in 1958 [7], the mass correction for the variation of the impact velocity of the jet is equal and opposite to the correction for the mass of the jet column, as $\Delta v = -g\Delta\tau$ (neglecting the air friction at the jet boundary), and the measurement model of equation (1) can be simplified as follows

$$\Delta m = \Delta m_{\mathrm{w}} \frac{1 - \frac{\rho_{\mathrm{a}}}{\rho_{\mathrm{c}}}}{1 - \frac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}}} + \delta m_{\mathrm{t}} + \delta m_{\mathrm{ev}}\ . \qquad (2)$$

It is worth observing that the rising level of the liquid inside the weightank is associated to a constant momentum component in the ascending vertical direction, which is normally cancelled in the evaluation of the mass variation Δm. This is true if the mass flow rate is kept constant during the collection interval and the cross sections of the weightank does not vary with the liquid level. The cross section of the jet is usually negligible compared to that of the weightank and its variation between the initial and final levels does not change the vertical momentum component associated to the rising liquid level.

Although the measurement model of equation (2) looks simple and easy to use, it should be noted that the mass correction δm_{t} and the associated uncertainty are difficult to be correctly estimated and may affect the accuracy and repeatability of measurements. In order to avoid the disturbances caused by air entrainment, dynamic methods based on immersed jet flows can be used.

2.2. The immersed jet method

The dynamic methods based on on the filling up of the weightank by means of a tube immersed under the liquid surface, are characterized by the variation of the hydrostatic buoyancy force acting on the immersed filled tube during the collection interval, caused by the rising liquid level inside the weightank.

Figure 2 shows the scheme of a dynamic weighing system adopting the immersed jet method, highlighting the volume of the submerged filled tube between the initial and final liquid levels, which must be considered for the hydrostatic buoyancy correction.

With the usual immersed jet methods, the flow rate may decrease as the back pressure on the outlet section of the submerged tube increases, owing to the gradual filling up of the weightank. However, if the flow control system of the facility is accurate enough, vertical momentum components exerted by the outflowing jet remain constant and no account

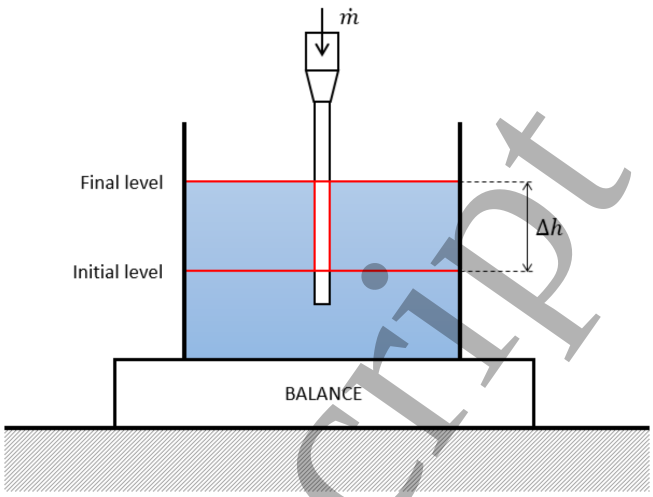

Figure 2. Scheme of a dynamic weighing system adopting the immersed jet method.

nor corrections must be made for them [10]. Moreover, in order to further reduce the vertical impact force of jet, in the immersed jet method it is also advisable to discharge the submerged liquid flow horizontally, through an outlet elbow.

An important advantage compared to the free jet method is that the contribution due to entrained air bubbles is avoided and the motion of the homogeneous fluid inside the weightank cannot cause a time-dependent net vertical motion of the center of mass of the liquid during the weighing period.

As well as for the free jet method, the rising level of the liquid inside the weightank is associated to a constant momentum component in the ascending vertical direction, which does not affect the flow measurement. This is true if the mass flow rate is kept constant during the collection interval and the cross sections of both the weightank and the immersed tube does not vary with the liquid level.

Thus, in the immersed jet method, flow measurements are usually not affected by net vertical momentum components, as they can be considered constant during the dynamic weighing process.

The effect of the surface tension of the liquid at the tube external surface may contribute to the repeatability of measurements, because of the changes of the meniscus shape during the level rise. For example, considering a tube diameter of about 6 mm, this contribution in terms of corresponding variation of mass readings may be in the order of tens of milligrams. However, provided that the level moves at constant velocity, the variation of the shape of the meniscus is mostly random, so that the corresponding systematic effect is negligible and its randomness may only contribute to the stability and repeatability of measurements.

This method is widely used for very low flow rate calibrations, where evaporation traps, or oil layers spread above the liquid surface are used to reduce evaporation losses [4,5,14].

The measurement model can be written as

$$\Delta m = \Delta m_{\mathrm{w}} \frac{1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{c}}}}{1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}}} - \frac{\rho_{\mathrm{w}} A_{\mathrm{p}} \Delta h}{1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}}} + \delta m_{\mathrm{s}} + \delta m_{\mathrm{ev}}\ . \quad (3)$$

Looking at the RHS of equation (3):

- the second term represents the mass correction for the variation of the hydrostatic buoyancy force exerted by the liquid on the immersed filled tube, where Δh is the variation of the immersion depth of the tube during the time period Δt, which can be expressed as follows

$$\Delta h = \frac{\Delta m}{\rho_{\mathrm{w}}\left(A_{\mathrm{t}} - A_{\mathrm{p}}\right)}\,, \quad (4)$$

where the cross section of the immersed tube A_{p} (calculated at its outer diameter) and the cross section of the weightank A_{t} are supposed not to vary with the liquid level. It is worth observing that the hydrostatic buoyancy force is exerted on the total immersed volume of the filled tube, which includes both the tube wall and inner liquid volume.

- the third term δm_{s} is the mass correction for the variation of the meniscus force at the tube outer surface during the dynamic weighing period. Since the variation of the shape of the meniscus is mostly random during the collection interval, δm_{s} can be regarded as a zero mean contribution, whose uncertainty may affect the accuracy and repeatability of measurements.

