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The spin Seebeck and spin Peltier reciprocal relation

Vittorio Basso, Michaela Kuepferling, Alessandro Sola, Patrizio Ansalone and Massimo Pasquale

Abstract—In this paper we derive the reciprocal relation between electric and thermal quantities of the spin Seebeck and the spin Peltier effects. For a Pt/YIG bilayer device, the reciprocity assumes the remarkably simple form $T\Delta V_{e,y}/I_{q,x} = \Delta T_{SP,x}/I_{e,y}$ where $\Delta V_{e,y}$ is the voltage generated at the Pt metal in a spin Seebeck experiment under the heat current $I_{q,x}$ and $\Delta T_{SP,x}$ is temperature difference generated in a spin Peltier experiment with an electric current $I_{e,y}$ flowing in the Pt metal. The ratios are related to intrinsic coefficients: the spin Hall angle θ_{SH} of Pt and the thermomagnetic power coefficient ϵ_M of YIG.

Index Terms—Spin caloritronics, reciprocal relations, Pt/YIG bilayers

I. INTRODUCTION

THE spin Seebeck and spin Peltier effects are the result of two independent physical mechanisms: the spin Hall effect in a metallic layer (i.e. Pt) and the thermomagnetic effect in a ferrimagnetic insulator (i.e. yttrium iron garnet, YIG) [1]–[4]. In the spin Seebeck effect, the presence of a temperature gradient in YIG generates a magnetic moment current (often referred to as spin current), carried by spin waves. At the interface between YIG and Pt the current is partially injected into the metal where it is carried by spin polarized electrons. In Pt the diffusing electrons are partially deflected in the perpendicular direction due to the inverse spin Hall effect and are revealed as a transverse electric voltage. In the spin Peltier effect the role of the two mechanisms is reversed: Pt is the active layer while YIG is the passive one. A transverse electric current in Pt is able to force the injection of a magnetic moment current into YIG. Thermomagnetic effects of YIG transform this current into a heat current which is finally revealed as a temperature difference. Both effects have already been experimentally observed [5]–[10] but, although the reciprocity of the spin caloritronic effects has been already discussed [11], [12], a quantitative reciprocal relation is still missing.

The Onsager reciprocal relations are an outcome of non-equilibrium thermodynamics [13]. For a thermoelectric device, the reciprocity consists in the fact that for a given thermoelectric conductor, the ratio between the measured voltage ΔV and the applied temperature difference ΔT in a Seebeck experiment, and the ratio between the measured heat flux I_q and the applied electric current I_e , in a Peltier one, are related by the absolute temperature T : $T\Delta V/\Delta T = I_q/I_e$ [14]. Furthermore the ratios are both related to the absolute thermoelectric power coefficient $\epsilon = \nabla V/\nabla T$ which is a material dependent property. In the case of a spin Seebeck and spin Peltier device, the formulation of the Onsager reciprocity

is not so straightforward, because there are two combined mechanisms: the spin Hall effect of the metal and the thermomagnetic effect of the insulator. Furthermore, unlike the electric charge, the magnetic moment is a non conserved quantity. This fact requires a proper thermodynamic treatment.

In this paper we derive the spin Seebeck and spin Peltier reciprocal relation on the base of the non-equilibrium thermodynamics of Johnson and Silsbee [15] following the application of the theory to ferromagnetic insulators of Refs. [16] and [17]. The result is that, for a Pt/YIG bilayer, the relation between the integrated quantities assumes the remarkably simple form

$$\frac{\Delta V_{e,y} T}{I_{q,x}} = \frac{\Delta T_{SP,x}}{I_{e,y}} \quad (1)$$

where the left hand side refers to the spin Seebeck experiment: $\Delta V_{e,y}$ is the voltage generated at the metal and $I_{q,x}$ is the heat current traversing the device, and the right hand side refers to the spin Peltier experiment: $\Delta T_{SP,x}$ is the temperature difference generated and $I_{e,y}$ is the electric current flowing into the metal. The theory also permits to relate the ratios of Eq.(1) to the intrinsic parameters: the spin Hall angle θ_{SH} of Pt, the thermomagnetic power coefficient ϵ_M of YIG and the magnetic moment conductance of the Pt/YIG interface v_p . In the paper we present the constitutive equations and the continuity equations for the two materials and we solve the magnetic moment conduction problem for the two cases of spin Seebeck and spin Peltier.

