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This is the author's accepted version of the contribution published as:
Original Risk of a false decision on conformity of an environmental compartment due to measurement uncertainty of concentrations of two or more pollutants / Pennecchi, Francesca R; Kuselman, Ilya; da Silva, Ricardo J N B; Hibbert, D Brynn In: CHEMOSPHERE ISSN 0045-6535 202:(2018), pp. 165-176. [10.1016/j.chemosphere.2018.03.054]
Availability: This version is available at: 11696/59853 since: 2020-09-30T17:33:14Z
Publisher: Elsevier
Published DOI:10.1016/j.chemosphere.2018.03.054
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(Article begins on next page)

Accepted Manuscript

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PII: S0045-6535(18)30468-5

DOI: 10.1016/j.chemosphere.2018.03.054

Reference: CHEM 20998

To appear in: ECSN

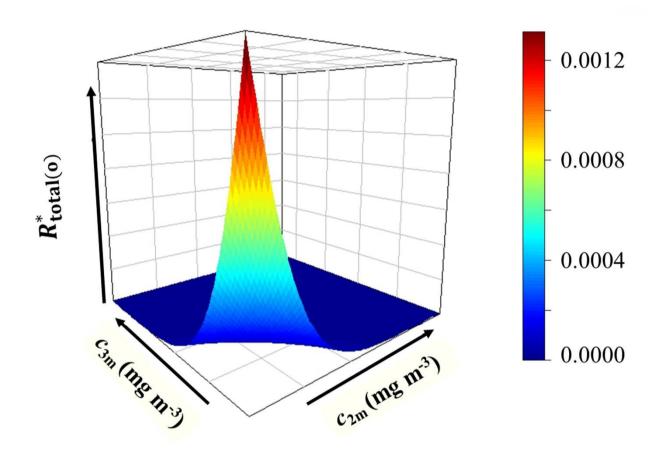
Received Date: 17 November 2017

Revised Date: 6 March 2018 Accepted Date: 7 March 2018

Please cite this article as: Pennecchi, F.R., Kuselman, I., da Silva, R.J.N.B., Hibbert, D.B., Risk of a false decision on conformity of an environmental compartment due to measurement uncertainty of concentrations of two or more pollutants, *Chemosphere* (2018), doi: 10.1016/i.chemosphere.2018.03.054.

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Dependence of the total risk of overestimation $R_{\rm total(o)}^*$ of suspended particulate matter concentration in ambient air on the measurement results $c_{\rm im}$ in proximity to the three quarries ($c_{\rm 1m}$ = 0.250 mg m⁻³; $c_{\rm 2m}$ and $c_{\rm 3m}$ are varying from 0.210 to 0.300 mg m⁻³).

1	Risk of a false decision on conformity of an environmental
2	compartment due to measurement uncertainty of concentrations of
3	two or more pollutants
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1. Introduction

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Actual ('true') concentration c_i of the i-th pollutant, i = 1, 2, ..., n, in an environmental compartment, e.g. ambient air (Duursma and Carroll (1996); TIMBRE project, Online), should not exceed a regulation or legal tolerance upper limit T_{Ui} . 'Concentration' is used here as a generic term (Cvitaš, 1996; Tolhurst, 2005; Fuentes-Arderiu, 2013). Comparing a chemical analytical test/measurement result c_{im} of the i-th pollutant concentration with the T_{Ui} value, one should decide whether the compartment conforms to the regulation or not. Since any result c_{im} has an associated measurement uncertainty (Ellison and Williams, 2012; Magnusson et al., 2012), several kinds of risk of a false decision on conformity of the compartment may arise. The probability of a decision that the actual pollutant concentration does not exceed the limit since $c_{im} \le T_{Ui}$, when it is not correct (i.e. $c_i > T_{Ui}$), is named 'consumer's risk'. The 'consumer' in the present paper is a habitant whose quality of life (including health) depends on adequate control of the pollutant. Thus, the consumer's risk is the probability of underestimation of c_i due to measurement uncertainty associated with c_{im} . On the other hand, the probability of falsely rejecting the decision on conformity of the compartment to the regulation (i.e. $c_{im} > T_{Ui}$ when $c_i \le T_{Ui}$) is the 'producer's risk'. The 'producer' here is a plant or another organization - a source of the environment pollution, obliged to pay a fine and/or to invest money for an unnecessary reduction of the pollutant concentration in the case of false nonconformity. The producer's risk is therefore the probability of overestimation of c_i due to measurement uncertainty in c_{im} . For a specified compartment, e.g. ambient air in a certain location at a certain time, such risks are referred to as the 'specific consumer's risk' of underestimation $R_{ci(\mathbf{u})}^*$ and the 'specific

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46 producer's risk' of overestimation $R_{ci(0)}^*$ for i-th particular pollutant concentration. The risks of 47 incorrect conformity assessment of a compartment randomly drawn from a statistical population 48 of such compartments are the 'global consumer's risk' of underestimation $R_{ci(u)}$ and the 'global producer's risk' of overestimation $R_{ci(0)}$, respectively, as they characterize the environmental 49 50 quality globally. Evaluation of the particular risks (both specific and global) is described in the JCGM 106 (2012) based on a Bayesian approach to conformity assessment. 51 However, when concentrations of two or more pollutants are controlled, pollutant-by-52 53 pollutant evaluation of the risks is not complete in general, as it does not give an answer to the 54 question of the probability of a false decision on the overall compartment conformity. If conformity assessment for each i-th pollutant concentration of a compartment is successful, i.e. 55 the particular specific R_{ci}^* or global R_{ci} risks of both under- and overestimation are small enough, 56

Fig. 1

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A scheme summarizing the used terminology is shown in Fig. 1, where the particular risks described in the JCGM 106 (2012) are shown at the top of the scheme. The *total risk evaluation*, as the task of the IUPAC Project (2016), is highlighted by an ellipse at the bottom of the scheme. Using the law of total probability for the case of independent quantities (pollutant concentration values and corresponding measurement results) the total risk of underestimation can be evaluated as a combination of the particular risks (Kuselman et al., 2017a). For example, for three pollutions i = 1, 2, 3, assuming independent actual values of each pollutant concentration c_i and independent corresponding measurement results c_{im} , the total specific risk of underestimation is:

the total probability of a false decision concerning conformity of the compartment as a whole

(the *total* specific R_{total}^* or *total* global R_{total} risk) might still be significant.