Since the cross section A_{p} is usually negligible compared to A_{t}, equation (3) can be approximated as follows

$$\Delta m = \Delta m_{\mathrm{w}} \frac{1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{c}}}}{1-\dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}}} - \Delta m_{\mathrm{w}} \frac{D_{\mathrm{p}}^2}{D_{\mathrm{t}}^2} + \delta m_{\mathrm{s}} + \delta m_{\mathrm{ev}}\ , \quad (5)$$

where D_{p} is the outer diameter of the immersed tube and D_{t} is the inner diameter of the weightank. Both equations (3) and (5) can be used indifferently in typical dynamic measurement systems [15,16], provided that A_{p} is negligible compared to A_{t}.

Neglecting the air buoyancy correction, it can be noted that the mass correction associated to the hydrostatic buoyancy force on the immersed tube, normalized with respect to the measured mass of liquid Δm, is equal to $A_{\mathrm{p}}/\left(A_{\mathrm{t}} - A_{\mathrm{p}}\right)$. Figure 3 shows the values of the normalized hydrostatic buoyancy correction, as a function of the ratio $D_{\mathrm{p}}/D_{\mathrm{t}}$.

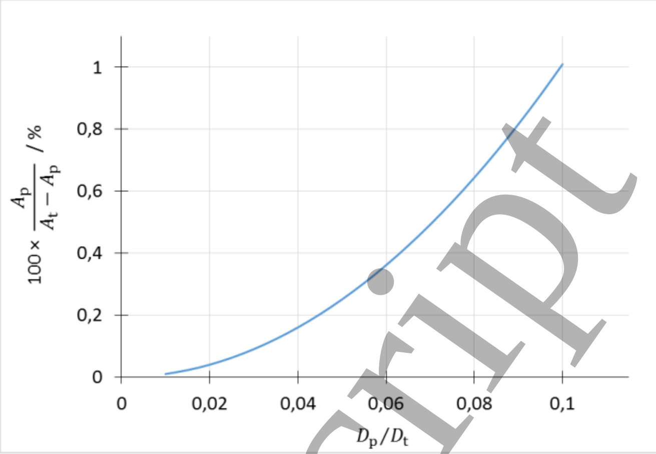

Figure 3. Normalized hydrostatic buoyancy correction as a function of the ratio between the outer tube diameter and the inner weightank diameter.

The uncertainty associated to hydrostatic buoyancy correction depends on the uncertainties associated to the measurements of the diameters D_{p} and D_{t}, which should be carried out at different heights; typically, it is lower than 0.01%. For example, considering D_{p}= 6 mm and D_{t}= 130 mm, with an uncertainty of the tube diameter equal to 0.02 mm and an uncertainty of the weightank diameter of 1 mm, the relative standard uncertainty contribution associated to the mass correction for the hydrostatic buoyancy force is around 0.004%.

The mass correction for the hydrostatic buoyancy force on the immersed filled tube can be also evaluated experimentally, by measuring, for different quantities of liquid collected into the weightank, the increase in the balance reading obtained after the immersion of the tube down to the position used during tests (the tube must be preliminary plugged keeping the same volume). The measured mass difference corresponds exactly to the mass correction for the hydrostatic buoyancy force, which can be usually expressed as a linear function of the mass reading provided by the balance. This method allows evaluating the hydrostatic buoyancy correction for any shape and layout of both the weightank and the immersed tube, provided that the system used for the positioning of the immersed tube is able to ensure the same alignment and position as used during tests, within a maximum acceptable uncertainty. This experimental method is effective in reducing the uncertainty associated with the estimation of the hydrostatic buoyancy correction; an example is shown in section 4.2.

3. Improved dynamic method for water flow rate measurements

An improved dynamic weighing method has been adopted at INRIM to be considered as the primary standard for water flow rate measurements lower than 40 kg h^{-1}, which is the current lower flow limit of the INRIM static gravimetric standard based on the flying start and stop method.

The proposed method is based on the immersed jet approach, but exhibits the advantage, also in terms of uncertainty evaluation, of not requiring the mass correction for the variation of the hydrostatic buoyancy force on the submerged tube.

3.1. Measurement model

In the proposed dynamic method, a constant and stable water flow rate is collected into the weightank by means of a tube kept at constant immersion depth. This is made possible as water overflows from a constant level container installed inside the weightank, in which the outlet tube is immersed. By this simple solution, both the hydrostatic buoyancy force exerted on the submerged tube and the vertical component of momentum are kept constant during the dynamic weighing process. This type of weightank, consisting of an inner overflow container, has been adopted in dynamic weighing for low liquid flows by Su et al. [11], who used a low volatility oil layer spread above the liquid surface to reduce evaporation losses. The approach of “overflowing an inner vessel inside a weighing vessel” has been also applied to both the static standing start and stop method [12], and the static flying start and stop method [13]. However, in static methods, the use of an inner overflow container installed inside the weightank introduces possible uncertainty sources due to the slight change of both the immersion depth of the feeding tube and the meniscus force, at the initial and final readings of the balance. Although, in static methods, some solutions need to be applied to avoid this uncertainty [12,13], on the other hand, in dynamic methods, the use of an inner overflow container is inherently safe from any change of immersion depth and meniscus force between the start and the end of tests, as the weighing is performed while the liquid is collected into the weightank.

Figure 4 shows the functional scheme of the proposed measurement system, where an evaporation trap is used to reduce evaporation losses.

With this method, the transport of momentum, determined by the liquid jet outflowing from the immersed tube into the weightank, induces a constant vertical force component on the balance during the collection interval, whose stability is only affected by the turbulent motion of the liquid inside the weightank. This is possible since the overflowing of the liquid from the constant level container allows keeping a constant back pressure at the outflow section of the immersed tube, thus ensuring a constant and stable liquid flow rate, i.e. a constant vertical momentum component over the dynamic weighing process. This represents an advantage compared to typical immersed jet methods, where the variation of the immersion depth of tube may cause a decrease of the flow rate circulating in the system.