II. CONSTITUTIVE EQUATIONS

A. Thermomagnetic effects in magnetic insulators

The constitutive equations for the joint transport of magnetic moment and heat in ferromagnetic insulators in one dimension ($\nabla_x = \partial/\partial x$) are

$$j_M = \sigma_M (\mu_0 \nabla_x H^* - \epsilon_M \nabla_x T) \quad (2)$$

$$j_q = \epsilon_M T j_M - \kappa \nabla_x T \quad (3)$$

In the previous equations j_M is the magnetic moment current density and j_q is the heat current density. Both currents are along the x direction therefore the corresponding subscript is omitted for simplicity. The coefficients are: the magnetic moment conductivity σ_M , the absolute thermomagnetic power ϵ_M and the thermal conductivity κ under zero magnetic moment current. The magnetic field H , the magnetization M and the transported magnetic moment are all directed along the z direction. The Johnson and Silsbee thermodynamic potential is $H^* = H - H_{eq}(M)$ given by the difference between the magnetic field H and the magnetic equation of state at equilibrium $H_{eq}(M)$. The gradient ∇H^* is the driving force

of magnetic moment currents [16]. As the magnetic moment is not conserved, the continuity equation is

$$\frac{\partial M}{\partial t} + \nabla_x j_M = \frac{H^*}{\tau_M}. \quad (4)$$

In non-equilibrium stationary states we impose the condition $\partial M/\partial t = 0$ to be true.

In a spin Seebeck-type experiment the temperature gradient $\nabla_x T$ is controlled by external conditions. For a constant $\nabla_x T$ profile, the magnetic moment current of Eq.(2) becomes

$$j_M = j_{MS} + \sigma_M \mu_0 \nabla_x H^* \quad (5)$$

where $j_{MS} = -\sigma_M \epsilon_M \nabla_x T$ is a magnetic moment current source. By using Eqs.(4) and (5) one finds the diffusion equation for the potential

$$l_M^2 \nabla_x^2 H^* = H^* \quad (6)$$

where $l_M = (\mu_0 \sigma_M \tau_M)^{1/2}$ is the diffusion length. The solution of this equation, with the appropriate boundary conditions, provides the profiles of $H^*(x)$ and $j_M(x)$ (see Appendix A).

In a spin Peltier-type experiment the temperature profile is the result of the energy balance equation in which one has to include the intrinsic thermal effect of YIG. As it is done in the classical treatment [13], from the expression of the entropy production rate, see Ref. [16], one finds, in stationary conditions $\partial T/\partial t = 0$, $\nabla_x (j_q - \mu_0 H^* j_M) = 0$. This equation has to be solved by using the continuity equation (4) and the constitutive equations (2) and (3). The result is

$$-\kappa \left((1 + \zeta_T) \nabla^2 T - \frac{\zeta_T}{\epsilon_M} \mu_0 \nabla^2 H^* \right) = \frac{j_M^2}{\sigma_M} + \frac{\mu_0 (H^*)^2}{\tau_M} \quad (7)$$

Equation (7) involves at the left hand side the second derivatives of the temperature T and of the potential H^* and the dimensionless parameter $\zeta_T = \sigma_M \epsilon_M^2 T / \kappa$ which is analogous of the ZT parameter of thermoelectrics [14]. On the right hand side, Eq.(7) contains the squares of the current and the potential. At this point two approximations can be made. First, for sufficiently small magnetic moment current j_M and potential H^* , the quadratic terms at the right hand side of Eq.(7) can be disregarded. Second, we make the approximation of small $\zeta_T \ll 1$. Both approximations are valid in the case of spin Seebeck and spin Peltier experiments [16]. The result is that the relation between temperature and potential becomes