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$$R_{\text{total(u)}}^* = R_{c1(u)}^* + R_{c2(u)}^* + R_{c3(u)}^* - R_{c1(u)}^* R_{c2(u)}^* - R_{c1(u)}^* R_{c3(u)}^* - R_{c2(u)}^* R_{c3(u)}^* + R_{c3(u)}^* R_{c3($$

$$70 R_{c1(u)}^* R_{c2(u)}^* R_{c3(u)}^*. (1)$$

71

- 72 E.g., for all the particular specific risks $R_{ci(u)}^* = 0.05$, the total specific risk by formula (1) is
- $R_{\text{total}}^* = 0.14$. Total global risk of underestimation for the three pollutants is:

74

- 75 $R_{\text{total(u)}} =$
- 76 $P(C_2)P(C_3)R_{c1(u)} + P(C_1)P(C_3)R_{c2(u)} + P(C_1)P(C_2)R_{c3(u)} P(C_3)R_{c1(u)}R_{c2(u)} -$
- 77 $P(C_2)R_{c1(u)}R_{c3(u)} P(C_1)R_{c2(u)}R_{c3(u)} + R_{c1(u)}R_{c2(u)}R_{c3(u)},$ (2)

- where $P(C_i)$ is the probability that a measurement result c_{im} is acceptable, i.e. $c_{im} \le T_{Ui}$. For
- 80 example, for the particular risks $R_{ci} = 0.05$ and probabilities $P(C_i) = 0.90$ for all i, formula (2)
- 81 gives $R_{\text{total}} = 0.12$.
- General expressions for evaluating the total risk of underestimation for any number n of the
- 83 material components (or pollutants of an environmental compartment) are also provided in the
- 84 mentioned above reference. Treatment of correlated measurement results for total risk evaluation is
- discussed in the paper by Kuselman et al. (2017b).
- In the present paper, the total risk of overestimation (producer's risk) is formulated in the same
- 87 Bayesian framework for uncorrelated test results as it was applied in the previous work (Kuselman
- 88 et al., 2017a) for underestimation (consumer's risk). Core code developed in R programming
- 89 environment (the R project, Online) for corresponding calculations is also provided. As a case
- 90 study, total risk values are calculated for conformity assessment of concentration of total suspended
- 91 particulate matter (TSPM) in ambient air from three independent stone quarries in Israel. In this

study TSPM contributed by the i-th quarry, i = 1, 2, 3, is considered as the i-th pollutant. While particular risk values of false decisions on conformity of the i-th TSPM concentration, evaluated earlier (Kuselman et al., 2012a), were related to each i-th pollutant (i-th quarry) separately, the total risk values discussed below allow characterization of conformity of the TSPM concentration in the region of the quarries as a whole. That is important as for the Regulator (the Ministry of Environmental Protection, Online) protecting the inhabitants' quality of life in the area surrounding the quarries, as for the Manufacturers Association (Online) acting in the interests of the stone producers in the country.

2. Methods

2.1. Raw data

105 2.1.1. Test method and likelihood functions

A measured TSPM concentration in ambient air c_{im} , mg m⁻³, is an averaged mass of particles with aerodynamic diameters of 100 μ m or less collected from the air drawn through a filter in a high-volume sampler over the sampling period of the test in proximity to the *i*-th stone quarry. The testing was organized at a distance of (1-3) km from each quarry during the quarry' work. Each test lasted 24 hours for collection of particles from about 2000 m³ of air (EPA IO-2.1, 1999). The distribution of the test/measurement results c_{im} at the actual concentration c_i was found to be normal with standard deviation equal to the standard measurement uncertainty u_i =

114 0.07 c_{im} and mean equal to c_i (Kuselman et al., 2012a). Corresponding likelihood functions are normal probability density functions (pdfs):

$$f(c_{im}|c_i) = \frac{1}{u_i \sqrt{2\pi}} \exp\left[-\frac{(c_{im} - c_i)^2}{2u_i^2}\right].$$
 (3)

2.1.2. Database and prior distributions of actual concentration values

The database of 496 test results obtained during a year and described in the work of Kuselman et al. (2012a) is considered again in the present paper. On the basis of the analysis of variances (ANOVA), it was shown that the wind from the desert did not influence the test results significantly, whereas anthropogenic contributions to TSPM concentration were dominant. No correlation among test results for different quarries was observed. The theoretical distributions of actual values of TSPM concentration c_i , fitting successfully the data collected close to quarry i, were lognormal distributions, used in the following as prior pdfs:

$$f(c_i) = \frac{1}{c_i \sigma_i \sqrt{2\pi}} \exp\left[-\frac{(\ln c_i - \mu_i)^2}{2\sigma_i^2}\right],\tag{4}$$

where standard deviations σ_i and means μ_i are for the first quarry (i=1) 0.434 and -2.326, respectively, on the logarithmic scale; for the second quarry (i=2) they are 0.280 and -2.031, respectively; and for the third quarry $\sigma_3 = 0.403$ and $\mu_3 = -2.338$.

2.2. Regulation and acceptance limits

There are national regulations of ambient air quality including upper regulation limits T_{Ui} for TSPM concentration depending on the period of sampling. In Israel, $T_{Ui} = 0.200$ mg m⁻³ for 24 hours, i.e. the same limit value is valid for any location in the country, also close to the *i*-th quarry.

Besides the regulation limit, a lower/stricter acceptance limits A_i could be applied for the test results with the purpose of decreasing the underestimation (inhabitant's) risks due to measurement uncertainty u_i . In such a case, the decision rules (is the air conforming or not?) are based on comparing the test results with the relevant *i*-th acceptance limit (JCGM 106, 2012; Ellison and Williams, 2007). The acceptance limits in the present study are taken as coincidental

2.3. Particular risks of under- and overestimation

148 2.3.1. Particular specific risks

with the regulation limits.

The particular specific risks of the pollutant concentration under- and overestimation are respectively

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$$R_{ci(u)}^* = \int_{T_{Ui}}^{\infty} f(c_i|c_{im}) dc_i$$
, for $c_{im} \le T_{Ui}$, and (5a)

155
$$R_{ci(0)}^* = \int_0^{T_{Ui}} f(c_i|c_{im}) dc_i$$
, for $c_{im} > T_{Ui}$, (5b)

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- where $f(c_i|c_{im})$ is the posterior pdf for the actual value of the TSPM concentration c_i
- 158 contributed by the *i*-th quarry, given the measurement result near the quarry c_{im} . From Bayes
- Law the posterior pdf is

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161 $f(c_i|c_{im}) = f(c_{im}|c_i)f(c_i) / \int_{-\infty}^{\infty} f(c_{im}|c_i)f(c_i) dc_i,$ (5c)

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where $f(c_{im}|c_i)$ is the likelihood function by eqn (3) and $f(c_i)$ is the prior pdf by eqn (4).