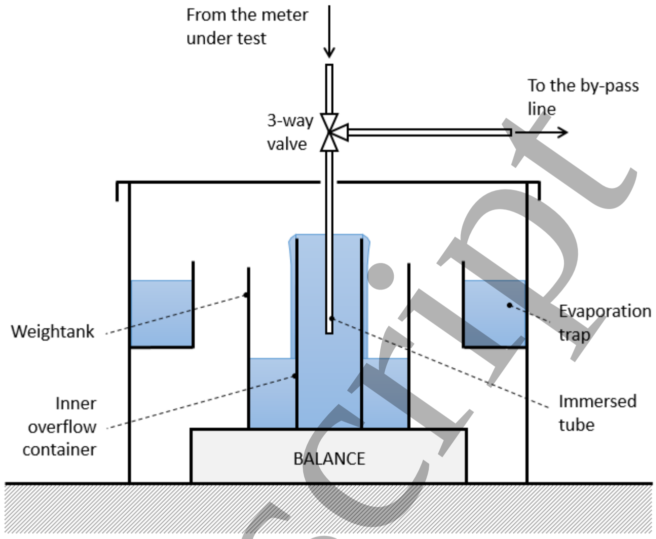

Figure 4. Scheme of the proposed dynamic weighing system.

In addition, another steady vertical force component is associated to the momentum of the annular liquid film flowing down the external surface of the inner overflow container and impinging on the rising liquid surface. This impact force can be considered constant during the weighing process, as the velocity profile of the falling film is fully developed along the most of the height of the container, except for a small portion of the surface close to the edge of the vessel, where the film takes shape. In particular, the formation of the fully developed velocity profile of the film is driven by the force balance between gravity, viscous stress, surface tension, and momentum flow, and is accomplished close to the top edge of the container, where the velocity gradients along the direction normal to the external surface of the vessel are higher. According to the modelling of falling film development in cylindrical coordinates [17], it can be found that the stream-wise distance from the top edge of the inner overflow container, which the annular liquid film must travel before it becomes fully developed, mainly depends on the flow rate and the initial film thickness at the top edge of the vessel. For example, considering the experimental setup of the proposed measurement system, where an inner overflow container of 60 mm diameter is used, and assuming an initial film thickness of 5 mm to remain on the safe side, the ratio between the local and the asymptotic annular film thickness δ/δ_∞ can be plotted as a function of the stream-wise distance from the top edge of the container for different water flow rates in the range from 10 kg h^{-1} to 60 kg h^{-1}, as shown in figure 5. It can be observed that the velocity profile of the annular film can be considered as fully developed, in the whole range of flow rates, after a distance of about 20 mm from the top edge of the vessel, where the film thickness reaches its asymptotic value (δ/δ_∞ < 1.002) and gravity is balanced by viscous stresses.

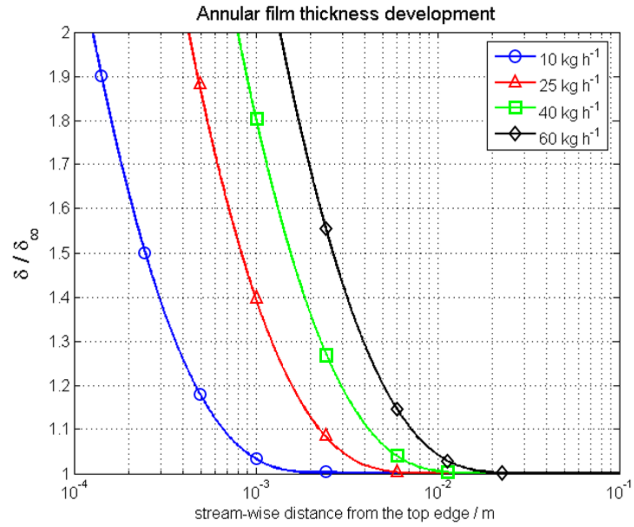

Figure 5. Estimation of the annular falling film thickness development as a function of the stream-wise distance from the top edge of the inner overflow container for different water flow rates (diameter of the cylindrical container equal to 60 mm and initial film thickness equal to 5 mm).

Thus, the measurement model can be written as follows

$$\Delta m = \Delta m_{\mathrm{w}} \frac{1 - \dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{c}}}}{1 - \dfrac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}}} + \delta m_{\mathrm{ev}} \,. \quad (6)$$

It is worth observing that the slow annular liquid film flowing down the external surface of the inner overflow container, and impinging on the rising liquid surface, does not cause air bubbles entrainment, so that the uncertainty contribution accounting for possible net vertical components of momentum δm_{t} can be neglected, as well as for typical immersed jet methods.

Another advantage with respect to typical immersed jet methods, is that the force exerted by the liquid meniscus formed on the tube is more stable, as the shape of the meniscus is not affected by the level rise. Therefore, the uncertainty contribution associated to the variation of the meniscus force during the weighing period δm_{s} can be neglected. However, the disturbances of fluid motion inside the weightank may cause slight variations of the shape of the meniscus during the weighing process, whose contribution is included in the stability and repeatability of measurements.

For this reason, the proposed dynamic method could be particularly appropriated for flow measurements lower than 10 g h^{-1}, where the effects due to the meniscus and the back pressure variation at the outlet section of the immersed tube may significantly contribute to the accuracy and repeatability of measurements.

3.2. Experimental apparatus and measurement procedure

The dynamic weighing of the water flow is carried out by means of an electromagnetic force compensation balance (10 kg full scale, 1 mg resolution), whose mass readings are acquired with a sampling frequency of 5 Hz. The acquisition of the time counter is synchronized with the balance readings much more easily compared to the static flying start and stop method, as time is triggered directly by the balance readouts during the collection of the liquid inside the weightank. However, timing errors are still present also in the dynamic method, as due to the particular response of the balance to the dynamic variation of the weighing force, the fluid motion inside the weightank, and possible synchronization errors in the acquisition of mass and time measurements. Anyway, such errors give a minor contribution on measurement accuracy, as the flow rate is obtained by the linear regression analysis of a sufficiently large number of mass and time readings acquired during the dynamic weighing process. In addition, timing errors can be further reduced by increasing the weighed amount of water.