$$\epsilon_M \nabla_x^2 T = \zeta_T \mu_0 \nabla_x^2 H^* \quad (8)$$

By substitution of Eq.(8) into the stationary Eq.(4) where one has used the appropriate constitutive equation (2) one finds that the potential H^* is still given by the solution of a diffusion equation of the type Eq.(6) but with a different diffusion length $(l'_M)^2 = \mu_0 \sigma_M \tau_M / (1 + \zeta_T)$ which is slightly smaller than the spin Seebeck case. From the integration of Eq.(8) we also obtain the spin Peltier temperature difference between the two sides of the YIG layer: $\Delta T_{SP} = (\zeta_T / \epsilon_M) \mu_0 \Delta H^*$ which is related to the difference of the potential ΔH^* between the two faces of YIG.

B. Spin Hall effect in non-magnetic metals

In order to define a set of constitutive equations for the spin Hall effect in a non magnetic metal with a negligible Hall effect, we select the conditions in which the electric current j_e is along y , the magnetic moment of the electrons is along z and the magnetic moment current j_M is along x . Then we have

$$j_{ey} = -\sigma_e \nabla_y V_e + \sigma_e \theta_{SH} \left(\frac{\mu_B}{e} \right) \mu_0 \nabla_x H^* \quad (9)$$

$$j_{Mx} = \sigma_e \theta_{SH} \left(\frac{\mu_B}{e} \right) \nabla_y V_e + \sigma_M \mu_0 \nabla_x H^* \quad (10)$$

where σ_e is the electric conductivity, V_e is the electric potential, e is the elementary charge, μ_B is the Bohr magneton and $\sigma_M = \sigma_e (\mu_B / e)^2$ is the conductivity for the magnetic moment current. The equations contain the spin Hall effect in the non diagonal terms which couple different directions and different currents. The spin Hall angle is θ_{SH} and for the magnetic moment has opposite sign with respect to the one for the spin angular momentum (i.e. θ_{SH} is negative for Pt and positive for W and Ta) [16]. Also in metals the magnetic moment is not a conserved quantity, therefore a continuity equation analogous to Eq.(4) holds.

In a spin Seebeck-type experiment the Pt is used to detect the magnetic moment current injected from YIG. For an open electric circuit along y ($j_{ey}=0$) we integrate Eq.(9) along x and find that the gradient of the electric potential along y , averaged over the thickness t_{Pt} , is

$$\nabla_y V_e = \theta_{SH} \left(\frac{\mu_B}{e} \right) \mu_0 \frac{\Delta H^*}{t_{Pt}} \quad (11)$$

where the difference of the potential ΔH^* between the two faces of the Pt layer is caused by the side YIG layer. The potential H^* in Pt is given by taking the continuity equation (4) in stationary conditions with the constitutive equation (10). The result is the diffusion equation (6) where $l_M = (\mu_0 \sigma_M \tau_M)^{1/2}$ is the diffusion length of Pt.

In a spin Peltier experiment Pt is used to generate a magnetic moment current. In the case of an imposed driving current density j_{ey} , the equation for the magnetic moment current is

$$j_M = j_{MS} + \sigma'_M \mu_0 \nabla_x H^* \quad (12)$$

with $j_{MS} = -\theta_{SH} (\mu_B / e) j_{ey}$ and $\sigma'_M = \sigma_M (1 + \theta_{SH}^2)$. The potential is given by the solution of the diffusion equation (6) with $l_M = (\mu_0 \sigma'_M \tau_M)^{1/2}$ being the diffusion length for the spin Hall metal.