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165 2.3.2. Particular global risks

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- The global risks of c_i under- and overestimation related to the TSPM regulation limit T_{Ui} , are
- 168 respectively

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170 $R_{ci(u)} = \int_{T_{Ui}}^{\infty} \int_{0}^{T_{Ui}} f(c_{im}|c_i) f(c_i) dc_{im} dc_i,$ (6a)

171

172 $R_{ci(0)} = \int_0^{T_{Ui}} \int_{T_{Ui}}^{\infty} f(c_{im}|c_i) f(c_i) dc_{im} dc_i.$ (6b)

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174 2.3.3. Probabilities of an acceptable test result and a conforming actual concentration value

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- Probability $P(C_i)$ of a conforming test/measurement result for the *i*-th pollutant $(c_{im} \le A_i =$
- T_{Ui}) is calculated by marginalization of the joint pdf of the measurement results and the actual
- values of TSPM concentration:

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180
$$P(C_i) = \int_0^\infty \int_0^{T_{Ui}} f(c_{im}|c_i) f(c_i) dc_{im} dc_i$$
 (7a)

181

Probability $P(\overline{B}_i)$ that the actual concentration value for the *i*-th pollutant is conforming

183 $(c_i \le T_{Ui})$ is calculated as:

184

185
$$P(\overline{B}_i) = \int_0^{T_{\text{U}i}} f(c_i) \, \mathrm{d}c_i \,. \tag{7b}$$

186

Note that the probability $P(\overline{B}_i)$ of a conforming actual (true) value c_i in eqn (7b) does not

depend on the measurement result c_{im} by definition. However, the vice versa holds: probability

189 $P(C_i)$ of a conforming measurement result c_{im} by eqn (7a) does depend on the relevant actual

190 value c_i .

191

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3. Modeling and calculation

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194 3.1. Total risks of overestimation

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196 3.1.1. Events

- Define the following events possible during testing concentrations of two or more pollutants
- in an environmental compartment:
- 200 \overline{B}_i : the actual concentration c_i of pollutant i does not exceed its regulation limit T_{Ui} ;
- 201 probability of this event $P(\overline{B}_i)$ is defined by formula (7b).

10

- \overline{B} : the actual concentration values c_i for any i do not exceed their own regulation limits T_{Ui} , $\overline{B} = \overline{B}_1 \cap \overline{B}_2 \cap ... \cap \overline{B}_n$; probability of this event is $P(\overline{B}) = \prod_{i=1}^n P(\overline{B}_i)$, if \overline{B}_i are mutually independent.
- B_i: the actual concentration c_i of pollutant i exceeds T_{Ui} , i.e. violates it; probability of this event is $P(B_i) = 1 P(\overline{B}_i)$.
- B: the actual concentration values c_i of one or more pollutants exceed their regulation limits T_{Ui} , $B = B_1 \cup B_2 \cup ... \cup B_n$; probability of this event is $P(B) = 1 P(\overline{B}) = 1 D(\overline{B}) = 1 D(\overline{B})$.
- 210 C_i : the test result c_{im} for i-th pollutant does not exceed its acceptance limit A_i ; probability of this event $P(C_i)$ is defined by formula (7a).
- C: the test results c_{im} for any i do not exceed their own acceptance limits A_i , $C = C_1 \cap C_2 \cap ... \cap C_n$; probability of this event is $P(C) = \prod_{i=1}^n P(C_i)$, if C_i are mutually independent.
- 215 \overline{C}_i : the test result c_{im} for i-th pollutant exceeds its acceptance limit A_i , i.e. such c_{im} is an out-of-specification test result (Kuselman et al., 2012b) as $A_i = T_{Ui}$ in the present study; probability of this event is $P(\overline{C}_i) = 1 P(C_i)$.
- 218 $\overline{\mathbb{C}}$: one or more test results c_{im} exceed their own A_i , $\overline{\mathbb{C}} = \overline{\mathbb{C}}_1 \cup \overline{\mathbb{C}}_2 \cup ... \cup \overline{\mathbb{C}}_n$; probability of this event is $P(\overline{\mathbb{C}}) = 1 P(\mathbb{C}) = 1 \prod_{i=1}^n P(\mathbb{C}_i)$.

221 3.1.2. Total specific risk

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When a specified environmental compartment is tested concerning concentrations of three pollutants, the total specific risk of overestimation $R_{\text{total(o)}}^*$ is the probability that the actual

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- concentrations of all pollutants in this compartment conform to their regulation limits ($\overline{B} = \overline{B}_1 \cap$
- 226 $\overline{B}_2 \cap \overline{B}_3$), whereas one or more test/measurement results c_{1m} , c_{2m} and c_{3m} exceed their
- acceptance limits. This event can occur when:
- 228 a) Just one measurement result out of the three, for example c_{1m} without losing generality,
- exceeds its acceptance limit, while the actual concentration c_1 does not exceed the
- regulation limit. In this case, the actual concentration c_1 will be overestimated. Hence, the
- total risk that the compartment is falsely considered as not conforming is equal to the
- particular specific risk concerning the first pollutant: $R_{\text{total(o)}}^* = P(\overline{B}_1|c_{1\text{m}}).$
- b) Two measurement results, e.g. c_{1m} and c_{2m} , exceed their acceptance limits. The total risk
- is $R_{\text{total(o)}}^* = P(\overline{B}_1 \cap \overline{B}_2 | c_{1m}, c_{2m}).$
- c) All the three measurement results exceed their acceptance limits. The total risk is
- 236 $R_{\text{total(o)}}^* = P(\overline{B}|c_{1m}, c_{2m}, c_{3m}) = P(\overline{B}_1 \cap \overline{B}_2 \cap \overline{B}_3 | c_{1m}, c_{2m}, c_{3m}).$
- If the events \overline{B}_i are conditionally independent, i.e. independent of the measurement results c_{im} ,
- 238 the total specific risk in each of the three considered situations is, respectively:

239

240 a)
$$R_{\text{total(o)}}^* = P(\overline{B}_1|c_{1m}),$$
 (8a)

241 b)
$$R_{\text{total(o)}}^* = \prod_{i=1}^2 P(\bar{B}_i | c_{\text{im}}),$$
 (8b)

242 c)
$$R_{\text{total(o)}}^* = \prod_{i=1}^3 P(\bar{B}_i | c_{\text{im}}),$$
 (8c)

243

- 244 where $P(\bar{B}_i|c_{im}) = R_{ci(o)}^*$ by formula (5b).
- For any number n of pollutants, $v \le n$ of which are characterized by the measurement results
- exceeding their acceptance limits, the total specific risk of overestimation is

248
$$R_{\text{total(o)}}^* = \prod_{i=1}^{v} R_{ci(o)}^*.$$
 (9)

Note again that $R_{ci(0)}^*$ in eqn (9) are related to the out-of-specification measurement results of concentrations of the pollutants, sorted as the first v from all n pollutants under control.

From eqn (9) it follows that any one of v particular specific risk of overestimation $R_{ci(o)}^*$ equal to zero will lead to $R_{total(o)}^* = 0$. That occurs when the actual concentration of the i-th pollutant c_i exceeds/violates the regulation limit unquestionably $(c_i > T_{Ui})$ at a given measurement result $c_{im} > T_{Ui}$ for this pollutant. In such a case, which does not depend on measurement results of concentrations of the other pollutants, the compartment as a whole is certainly not conforming. Therefore, the producer(s) should take action to reduce the i-th pollutant concentration and/or to pay a fine.