The weightank consists of a cylindrical glass beaker of 130 mm diameter and 170 mm height, into which an inner overflow cylindrical glass container of 60 mm diameter and 180 mm height has been installed. The top circumferential rim of the inner overflow container is sharp-edged in order to allow forming a uniform annular water film flowing down the external surface of the container.

In order to avoid water losses due to evaporation during the weighing process, the balance and the weightank are enclosed inside an evaporation trap, which allows maintaining an almost water vapour saturated environment in the surrounding of the vessel.

The flow generation and control system is that of the INRIM static gravimetric standard, in which a centrifugal pump is used to generate a steady flow of demineralized water from the reservoir to the weightank. The constant level of the large volume reservoir allows keeping a constant pressure at the pump inlet section during measurements. The water flow and pressure are regulated by means of the electronic speed control of the pump, a flow recirculation bypass, and a control valve installed just before the outlet flow section of the immersed tube. The water temperature is controlled by a heating/cooling system.

The water flow rate outflows vertically into the weighing vessel from a stainless steel tube (6.3 mm outer diameter and 4.3 mm inner diameter), which is immersed into the inner overflow container at about 100 mm immersion depth. A three-way valve is used to bypass the water flow to the reservoir when the weightank must be emptied. Figure 6 shows a picture with the detail of the proposed dynamic weighing system.

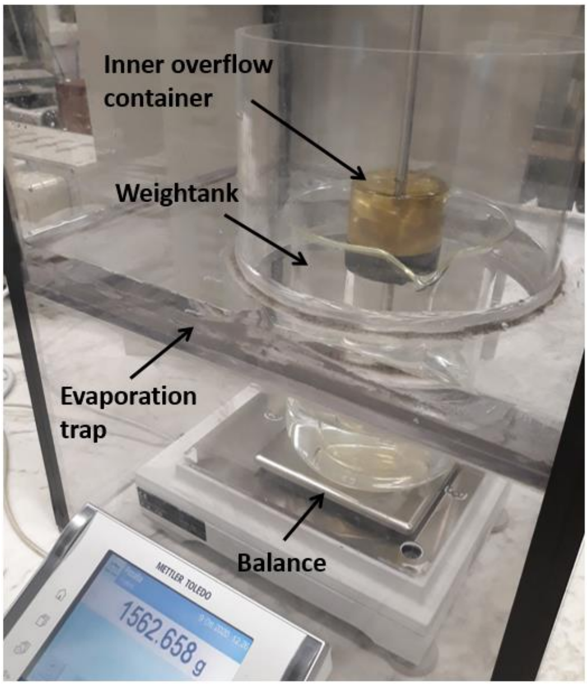

Figure 6. Picture of the dynamic weighing system.

The system has also been tested with the water jet outflowing radially from the immersed tube, in order to reduce the vertical velocity component inside the weightank. The difference between radial and axial submerged jet flows has been observed to be negligible in the range between 10 kg h^{-1} and 60 kg h^{-1}.

Once water flow rate and temperature have reached the desired stability, the tests are started by operating the three-way valve to direct the flow into the weightank. Balance readings, as well as those of the time counter, are acquired after a while, as soon as the liquid has overflowed the inner container and has passed a minimum level with respect to the base of the weightank (about 40 mm to avoid disturbances on balance readings due to the impingement of the annular falling film on the rising water level at the beginning of the dynamic weighing process).

In order to select the proper duration for the acquisition of mass readings, it is important to take into account the timing error contribution due to both the dynamic response of the balance during water collection, and the fluid motion inside the weightank. The maximum errors could occur if the mass flow rate were determined by considering only the initial and final readings of both the balance and the time counter over the test. To avoid this, we determine the mass flow rate $\dot{m}$ by the linear regression analysis of a number of mass and time readings acquired almost simultaneously during the whole dynamic weighing process. The liquid flow rate is obtained by multiplying the slope of the linear regression model by the air buoyancy correction factor and correcting for evaporation losses. In this way, the fluctuations of the mass readings, due to both the turbulence inside the weightank and the dynamic response of the balance, are averaged over the whole weighing process and the timing error contribution can be evaluated as the uncertainty associated to the slope of the linear fit.

The maximum water level inside the weightank, at which the acquisition of the balance readings and the time counter can be stopped, must be within the region of the annular water film characterized by a fully developed velocity profile, i.e. at a minimum distance of 20 mm from the top edge of inner overflow container, in order to keep constant the vertical component of momentum during the weighing process.

3.3. Uncertainty analysis

The uncertainty associated to the measurement of mass flow rate provided by the proposed dynamic method is evaluated by taking into account the measurement model of equation (6) and the linear regression analysis of the measurement data provided by the balance and the time counter during the dynamic weighing process. The main uncertainty sources are described and the typical values of the corresponding uncertainty contributions are given.

3.3.1. Dynamic response of the balance and fluid motion disturbances. As observed in section 3.2, the fluctuation of the mass readings provided by the balance during the dynamic weighing process are due to both the dynamic response of the balance to the continuous variation of the weighing force, and the effects of turbulence and possible entrainment of air bubbles in the collected liquid volume.

The type of dynamic response of the balance depends on the measuring principle used for the measurement of the weighing force. In particular, active measuring principles, like the one adopted by the electromagnetic force compensation balance used in the proposed measurement system, are usually characterized by a slow dynamic response to the variation of the weighing force, due to the active adjustment of the equilibrium position of the balance pan. On the other hand, passive measuring principles, like the ones used by load cells, are based on the relation between the applied force and the corresponding deformation of the sensing element and are usually characterized by a fast response to the dynamic variation of the weighing force, even if the linearity is worse. However, even if, in principle, load cells are more suitable for dynamic weighing systems for liquid flow measurement, the electromagnetic force compensation balance used in the proposed measuring system is appropriate for the measurement of low and stable liquid flows, provided that a sufficient number of mass readings is acquired during the dynamic weighing process. In fact, at low and stable flow conditions, the dynamic response of this kind of balance is such that the fluctuation of mass readings is mostly random with respect to the actual collected mass of liquid, and the corresponding uncertainty contribution can be reduced by increasing the number of measurements. In addition, the sampling frequency used for the acquisition of mass readings should be as high as possible in order to avoid aliasing effects on the sampled signal.