III. SPIN SEEBECK EFFECT

The magnetic moment conduction in the Pt/YIG bilayer is described by taking the solution of the diffusion equation (6) for each layer with the magnetic current source $j_{MS} = -\sigma_{YIG} \epsilon_{YIG} \nabla_x T$ in YIG and setting the appropriate boundary conditions (see Appendix A). An example of the resulting profile is shown in Fig. 1. The resulting electric effect in Pt, Eq.(11), is

$$\nabla_y V_e = -\theta_{SH} \mu_0 \left(\frac{\mu_B}{e} \right) \frac{1}{v_p} \frac{\epsilon_{YIG} \sigma_{YIG}}{t_{Pt}} \nabla_x T \quad (13)$$

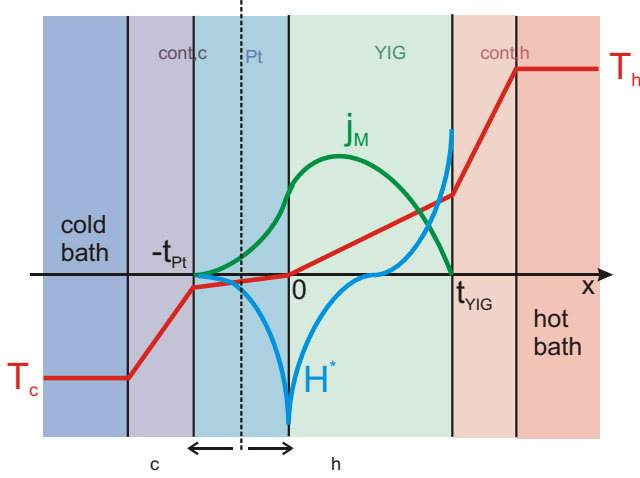


Fig. 1. Spin Seebeck effect. Sketch of the cross section of the whole stack including thermal reservoirs, thermal resistances ϱ_h and ϱ_c and Pt and YIG layers, along the diffusion direction x of the heat and magnetic moment current. The profiles of the magnetic moment current density j_M and the potential H^* are shown for a uniform temperature gradient in YIG.

where

$$\frac{1}{v_p} = \frac{v_{eff}}{v_{Pt} \coth(t_{Pt}/(2l_{Pt})) v_{YIG} \coth(t_{YIG}/(2l_{YIG}))} \quad (14)$$

and

$$\frac{1}{v_{eff}} = \frac{1}{v_{Pt} \tanh(t_{Pt}/l_{Pt})} + \frac{1}{v_{YIG} \tanh(t_{YIG}/l_{YIG})} \quad (15)$$

are magnetic moment conductances for the bilayer (see Appendix A) depending on the intrinsic conductances of YIG, $v_{YIG} = l_{YIG}/\tau_{YIG}$, and Pt, $v_{Pt} = l_{Pt}/\tau_{Pt}$, and on the thicknesses t_{YIG} and t_{Pt} .

IV. SPIN PELTIER EFFECT

For the spin Peltier case the procedure is similar to the previous case but the magnetic current source $j_{MS} = -\theta_{SH}(\mu_B/e)j_e$ is now in the Pt layer. A sketch of the resulting profiles are shown in Fig.2 and the analytic expressions are given in Appendix A. The temperature profile in YIG is then obtained by integrating Eq.(8) and setting the thermal boundary conditions. For example $T(0) = T_0$ and $T(t_{YIG}) = T_2$. The simplest case is the one in which the temperatures at the boundaries are not constrained. In that case the difference between the boundaries is $T_2 - T_0 = \Delta T_{SP}$ with

$$\Delta T_{SP} = \theta_{SH} \mu_0 \left(\frac{\mu_B}{e} \right) \frac{1}{v_p} \frac{\epsilon_{YIG} \sigma_{YIG} T}{\kappa_{YIG}} j_e \quad (16)$$

This is the temperature difference that one would measure in presence of the spin Peltier effect alone. However in presence of the Pt layer, one has to add the Joule heat dissipation caused by the flowing electric current. As a first approximation we neglect the dissipation due to magnetic moment current and obtain directly the temperature profile in Pt as due to the conventional electric Joule heat