In the opposite case of a particular specific risk value $R_{ci(o)}^* = 1$, although c_{im} exceeds its acceptance limit, the actual concentration c_i certainly conforms. Such $R_{ci(o)}^*$ would not influence the total specific risk $R_{total(o)}^*$ by eqn (9). In this case, the number n of pollutants is de-facto decreased by one.

Another property of eqn (9) is reduction of $R_{\text{total(o)}}^*$ with increasing number v of pollutants for which the measurement results are out-of-specification. The logic is that the more such measurement results, the smaller is the total probability of the overestimation. Thus, the greater is the probability that the compartment as a whole does not conform.

Note also that the model used in the work of Subaric-Leitis (2010) and adopted later in the EURAMET guide (Pendrill et al., 2015) leads to an expression equivalent to eqn (9) when the variables (concentrations of the pollutants in our task) are independent, hence validating the model proposed in the present work.

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272 3.1.3. Total global risk

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- Particular global risk $R_{ci(0)}$ of overestimation for the *i*-th pollutant (i = 1, 2, 3) is the
- 275 probability of false nonconformance when the corresponding test result exceeds its acceptance
- limit A_i , while the actual value does not exceed the regulation limit T_{Ui} :

277

$$R_{ci(0)} = P(\overline{B}_i \cap \overline{C}_i). \tag{10}$$

279

- The total global risk $R_{total(o)}$ of overestimation is the risk of having the actual concentrations
- of the three pollutants within their regulation limits T_{Ui} , when at least one of test results are
- outside its acceptance limits (that is outside the three-dimensional domain $A_1 \times A_2 \times A_3$), i.e.
- 283 $R_{\text{total(o)}} = P(\overline{B} \cap \overline{C})$, where

284

$$285 \qquad \overline{B} \cap \overline{C} = \overline{B}_1 \cap \overline{B}_2 \cap \overline{B}_3 \cap (\overline{C}_1 \cup \overline{C}_2 \cup \overline{C}_3) = (\overline{B}_1 \cap \overline{B}_2 \cap \overline{B}_3 \cap \overline{C}_1) \cup (\overline{B}_1 \cap \overline{B}_2 \cap \overline{B}_3 \cap \overline{C}_2) \cup (\overline{B}_1 \cap \overline{B}_2 \cap \overline{B}_3 \cap \overline{C}_3) \cup (\overline{B}_1 \cap \overline{C}_3 \cap \overline{C}_3) \cup (\overline{C}_3 \cap \overline{C}_3 \cap \overline{C}_3 \cap \overline{C}_3 \cap \overline{C}_3 \cap \overline{C}_3) \cup (\overline{C}_3 \cap \overline{C}_3 \cap \overline{C}_3$$

$$\overline{B}_2 \cap \overline{B}_3 \cap \overline{C}_3). \tag{11}$$

287

288 The total global risk of overestimation is thus:

289

$$290 \qquad R_{\mathsf{total}(\mathsf{o})} = P(\overline{\mathsf{B}}_1 \cap \overline{\mathsf{B}}_2 \cap \overline{\mathsf{B}}_3 \cap \overline{\mathsf{C}}_1) + P(\overline{\mathsf{B}}_1 \cap \overline{\mathsf{B}}_2 \cap \overline{\mathsf{B}}_3 \cap \overline{\mathsf{C}}_2) + P(\overline{\mathsf{B}}_1 \cap \overline{\mathsf{B}}_2 \cap \overline{\mathsf{B}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{B}}_1 \cap \overline{\mathsf{B}}_2 \cap \overline{\mathsf{B}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{B}}_1 \cap \overline{\mathsf{B}}_3 \cap \overline{\mathsf{C}}_3) + P(\overline{\mathsf{B}}_1 \cap \overline{\mathsf{B}}_2 \cap \overline{\mathsf{B}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{B}}_1 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{B}}_1 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3) - P(\overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_3 \cap \overline{\mathsf{C}}_$$

- $291 \quad \overline{\mathbb{B}}_2 \cap \overline{\mathbb{B}}_3 \cap \overline{\mathbb{C}}_1 \cap \overline{\mathbb{C}}_2) P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{B}}_2 \cap \overline{\mathbb{B}}_3 \cap \overline{\mathbb{C}}_1 \cap \overline{\mathbb{C}}_3) P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{B}}_2 \cap \overline{\mathbb{B}}_3 \cap \overline{\mathbb{C}}_2 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{B}}_2 \cap \overline{\mathbb{B}}_3 \cap \overline{\mathbb{C}}_2 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{B}}_2 \cap \overline{\mathbb{B}}_3 \cap \overline{\mathbb{C}}_2 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{B}}_2 \cap \overline{\mathbb{B}}_3 \cap \overline{\mathbb{C}}_2 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{B}}_1 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{C}}_1 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{C}}_1 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3 \cap \overline{\mathbb{C}}_3) + P(\overline{\mathbb{C}}_1 \cap \overline{\mathbb{C}}_3 \cap \overline$
- $\overline{B}_2 \cap \overline{B}_3 \cap \overline{C}_1 \cap \overline{C}_2 \cap \overline{C}_3). \tag{12}$

14

- Whenever \overline{B}_1 , \overline{B}_2 and \overline{B}_3 , as well as \overline{C}_1 , \overline{C}_2 , and \overline{C}_3 , are mutually independent, events $\overline{B}_1 \cap \overline{C}_1$,
- 295 $\overline{B}_2 \cap \overline{C}_2$ and $\overline{B}_3 \cap \overline{C}_3$ are also independent and equation (12) can be rewritten using notation (10)
- in the following way:

297

- 298 $R_{\text{total(o)}} =$
- $299 \qquad P(\overline{\mathbb{B}}_2)P(\overline{\mathbb{B}}_3)R_{c1(0)} + P(\overline{\mathbb{B}}_1)P(\overline{\mathbb{B}}_3)R_{c2(0)} + P(\overline{\mathbb{B}}_1)P(\overline{\mathbb{B}}_2)R_{c3(0)} P(\overline{\mathbb{B}}_3)R_{c1(0)}R_{c2(0)} P(\overline{\mathbb{B}}_3)R_{c3(0)} P(\overline{\mathbb{B}_3)R_{c3(0)} P(\overline{\mathbb{B}}_3)R_{c3(0)} P($
- 300 $P(\overline{B}_2)R_{c1(0)}R_{c3(0)} P(\overline{B}_1)R_{c2(0)}R_{c3(0)} + R_{c1(0)}R_{c2(0)}R_{c3(0)}.$ (13)

301

- Note that eqn (13) is similar to eqn (2) for the total global risk of underestimation. However,
- it involves probabilities of different events and different particular risks.
- 304 In general, for any number n of pollutants

305

- $R_{\text{total(o)}} =$
- 307 $\sum_{i=1}^{n} (\prod_{l \neq i} P(\overline{B}_l)) R_{ci(o)} -$
- 308 $\sum_{i=1}^{n} \sum_{j>i} \left(\prod_{l \neq i,j} P(\overline{B}_l) \right) \left(\prod_{q=i,j} R_{cq(0)} \right) +$
- 310 $(-1)^{n-1} \prod_{q=1}^{n} R_{cq(0)},$ (14)

311

where i, j, k, l and q are subscripts of the pollutant in the range (1, ..., n).