In order to reduce the uncertainty contribution associated with the fluctuations of mass readings, the water flow rate is determined by the linear fit of all the measurement data of mass and corresponding time acquired at regular intervals during the test, using the ordinary least squares method. Thus, the standard uncertainty contribution on water flow rate measurement $u_\mathrm{d}(\dot{m})$, due to the dynamic response of the balance and the fluid motion disturbances, can be evaluated as the uncertainty of the slope of the linear regression model

$$u_\mathrm{d}(\dot{m}) = \frac{\sigma(e_i)}{\sqrt{N}\,\sigma(t_i)}\,, \qquad (7)$$

where $\sigma(e_i)$ is the standard deviation of the residuals $e_i = \tilde{m}_{\mathrm{w},i} - m_{\mathrm{w},i}$, being $\tilde{m}_{\mathrm{w},i}$ the mass calculated by the linear regression model and $m_{\mathrm{w},i}$ the mass reading provided by the balance at the i-th time instant t_i during the dynamic weighing period, $\sigma(t_i)$ is the standard deviation of the readings provided by the time counter, and N is the number of readings. It is worth observing that the standard deviation of fit is calculated as $\sigma(e_i) = \sqrt{SSE/(N-2)}$, where SSE is the sum of the squared residuals.

In the case of measurement of the totalized mass of liquid flowed through an upstream meter under test, the mass readings provided by the balance can be directly fitted against the totalized pulses generated by the meter, and the resulting slope of the linear fit, corrected for air buoyancy and evaporation losses, represents the unit value of the pulse. The uncertainty contribution associated with the fluctuations of mass readings is evaluated by equation (7) as well, by substituting the time readings with the pulse readings.

Figure 7 shows the residuals of fit obtained for a water flow rate measurement of 10 kg h^{-1}, carried out at ambient temperature conditions by means of the proposed dynamic method. The residuals are plotted over a 10 s time window within the dynamic weighing period.

From figure 7, it can be observed that the residuals of fit are distributed along parallel straight lines according to a saw-tooth trend, as also observed by Engel [10] using electromagnetic force compensation balances. This highlights the effect of the active control system of the balance, which is based on the continuous adjustment of the equilibrium position of the balance pan. The standard deviation of residuals has been observed to be directly proportional to the flow rate, being equal to about 100 mg at 10 kg h^{-1}, 250 mg at 25 kg h^{-1}, 400 mg at 40 kg h^{-1}, and 600

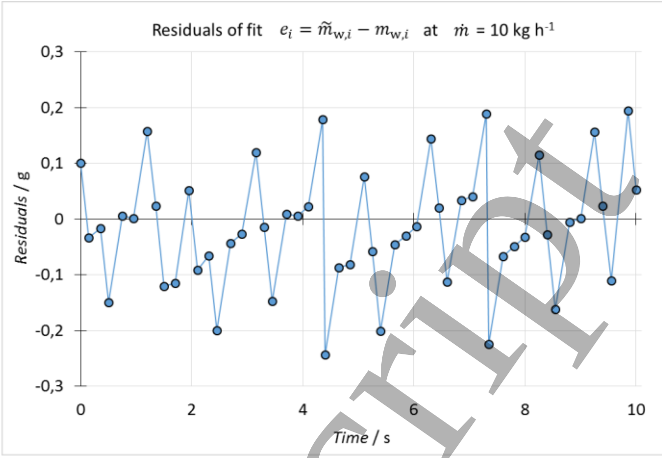

Figure 7. Residuals of fit for 10 kg h^{-1} water flow rate measurement.

mg at 60 kg h^{-1}, preserving the same trend of residuals as observed in figure 7. This confirms the effect of the active control system of the balance on its dynamic response, maybe due to a fixed stepwise increment of the electromagnetic force used to adjust the equilibrium position of the balance pan.

The uncertainty contribution $u_\mathrm{d}(\dot{m})$ can be reduced by increasing the number of readings N, i.e. by increasing the weighing period as much as possible, until a further increase of N does not produce a significant improvement of measurement repeatability, which can be evaluated as the standard deviation of the mean of several repeated flow rate measurements. In the tests, at least five repetitions have been performed. As observed from measurements in the range from 10 kg h^{-1} to 60 kg h^{-1} at ambient temperature conditions, the relative standard uncertainty contribution due to repeatability settles around 0.005%.

3.3.2. Mass and time measurement. Considering that the balance used in the proposed measurement system is characterized by a resolution of 1 mg and a calibration standard uncertainty of 2 mg, and that the weighed amount of water is higher than 0.5 kg for water flow rates from 10 kg h^{-1} to 60 kg h^{-1}, it is clear that the uncertainty associated to mass measurement is dominated by the dynamic response of the balance, as described in section 3.3.1.

Concerning time measurement, as the calibration uncertainty of the time counter is lower than 0.001% and the flow rate is determined by the regression analysis of mass and time measurement data, the corresponding uncertainty contribution can be reasonably neglected.

It should be noted that the possible synchronization errors in the acquisition of mass and time measurements are such that the slight delay between the readings of the balance and the time counter is approximately constant over the acquisition interval, entailing a negligible uncertainty contribution on flow measurement. This can be considered as already included in the uncertainty contribution $u_{\mathrm{d}}(\dot{m})$, evaluated according to equation (7).

The stability of air temperature conditions (temperature, pressure, and relative humidity) in proximity of the weightank during the tests represents the main uncertainty contribution in the determination of the air buoyancy correction factor. However, even considering large variations of air conditions and high humidity values inside the weighing chamber during the dynamic weighing process, the uncertainty associated to the air buoyancy correction is negligible compared to the other contributions. For instance, considering a stability of air temperature, pressure, and relative humidity equal to 0.5 °C, 200 Pa, and 10%, respectively, the corresponding relative uncertainty associated to the air buoyancy correction factor is lower than 0.001%.