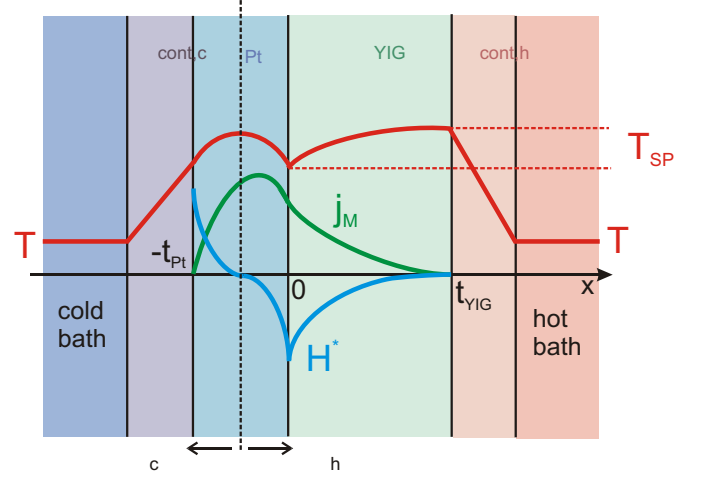


Fig. 2. Spin Peltier effect. Sketch of the cross section of the whole stack as in Fig.1. The profiles of the magnetic moment current density j_M , the potential H^* and the temperature T are shown for a uniform electric current density in the Pt layer.

$$\nabla_x^2 T = -\frac{j_e^2}{\sigma_e \kappa_{Pt}} \quad (17)$$

where κ_{Pt} is the thermal conductivity of Pt. The temperature profile in Pt is then obtained by integrating the previous equation and setting the boundary conditions $T(-t_{Pt}) = T_1$ and $T(0) = T_0$. We have therefore obtained the temperature and the heat current profiles in both YIG and Pt layers. We impose now the continuity of both quantities at the interfaces. In order to obtain a sensible estimate, we assume perfect thermal interface between Pt and YIG, while we add non ideal thermal contacts between the bilayer itself and the external thermal baths. We introduce the thermal contact resistance $\varrho_{cont,c}$ between the Pt side and the cold reservoir at T_c and $\varrho_{cont,h}$ between the YIG side and the hot reservoir at T_h (see Fig.2). If $T_h = T_c = T$ we have

$$j_{q,c} = \frac{1}{\varrho} \Delta T_{SP} - \frac{\varrho_h}{\varrho} j_{q,JH} \quad (18)$$

$$j_{q,h} = \frac{1}{\varrho} \Delta T_{SP} + \frac{\varrho_c}{\varrho} j_{q,JH} \quad (19)$$

where $\varrho_h = \varrho_{YIG} + \varrho_{cont,h} + \varrho_{Pt}/2$, $\varrho_c = \varrho_{cont,c} + \varrho_{Pt}/2$, $\varrho = \varrho_c + \varrho_h$ and $j_{q,JH} = t_{Pt} j_e^2 / \sigma_e$. The thermal resistances are expressed per unit surface area, i.e. $\varrho_i = t_i / \kappa_i$. The ΔT_{SP} and the Joule heat $j_{q,JH}$ can then be extracted and separated by the joint measurement of the two heat currents $j_{q,c}$ and $j_{q,h}$ once the thermal resistances ϱ_c and ϱ_h are known.

V. RECIPROCAL RELATION

From the equations for the spin Seebeck, Eq.(13), with $j_q = -\kappa_{YIG} \nabla T$, and the equation for the spin Peltier, Eq.(16), we find the reciprocal relation sought

$$\frac{\nabla_y V_e}{j_q} t_{Pt} = \frac{1}{T} \frac{\Delta T_{SP}}{j_e} = \theta_{SH} \mu_0 \left(\frac{\mu_B}{e} \right) \frac{1}{v_p} \frac{\epsilon_{YIG} \sigma_{YIG}}{\kappa_{YIG}} \quad (20)$$

that is expressed in terms of current densities and gradients. To move to integrated quantities we have to specify the dimensions of the sample. We use a parallelepiped with electric effects occurring along the length L_y and through the cross section $L_z \times t_{Pt}$ and with thermal effects occurring along the thickness t_{YIG} and through the cross section $L_y \times L_z$. As the spin Seebeck is characterized by the transverse electric voltage, $\Delta V_{e,y} = L_y \nabla_y V_e$ and by the heat current $I_{q,x} = L_y L_z j_q$ and the spin Peltier effect is driven by the electric current $I_{e,y} = L_z t_{Pt} j_e$, we find Eq.(1) which is valid for a given bilayer device.