313

3.2. Calculation

316	When the likelihood function is a normal distribution and the prior pdf is lognormal, the
317	posterior pdf cannot be easily described by an analytical closed form. Therefore, the posterior
318	pdf was obtained by numerical integration (and subsequent normalization) of the product of the
319	prior and the likelihood. The under- and overestimation particular risks were calculated as the
320	fraction of the (approximated) posterior pdf lying outside/inside the tolerance limit, respectively.
321	Core code developed in R programming environment for calculation of the risks is reported in
322	Annex A. Calculation of total specific risks of under- and overestimation by eqns (1) and (8),
323	respectively, using corresponding particular specific risk values by eqns (5), is shown in Section
324	A-1. Time spent for calculation of the total specific risks with a regular PC (Intel® Core TM i5-
325	3470 Processor, CPU @ 3.20 GHz, Windows 7 Professional 64 bit) is about one second. While
326	increasing (doubling, for example) the number of the involved components does not affect the
327	calculation time, decreasing the numerical integration parameter (stepsize) from 0.001 to 0.0001,
328	increases the execution time up to 6 seconds.
329	Calculation of total global risks of under- and overestimation by eqns (2) and (13),
330	respectively, using particular global risk values by eqns (6), probabilities of conforming
331	measurement results by eqn (7a) and probabilities of conforming actual concentration values by
332	eqn (7b), is detailed in Section A-2. Time spent for calculation of the total global risks with the
333	same PC is about 5 seconds. In this case, doubling the number of components doubles the
334	required time, whereas decreasing the integration parameter (step) from 0.00001 to 0.000001
335	increases the computational time up to about 37 seconds.

4. Results and discussion

4.1. Total specific risks of under- and overestimation

 c_{1m} = 0.194 mg m⁻³ (indicated in Fig 2a by dotted lines).

340

361

339

341 Dependence of the total specific risks of underestimation of TSPM concentration in air on the measurement results c_{im} is demonstrated in Fig. 2. A case when only the first quarry is active and 342 Fig. 2 the total risk $R_{\text{total(u)}}^*$ equals to the particular risk $R_{c1(u)}^*$, is shown in Fig. 2a by solid line 1. Dotted lines 3 and 2 point a measured TSPM concentration $c_{1m} = 0.194$ mg m⁻³ and 344 corresponding risk value $R_{c1(u)}^* = 0.211$, as an instance. One can see in Fig. 2a that $R_{c1(u)}^*$ is 345 close to zero (negligible) at $c_{1m} < 0.170$ mg m⁻³, however significantly increasing with c_{1m} 346 approaching the tolerance limit $T_{\rm U1} = 0.200 \text{ mg m}^{-3}$. 347 348 A case when only the second and the third quarries are active, is represented in Fig. 2b, where the total risk, $R_{\text{total(u)}}^*$, shown as a surface, depends on both c_{2m} and c_{3m} in the range [0.010, 349 0.200] mg m⁻³. The surface lies mostly on the bottom of the three-dimensional region where 350 351 $R_{\text{total(u)}}^*$ is close to zero, as in Fig. 2a, increasing with c_{2m} and c_{3m} approaching their tolerance limits $T_{U1} = T_{U2} = 0.200$ mg m⁻³. When both c_{2m} and c_{3m} simultaneously approach 0.200 mg m⁻³, 352 353 this leads to a 'protuberance' in the total risk surface. The same dependence of $R_{\text{total(u)}}^*$ on c_{2m} and c_{3m} is observed when all the three quarries are 354 active simultaneously, but $c_{1m} < 0.170 \text{ mg m}^{-3}$: the contribution of the particular risk $R_{c1(u)}^*$ to the 355 356 total one in such a case is negligible as shown in Fig. 2a. For comparison, Fig. 2c illustrates a scenario when all the three quarries are active and $R_{\text{total(u)}}^*$ - the surface - is depending on c_{2m} 357 and c_{3m} in the range [0.010, 0.200] mg m⁻³ as in Fig. 2b, whereas $c_{1m} = 0.194$ mg m⁻³. Fig. 2c 358 359 seems very similar to Fig. 2b. However, the color scales of the $R_{\text{total(u)}}^*$ surfaces are different, since the scale in Fig. 2c is greater because of the significant contribution of $R_{c1(u)}^* = 0.211$ at 360

362	Dependence of the total specific risks of overestimation of the actual TSPM concentration in
Fig. 3	air on measurement results, when they are out-of-specification ($c_{im} > T_{Ui}$), is detailed in Fig. 3.
364	A case when only the first quarry is active, and the total risk $R_{\text{total}(o)}^*$ is equal to the particular
365	risk $R_{c1(0)}^*$, is shown in Fig. 3a by solid line 1. Dotted lines 3 and 2 point a measured TSPM
366	concentration $c_{1m} = 0.250$ mg m ⁻³ and corresponding risk value $R_{c1(0)}^* = 0.008$, as an example.
367	Naturally, the risk of overestimation increases as $c_{\rm 1m}$ approaches 0.200 mg m ⁻³ (the tolerance
368	limit), and is close to zero for $c_{1m} > 0.260 \text{ mg m}^{-3}$.
369	The case when only the second and the third quarries are active, as in Fig 2b, and $R^*_{total(o)}$
370	value depending on both c_{2m} and c_{3m} in the range [0.210, 0.300] mg m ⁻³ , is shown in Fig. 3b. The
371	maximum $R_{\text{total(o)}}^*$ value is observed as c_{2m} and c_{3m} near the tolerance limit simultaneously.
372	Fig. 3c illustrates a case when all the three quarries are active, as in Fig. 2c, but $c_{1m} = 0.250$
373	mg m ⁻³ . The scale of the $R_{\text{total(u)}}^*$ surface, shown by the color bar, is two orders less than in Fig.
374	3b. The reason is that the total risk of overestimation, defined as a product of the three particular
375	risks, is influenced by the contribution of $R_{c1(0)}^* = 0.008$ at $c_{1m} = 0.250$ mg m ⁻³ (indicated in Fig
376	3a by dotted lines). In other words, if an out-of-specification measurement result is significantly
377	greater than the tolerance limit, the probability of violation of the regulation is high and the
378	particular risk of overestimation is low. Therefore the total specific risk of overestimation is low
379	also.
380	
381	4.2. Total global risks of under- and overestimation
382	
383	The particular global risks of underestimation $R_{c1(u)} = 0.006$, $R_{c2(u)} = 0.010$ and $R_{c3(u)} = 0.006$
384	0.005 obtained here are equal to the values published earlier (Kuselman et al., 2012a). They are