The water loss by evaporation is strongly reduced by the use of the evaporation trap, as it has been observed that an evaporation of less than 20 mg usually occurs for a time period of about 1 min. Hence, the mass correction δm_{ev} has been evaluated as uniformly distributed between zero and the product between the maximum evaporation rate and the collection interval.

3.3.3. Meter under test. When the reference flow rate measurement is compared against the measurement provided by a flow meter under calibration, it is important to take into account the uncertainty source related to the acquisition of the output signal of the meter under test. In the proposed measurement system, a digital multimeter is used for the acquisition of the output signals, ensuring a relative uncertainty of frequency, current and voltage measurements lower than 0.001%. In the case of calibration of the pulse output signal of the meter under test, the uncertainty contribution associated with the resolution of the signal must be considered. The resolution error can be limited by counting a sufficient number of pulses during the dynamic flow measurement.

Another uncertainty contribution that arises when the flow rate measured by the reference standard is compared against the measurement provided by a flow meter under calibration, consists in the variation of the liquid volume inside the inter-connecting pipework (between the meter and the weighing system) during the test [18]. This variation of liquid volume depends on temperature and pressure variations during the test. However, considering that the liquid pressure is usually constant during the test, the liquid temperature is sufficiently stable because of the short duration of the dynamic weighing process, and the inter-connecting pipework is not excessively long, no correction of the measurement is usually required.

In addition, it is worth noting that the repeatability of measurements may worsen as due to the additional repeatability contribution introduced by the meter under test.

4. Experimental results

The proposed dynamic weighing method has been validated by comparison against the gravimetric static flying start and stop method adopted at the INRIM national standard for liquid flow rate measurements, using as transfer standard a DN3 Coriolis mass flow meter.

The proposed method has been also compared, by means of the same transfer standard, against the typical dynamic method based on the immersed jet flow as well, but characterized by a variable immersion depth, as due to the rising water level inside the weightank.

4.1. Validation of the proposed dynamic method

The validation of the proposed method has been carried out at two mass flow rates, namely the minimum flow rate of the INRIM gravimetric standard, equal to 40 kg h^{-1}, and the maximum flow rate at which the proposed dynamic weighing system can operate, equal to 60 kg h^{-1}. In this range, the flow rate measurements carried out by the flying start and stop method are characterized by a weighed amount of water of 20 kg at least (balance capacity of 150 kg with associated uncertainty of 4 g), which is necessary in order to reduce the uncertainty down to a level comparable to the dynamic method. Consequently, because of the long weighing durations (up to about 30 min for the lower flow rate), the correction due to evaporation losses has been accurately estimated. In particular, the water loss by evaporation has been evaluated by emptying a 20 L water tank into the weightank, and weighing both tanks before and after the water pouring. The water discharge from the 20 L tank into the weightank has been performed in about 30 min using the same flow conveyor and weightank as in the static gravimetric system, and ensuring almost saturated air conditions around the 20 L tank.

The calibration of the transfer standard by means of the proposed dynamic method is carried out keeping the meter installed on the hydraulic circuit of the INRIM flow rate standard and connecting the outlet section of the meter itself to the weighing system described in section 3.2. In such a way, possible systematic errors that may arise from the use of different hydraulic distribution circuits are avoided and the dynamic method can be effectively validated. During tests at different flow rates, the water temperature was kept at (20.3 ± 0.3) °C and the downstream pressure with respect to the Coriolis transfer standard was maintained at (0.33 ± 0.03) MPa.

The Coriolis transfer standard has been calibrated as mass flow counter, configuring its pulse output with a pulse value K_{T} of 10 mg. The meter error at zero flow has been adjusted before starting the calibration and checked again at the end of tests, to evaluate possible zero drift of the instrument. The stability of the transfer standard has been evaluated by

Table 1. Results of the experimental validation of the proposed dynamic method.

$\dot{m}$ / kg h^{-1}	K_d / mg	ε_d / %	$u(\varepsilon_d)$ / %	K_s / mg	ε_s / %	$u(\varepsilon_s)$ / %	E_n
40	9.989 60	0.104	0.009	9.991 10	0.089	0.024	0.29
60	9.989 52	0.105	0.016	9.991 95	0.081	0.026	0.39

intercomparing the calibration results obtained by the static flying start and stop method, before and after the calibration performed by the proposed dynamic method. The maximum difference between the relative errors of the meter obtained at the initial and final calibration has been observed to be within the repeatability of measurements.

The quantity to be measured is the relative percentage error ε associated to the pulse value configured on the transfer standard K_T, with respect to pulse value measured by the reference standard K

$$\varepsilon = 100\frac{K_T - K}{K}, \quad (8)$$

where K is calculated as the ratio between the mass of water measured by the reference standard Δm and the corresponding number of pulses Δn totalized by the transfer standard

$$K = \frac{\Delta m}{\Delta n}. \quad (9)$$

The acquisition of the pulses generated by the transfer standard is synchronized with the mass readings provided by the balance during the dynamic weighing process, so that the pulse counter and the balance will provide, respectively, the number of pulses n_i and the mass readings $m_{w,i}$ related to the same *i*-th time instant. Thus, the pulse value is obtained from the linear regression analysis of the couples ($m_{w,i}$, n_i), multiplying the slope of the linear regression model by the air buoyancy correction factor and correcting for evaporation losses.

The pulse value K_T configured on the Coriolis transfer standard ensures a maximum resolution error of the pulse counter always lower than 0.002%, the totalized number of pulses being higher than 5×10^4 for each test.

The degree of equivalence between the proposed dynamic method and the static flying start and stop method is evaluated by means of the normalized deviation E_n, defined as

$$E_n = \frac{|\varepsilon_d - \varepsilon_s|}{2\sqrt{u^2(\varepsilon_d) + u^2(\varepsilon_s)}}, \quad (10)$$

where ε_d and ε_s are the relative errors obtained from the pulse values K_d and K_s measured by the proposed dynamic method and the static flying start and stop method, respectively, whereas $u(\varepsilon_d)$ and $u(\varepsilon_s)$ are the corresponding standard uncertainties.