VI. CONCLUSION

The main result of this paper is the derivation of the reciprocal relation between the spin Seebeck and the spin Peltier effects. The relation can be expressed both in terms of current densities and gradient of the electric potential, Eq.(20), which is independent of the size of the device, and in its integral form, expressed as a function of currents and voltage, Eq.(1), which depends on the size of the device. The merit of Eq.(20) is to express both the spin Seebeck and the spin Peltier effects as a function of the intrinsic coefficients: the spin Hall angle for the metal θ_{SH} , the thermomagnetic power coefficient ϵ_M , the magnetic moment conductivity σ_M , the thermal conductivity κ of YIG and the magnetic moment conductance of the Pt/YIG interface v_p . The maximization of Eq.(20) will lead to the determination of the optimal condition for both effects. This will be the subject of a future work. On the other hand Eq.(1) permits to pass from the spin Seebeck effect to the spin Peltier one for a given device. Work is in progress to test the experimental validity of the integrated relation on a bilayer device by using the setup described in Ref. [10].

APPENDIX A

SOLUTION OF THE MAGNETIC MOMENT DIFFUSION PROBLEM IN ONE DIMENSION

The magnetic moment current j_M and the potential H^* of a layer are given by the solution of Eq.(6) with the boundary conditions given by Eqs.(5) or (12). We consider a generic layer (i) from $x = -t/2$ to $t/2$ and take magnetic moment current at the boundaries as $j_- = j(-t/2)$, $j_+ = j(t/2)$ (we have dropped subscript M for compactness of the symbols). The solutions are

$$j(x) = j_{MS} + (j_s - j_{MS}) \frac{\cosh(x/l)}{\cosh(t/(2l))} + j_d \frac{\sinh(x/l)}{\sinh(t/(2l))} \quad (21)$$

$$H^*(x) = (j_s - j_{MS}) \frac{1}{v} \frac{\sinh(x/l)}{\cosh(t/(2l))} + j_d \frac{1}{v} \frac{\cosh(x/l)}{\sinh(t/(2l))} \quad (22)$$

with $j_s = (j_+ + j_-)/2$, $j_d = (j_+ - j_-)/2$ and $v = l_M/\tau_M$. With reference to the bilayer of Figs.1 and 2 we set layer 1=Pt and layer 2=YIG and the boundary conditions: $j_{1-} = j_{2+} = 0$, $j_{1+} = j_{2-} = j_0$ and $H_{1+}^* = H_{2-}^* = H_0^*$. The solution for the bilayer is

$$j_0 = v_{eff} \left(\frac{j_{MS,1}}{v_1 \coth(t_1/(2l_1))} + \frac{j_{MS,2}}{v_2 \coth(t_2/(2l_2))} \right) \quad (23)$$

with v_{eff} given by the Eq.(15). The spin Seebeck effect corresponds to a magnetic moment current source in 2 (YIG), so $j_{MS,2} = -\sigma_2 \epsilon_2 \nabla_x T$ while the layer 1 (Pt) is passive ($j_{MS,1} = 0$). The quantity of interest is ΔH^* in layer 1. We find

$$\Delta H_1^* = \frac{1}{v_p} j_{MS,2} \quad (24)$$

with v_p given by the Eq.(14). The spin Peltier effect corresponds to a magnetic moment current source in 1 (Pt), $j_{MS,1} = -\theta_{SH} (\mu_B/e) j_{ey}$ while the layer 2 (YIG) is passive ($j_{MS,2} = 0$). The quantity of interest is ΔH^* in layer 2. We find

$$\Delta H_2^* = \frac{1}{v_p} j_{MS,1} \quad (25)$$

with v_p given by the Eq.(14).

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