used as a part of the validation process of the current calculations. The probabilities of conforming measurement results are $P(C_1) = 0.949$, $P(C_2) = 0.929$ and $P(C_3) = 0.963$. The total risk of underestimation, evaluated in the present work for the first time, is $R_{\text{total(u)}} = 0.019$, hence greater than the particular risk contributed by each quarry. The particular global risks of overestimation are $R_{c1(0)} = 0.007$, $R_{c2(0)} = 0.015$ and $R_{c3(0)} =$ 0.006. They are also equal to those published by Kuselman et al. (2012a). The probabilities of conforming actual concentration values calculated are $P(\overline{B}_1) = 0.951$, $P(\overline{B}_2) = 0.934$ and $P(\overline{B}_3) = 0.965$. The total risk of overestimation, evaluated in the present work for the first time as well, is $R_{\text{total}(0)} = 0.026$, again greater than each $R_{ci(0)}$. The total risk of overestimation $R_{\text{total(u)}}$, exceeds the total risk of underestimation $R_{\text{total(u)}}$, which implies that there is a reasonable balance between the requirements of an inhabitant's quality of life and the producer's expenditure on environmental protection.

5. Conclusions

Quantification of risks of false decisions in conformity assessment of an environmental compartment due to measurement uncertainty of concentrations of two or more pollutants, is developed. Even if the assessment of conformity for each pollutant in the compartment is successful, the total probability of a false decision concerning the compartment as a whole might still be significant.

A model of the total probability of a false decision, formulated on the basis of the law of total probability, is used for a study of test results of total suspended particulate matter concentration in ambient air from three independent stone quarries in Israel. Total probabilities of

408	underestimation of the particulate matter concentration (total risk of the inhabitants) and
409	overestimation (total risk of the stone producers) are evaluated as a combination of the particular
410	risks of air conformity assessment near to each quarry.
411	It is shown that the total global risk of underestimation of the particulate matter concentration
412	is smaller than the total risk of its overestimation. That is a reasonable balance between the
413	requirements of an inhabitant's quality of life and the producer's expenditure on environmental
414	protection.
415	
416	Acknowledgment
417	
418	This research was supported in part by the International Union of Pure and Applied
419	Chemistry (IUPAC Project 2016-007-1-500).
420	
421	Appendix A. Core of the R code
422	
423	A-1. Calculation of the total specific risks
424	
425 426 427 428	######################################
429 430 431 432 433 434 435	# Input data for the quarries mu1 = -2.326 # Prior location parameter for Q1 mu2 = -2.031 # Prior location parameter for Q2 mu3 = -2.338 # Prior location parameter for Q3 sigma1 = 0.434 # Prior scale parameter for Q1 sigma2 = 0.280 # Prior scale parameter for Q2 sigma3 = 0.403 # Prior scale parameter for Q3

```
436
       Rsigmam = 0.07
                            # Relative measurement uncertainty
437
       TU = 0.2
                            # Tolerance limit
438
439
       # Settings for numerical integrations
440
       stepsize <- 0.001
441
       obsvalues = seq(0.01,TU,stepsize)
442
       postmean = rep(0, length(obsvalues))
443
       poststd = rep(0, length(obsvalues))
444
       Rspec1 = rep(0, length(obsvalues))
445
       Rspec2 = rep(0, length(obsvalues))
446
       Rspec3 = rep(0, length(obsvalues))
447
       c = seq(0,0.5,stepsize)
448
449
       450
       # Consumer specific risk for each observed value in [0.01, TU]
451
       # Normal Likelihood and Lognormal prior
452
453
       #Q1
454
       i = 1
455
       prior <- dlnorm(c, meanlog = mu1, sdlog = sigma1)
456
       logprior <- log(prior)
       for(obs in obsvalues)
457
458
       {
       loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
459
460
       logpos <- logprior + loglik
461
       posterior <- exp(logpos)
       posterior <- posterior/(sum(posterior)*stepsize)</pre>
462
463
       postmean[i] <- sum(posterior*c)*stepsize</pre>
464
       postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
       poststd[i] = sqrt(postvar)
465
466
       Rspec1[i] = stepsize*sum(posterior[c>TU])
467
       i = i+1
468
       }
469
470
       # Q2
471
       i = 1
       prior <- dlnorm(c, meanlog = mu2, sdlog = sigma2)</pre>
472
473
       logprior <- log(prior)
474
       for(obs in obsvalues)
475
476
       loglik < -dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
       logpos <- logprior + loglik
477
478
       posterior <- exp(logpos)
479
       posterior <- posterior/(sum(posterior)*stepsize)</pre>
480
       postmean[i] <- sum(posterior*c)*stepsize
481
       postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
```

```
482
       poststd[i] = sqrt(postvar)
483
       Rspec2[i] = stepsize*sum(posterior[c>TU])
484
       i = i+1
485
       }
486
487
      # Q3
488
      i = 1
489
      prior <- dlnorm(c, meanlog = mu3, sdlog = sigma3)
490
      logprior <- log(prior)
491
      for(obs in obsvalues)
492
493
       loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
494
       logpos <- logprior + loglik
495
       posterior <- exp(logpos)
496
       posterior <- posterior/(sum(posterior)*stepsize)</pre>
497
       postmean[i] <- sum(posterior*c)*stepsize
498
       postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
499
       poststd[i] = sqrt(postvar)
500
       Rspec3[i] = stepsize*sum(posterior[c>TU])
501
       i = i+1
502
       }
503
504
      # Total specific consumer risk for the particular case obs1=obs2=obs3
505
      Rtotu = Rspec1 + Rspec2 + Rspec3 - Rspec1*Rspec2 - Rspec1*Rspec3 - Rspec2*Rspec3 +
506
      Rspec1*Rspec2*Rspec3
507
508
      509
      # Producer specific risk for each observed value in [0.21, 0.3]
510
      # Settings for numerical integrations
511
512
      obsvalues = seq(0.21,0.3,stepsize)
513
      postmean = rep(0, length(obsvalues))
514
      poststd = rep(0, length(obsvalues))
515
      Rspec1 = rep(0, length(obsvalues))
516
      Rspec2 = rep(0, length(obsvalues))
517
      Rspec3 = rep(0, length(obsvalues))
518
519
      #Q1
520
521
      prior <- dlnorm(c, meanlog = mu1, sdlog = sigma1)
522
      logprior <- log(prior)
523
      for(obs in obsvalues)
524
525
       loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
526
       logpos <- logprior + loglik
527
       posterior <- exp(logpos)
```

```
528
        posterior <- posterior/(sum(posterior)*stepsize)</pre>
529
        postmean[i] <- sum(posterior*c)*stepsize</pre>
530
        postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
531
        poststd[i] = sqrt(postvar)
532
        Rspec1[i] = stepsize*sum(posterior[c<=TU])
533
        i = i+1
534
       }
535
536
       # Q2
       i = 1
537
538
       prior <- dlnorm(c, meanlog = mu2, sdlog = sigma2)
539
       logprior <- log(prior)
540
       for(obs in obsvalues)
541
542
        loglik < -dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
543
        logpos <- logprior + loglik
544
        posterior <- exp(logpos)
545
        posterior <- posterior/(sum(posterior)*stepsize)</pre>
546
        postmean[i] <- sum(posterior*c)*stepsize</pre>
547
        postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
548
        poststd[i] = sqrt(postvar)
549
        Rspec2[i] = stepsize*sum(posterior[c<=TU])
550
        i = i+1
551
       }
552
553
       # Q3
554
       i = 1
555
       prior <- dlnorm(c, meanlog = mu3, sdlog = sigma3)
556
       logprior <- log(prior)
557
       for(obs in obsvalues)
558
559
        loglik < -dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
560
        logpos <- logprior + loglik
561
        posterior <- exp(logpos)
        posterior <- posterior/(sum(posterior)*stepsize)</pre>
562
        postmean[i] <- sum(posterior*c)*stepsize</pre>
563
564
        postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
565
        poststd[i] = sqrt(postvar)
        Rspec3[i] = stepsize*sum(posterior[c<=TU])
566
567
        i = i+1
568
       }
569
570
       # Total specific producer risk for the particular case obs1=obs2=obs3
571
       Rtoto = Rspec1*Rspec2*Rspec3
572
```