The results of the experimental validation are shown in table 1, where the normalized deviations E_n are given for each water flow rate, from which the consistency between the two methods can be pointed out.

Despite the much higher uncertainty of the static method, whose main contribution is due to the weighing (0.020%), it is important to highlight that the repeatability contribution, evaluated as the standard deviation of the mean of five repeated measurements of the relative errors ε_d and ε_s, is equal to about 0.003% for the static flying start and stop method, and 0.005% for the proposed dynamic method. This slight difference could be due to the effects of turbulence and fluid motion disturbances inside the weightank during the dynamic weighing process.

In addition, concerning the reproducibility of the method, the deviations of calibration results repeated over different weeks have been observed to be within the typical standard deviation obtained during a specific flow measurement.

4.2. Comparison against a typical dynamic method

In order to extend the validation to flow rates lower than 40 kg h^{-1}, the proposed method has been compared against a typical dynamic method based on the immersed tube as well, but characterized by a variable immersion depth, as due to the rising water level inside the weightank. For such a method, as discussed in section 2.2, the mass correction associated to the hydrostatic buoyancy force on the immersed tube can be either calculated according to equation (5), or determined experimentally. In this last case, which is the one here adopted, the correction has been preliminarily determined according to the following procedure and considering the same tube and weightank used in the measurement system of the proposed dynamic method:

1. the weightank is filled up to a minimum water level and the stable mass reading of the balance $m_{w,ET}$ is acquired;
2. the tube, whose outlet section has been preliminary plugged without increasing the tube volume, is immersed under the water surface down to the same position as used during the dynamic flow measurement;
3. the stable mass reading of the balance $m_{w,IT}$ is acquired with the tube immersed under the water

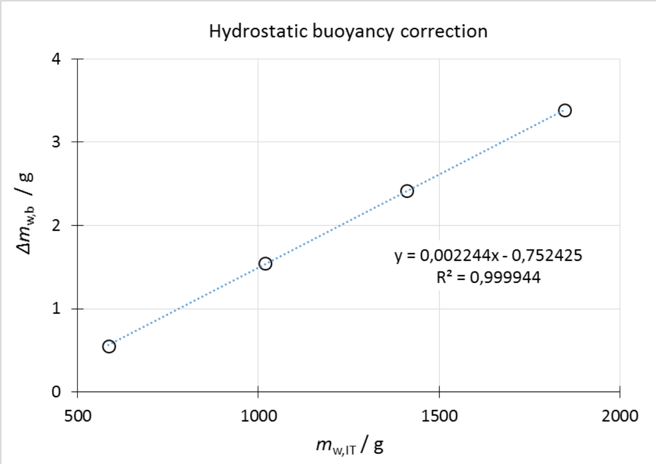

Figure 8. Experimental corrections for the hydrostatic buoyancy force on the immersed tube, as a function of the balance reading.

surface: the difference $\Delta m_{\mathrm{w,b}} = m_{\mathrm{w,IT}} - m_{\mathrm{w,ET}}$ corresponds to the mass correction associated to the hydrostatic buoyancy force on the immersed tube;

4. the tube is extracted from water and the procedure is repeated for two other water levels at least, in order to get the values of the mass correction $\Delta m_{\mathrm{w,b}}$ as functions of the balance readings $m_{\mathrm{w,IT}}$ up to the maximum immersion depth of the tube.

Figure 8 shows the experimental mass corrections $\Delta m_{\mathrm{w,b}}$ associated to the hydrostatic buoyancy force on the immersed tube (6.3 mm outer diameter), as a function of the balance readings $m_{\mathrm{w,IT}}$.

While possible drifts of the balance are easily corrected, the uncertainty associated to the experimental determination of the hydrostatic buoyancy correction mainly depends on the uncertainties related to the vertical alignment and positioning of the immersed tube and to the variability of meniscus shape. These uncertainty contributions can be considered as included in the residuals of the linear regression model, which describes the relation between the hydrostatic buoyancy corrections and the balance readings. Concerning the linear model shown in figure 8, the relative standard uncertainty contribution due to the hydrostatic buoyancy correction is around 0.002%.

The comparison between the two dynamic methods has been carried out by means of the Coriolis transfer standard, used as mass flow counter and configured as described in section 4.1. The tests are performed in the flow range from 10 kg h^{-1} to 60 kg h^{-1}, at the same water temperature and pressure conditions of the experimental validation against the flying start and stop method.

In table 2, the results of the comparison are shown, highlighting the pulse values K_{d} and K_{v}, the relative errors ε_{d} and ε_{v}, and the associated standard uncertainties $u(\varepsilon_{\mathrm{d}})$ and $u(\varepsilon_{\mathrm{v}})$, obtained, respectively, by the proposed method and the variable immersion depth dynamic method. The consistency between the two methods is demonstrated by the values of the normalized deviations E_{n}, which are evaluated between the relative errors ε_{d} and ε_{v}, taking into account their associated uncertainties. It can be observed that the proposed dynamic method is characterized by lower uncertainty values compared to the variable immersion depth dynamic method, particularly for lower flow rates.

Table 3 shows the uncertainty budget associated to the proposed dynamic method, in which the contributions of the different uncertainty sources are detailed. It must be noted

Table 2. Results of the experimental comparison between the proposed method and the variable immersion depth dynamic method.

$\dot{m}$ / kg h^{-1}	K_{d} / mg	ε_{d} / %	$u(\varepsilon_{\mathrm{d}})$ / %	K_{v} / mg	ε_{v} / %	$u(\varepsilon_{\mathrm{v}})$ / %	E_{n}
10	9.992 31	0.077	0.006	9.993 40	0.066	0.008	0.54
25	9.991 74	0.083	0.006	9.991 48	0.085	0.008	0.13
40	9.989 60	0.104	0.009	9.990 59	0.094	0.010	0.37
60	9.989 52	0.105	0.016	9.989 22	0.108	0.017	0.06

Table 3. Uncertainty budget for the proposed dynamic method.