23

A-2. Calculation of the total global risks

```
574
       ###############
575
576
       # Global risks #
       ###############
577
578
579
       # Input data for the quarries
580
       mu1 = -2.326
                            # Prior location parameter for Q1
581
       mu2 = -2.031
                            # Prior location parameter for Q2
582
                            # Prior location parameter for Q3
       mu3 = -2.338
583
       sigma1 = 0.434
                            # Prior scale parameter for Q1
                            # Prior scale parameter for Q2
584
       sigma2 = 0.280
585
       sigma3 = 0.403
                            # Prior scale parameter for Q3
                            # Relative measurement uncertainty
586
       um = 0.07
587
       T = 0.2
                            # Tolerance limit
588
       A = T
                            # Acceptance limit
589
590
       # Consumer's risk Rc and the producer's risk Rp
591
       # Normal Likelihood and Lognormal prior
592
       # Initializations
593
       step = 0.00001
                                    # Integral domain [T, infinity]
594
       etac = seq(T,10,step)
595
       etap = seq(step, T, step)
                                    # Integral domain [0, T]
       etacinf = seq(step, 10, step)
596
                                    # Integral domain [0, infinity]
597
598
       #Q1
599
       ymeanlogQ1 = mu1
600
       ystdlogQ1 = sigma1
       RcQ1 = sum( (pnorm((A-etac)/(um*etac)) - pnorm(-etac/(um*etac))) *
601
602
       dlnorm(etac,ymeanlogQ1,ystdlogQ1) * step)
603
       PC1 = sum( (pnorm((A-etacinf)/(um*etacinf)) - pnorm(-etacinf/(um*etacinf))) *
       dlnorm(etacinf,ymeanlogQ1,ystdlogQ1) * step)
604
605
       RpQ1 = sum((1-pnorm((A-etap)/(um*etap))) * dlnorm(etap,ymeanlogQ1,ystdlogQ1) * step)
606
       PBcompl1 = plnorm(T,ymeanlogQ1,ystdlogQ1)
       c(RcQ1,RpQ1,PC1,PBcompl1)
607
608
       # [1] 0.005769988 0.007368876 0.949038432 0.950637320
609
610
       # Q2
611
       ymeanlogQ2 = mu2
       ystdlogQ2 = sigma2
612
613
       RcQ2 = sum((pnorm((A-etac)/(um*etac)) - pnorm(-etac/(um*etac))) *
       dlnorm(etac,ymeanlogQ2,ystdlogQ2) * step)
614
       PC2 = sum( (pnorm((A-etacinf)/(um*etacinf)) - pnorm(-etacinf/(um*etacinf))) *
615
616
       dlnorm(etacinf,ymeanlogQ2,ystdlogQ2) * step)
```