Uncertainty source	Standard uncertainty contribution, $u_i(\varepsilon_{\mathrm{d}})$ / %			
	10 kg h^{-1}	25 kg h^{-1}	40 kg h^{-1}	60 kg h^{-1}
Linear fit of mass and pulse readings	0.002 0	0.003 5	0.007 0	0.015 0
Balance calibration	0.000 6	0.000 3	0.000 3	0.000 3
Pulse counter resolution	0.002 0	0.001 0	0.001 0	0.001 0
Air buoyancy correction	0.000 6	0.000 6	0.000 6	0.000 6
Evaporation	0.003 0	0.001 2	0.000 8	0.000 5
Repeatability	0.005 0	0.005 0	0.005 0	0.006 0
$u(\varepsilon_{\mathrm{d}})$ / %	0.006	0.006	0.009	0.016

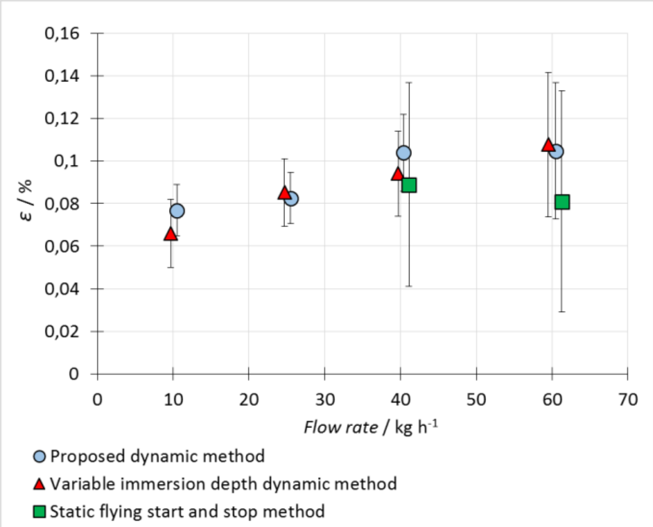

Figure 9. Synthesis of test results: relative errors of the transfer standard with the corresponding expanded uncertainty bars, as a function of the water flow rate.

that the weighed amount of water was of about 0.5 kg at 10 kg h^{-1}, and 1 kg for the other flow rates.

It can be observed that at lower flow rates, the main uncertainty contributions are the ones associated to evaporation and repeatability, whereas at higher flow rates the contribution associated to the linear fit of mass and pulse readings becomes dominant, the number of readings N being reduced from about 940 at 10 kg h^{-1} (i.e. about 3 min acquisition time), down to about 260 at 60 kg h^{-1} (i.e. about 1 min acquisition time). Of course, the higher uncertainty values obtained at higher flow rates can be easily reduced by increasing the weighed amount of water, i.e. by increasing the number of measurement points for the linear fit.

The experimental results concerning both the validation of the proposed method against the static flying start and stop method, and the comparison against the variable immersion depth dynamic method, are shown in figure 9, where the relative errors ε_d, ε_v, and ε_s are plotted with the corresponding expanded uncertainty bars.

Concerning the variable immersion depth method, it is important to observe that the higher uncertainties compared to the proposed dynamic method are basically due to the additional uncertainty contribution associated to the hydrostatic buoyancy correction, which is equal to 0.002%, and to the higher repeatability contribution, equal to 0.007% for each flow rate. In particular, the worse repeatability is probably due to the uncorrected effects associated to the slight variations of vertical momentum components (variation of the back pressure at the outlet section of the immersed tube) and meniscus force (variation of the shape of the meniscus) during the dynamic weighing period, as described in section 2.2. Moreover, it is worth observing that the uncertainty contribution associated to the linear fit of mass and pulse readings is approximately the same as the one evaluated for the proposed dynamic method, since it mainly depends on the characteristic dynamic response of the balance.

5. Conclusions

An improved dynamic method for liquid flow rate measurements has been adopted at INRIM to be consider as the primary standard for water flows lower than 40 kg h^{-1}. It is based on the immersed jet approach, but thanks to the constant immersion depth of the submerged tube, it has the advantage, also in terms of uncertainty evaluation, that measurements do not need to be corrected for hydrostatic buoyancy. This is possible as water overflows from a constant level container realized inside the weightank, in which the tube is immersed. Hence, the measurement model is significantly simplified, reducing, consequently, the uncertainty contributions that affect the calibration.

An important advantage of the proposed method, particularly for low flow rates, is that it allows keeping a constant back pressure at the outlet flow section of the immersed tube and a stable shape of the meniscus during the dynamic weighing process, as due to the constant immersion depth of the submerged tube. This allows improving the accuracy and repeatability of measurements, avoiding the effects associated to the time variation of the mass flow rate and the instability of the meniscus force, which may arise if the rise of the liquid level changes the immersion depth of the tube.

The proposed dynamic method has shown a significant improvement both in terms of uncertainty and test execution time, with respect to the static method adopted by the INRIM national standard for liquid flow. In order to evaluate the measurement capabilities also at lower flow rates, the proposed method has been compared against a typical dynamic method based on the submerged jet approach as well, but characterized by a variable immersion depth of the submerged tube. The comparison has been carried out in the water flow range from 10 kg h^{-1} to 60 kg h^{-1} and has pointed out a good degree of equivalence between the two dynamic methods, highlighting a lower uncertainty of the proposed method, particularly for flow rates lower than 25 kg h^{-1}, where uncertainties of 0.006% can be obtained.

Future developments will aim to improve the uncertainty associated with flow measurements, by trying to reduce the uncertainty contribution due to the dynamic response of the balance. To this aim, other types of balances will be tested, e.g. load cells, and the capacity of the weightank will be increased to reduce the uncertainty. In addition, the measurement system will be also scaled and modified, in order to test the performance of the proposed method at very low flow rates, even lower than 10 g h^{-1}.

Acknowledgements

The authors wish to thank Dr. Giorgio Cignolo for valuable comments and suggestions during the preparation of this paper.

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