24

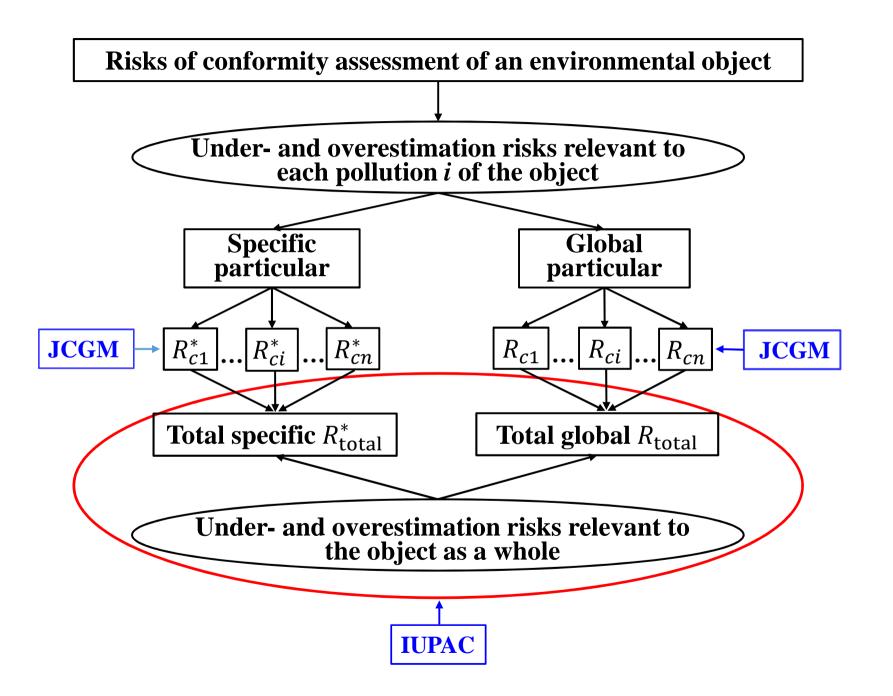
```
617
      RpQ2 = sum((1-pnorm((A-etap)/(um*etap))) * dlnorm(etap,ymeanlogQ2,ystdlogQ2) * step)
618
      PBcompl2 = plnorm(T,ymeanlogQ2,ystdlogQ2)
619
      c(RcQ2,RpQ2,PC2,PBcompl2)
620
      #[1] 0.01045913 0.01525355 0.92911792 0.93391234
621
622
      # Q3
623
      ymeanlogQ3 = mu3
624
      ystdlogQ3 = sigma3
      RcQ3 = sum((pnorm((A-etac)/(um*etac)) - pnorm(-etac/(um*etac))) *
625
626
      dlnorm(etac,ymeanlogQ3,ystdlogQ3) * step)
627
      PC3 = sum( (pnorm((A-etacinf)/(um*etacinf)) - pnorm(-etacinf/(um*etacinf))) *
      dlnorm(etacinf,ymeanlogO3,ystdlogO3) * step)
628
629
      RpQ3 = sum((1-pnorm((A-etap)/(um*etap))) * dlnorm(etap,ymeanlogQ3,ystdlogQ3) * step)
630
      PBcompl3 = plnorm(T,ymeanlogQ3,ystdlogQ3)
631
      c(RcQ3,RpQ3,PC3,PBcompl3)
      #[1] 0.004602961 0.006233814 0.963053939 0.964684793
632
633
634
      # TOTAL global consumer risk (underestimation risk)
635
      c(PC1,PC2,PC3)
636
      #[1] 0.9490384 0.9291179 0.9630539
637
      c(RcO1,RcO2,RcO3)
      #[1] 0.005769988 0.010459133 0.004602961
638
      Rtotu = PC2*PC3*RcQ1 + PC1*PC3*RcQ2 + PC1*PC2*RcQ3 - PC3*RcQ1*RcQ2 -
639
640
      PC2*RcQ1*RcQ3 - PC1*RcQ2*RcQ3 + RcQ1*RcQ2*RcQ3
641
      Rtotu # 0.01865286, for step = 0.00001
642
643
      # TOTAL global producer risk (overestimation risk)
644
      c(PBcompl1,PBcompl2,PBcompl3)
645
      #[1] 0.9506373 0.9339123 0.9646848
646
      c(RpQ1,RpQ2,RpQ3)
647
      #[1] 0.007368876 0.015253553 0.006233814
648
      Rtoto = PBcompl2*PBcompl3*RpQ1 + PBcompl1*PBcompl3*RpQ2 +
649
      PBcompl1*PBcompl2*RpQ3 - PBcompl3*RpQ1*RpQ2 - PBcompl2*RpQ1*RpQ3 -
650
      PBcompl1*RpQ2*RpQ3 + RpQ1*RpQ2*RcQ3
651
      Rtoto # 0.0259206, for step = 0.00001
652
      References
653
654
```

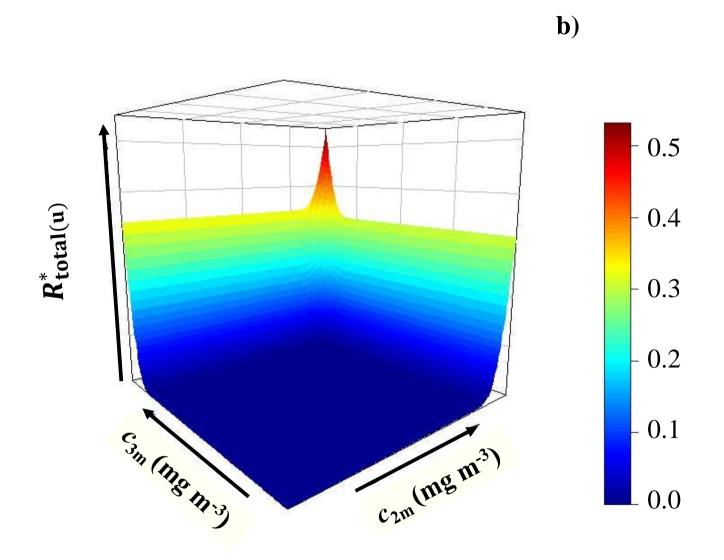
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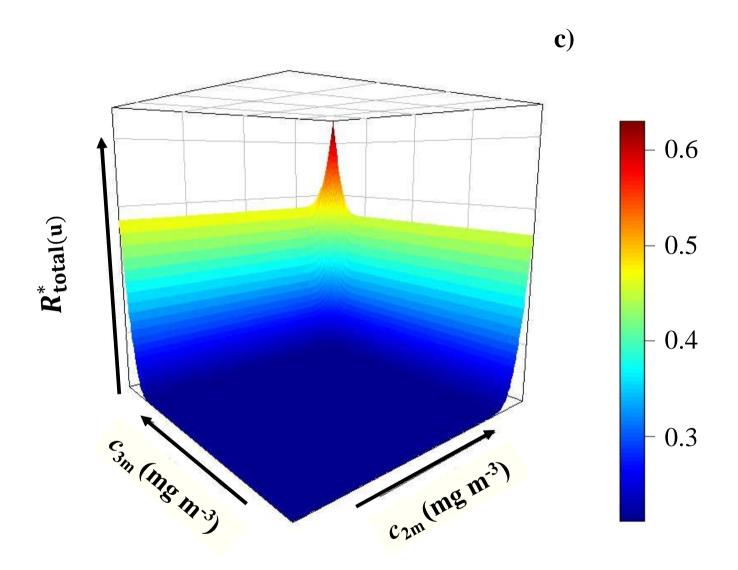
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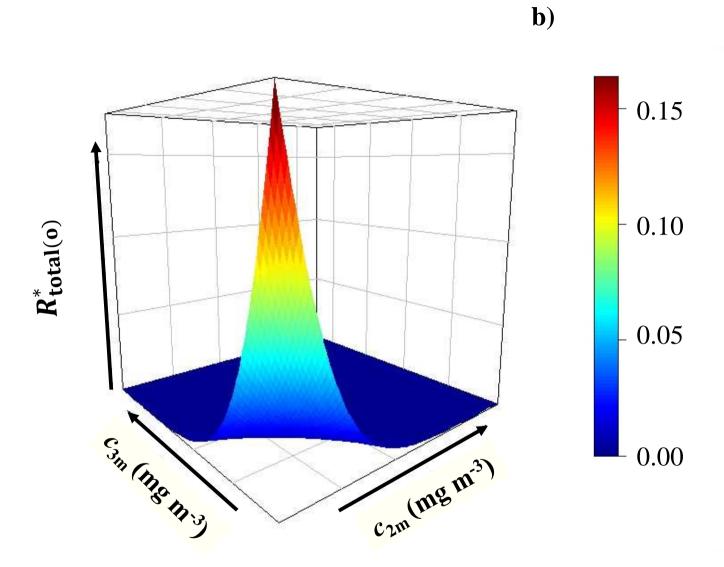
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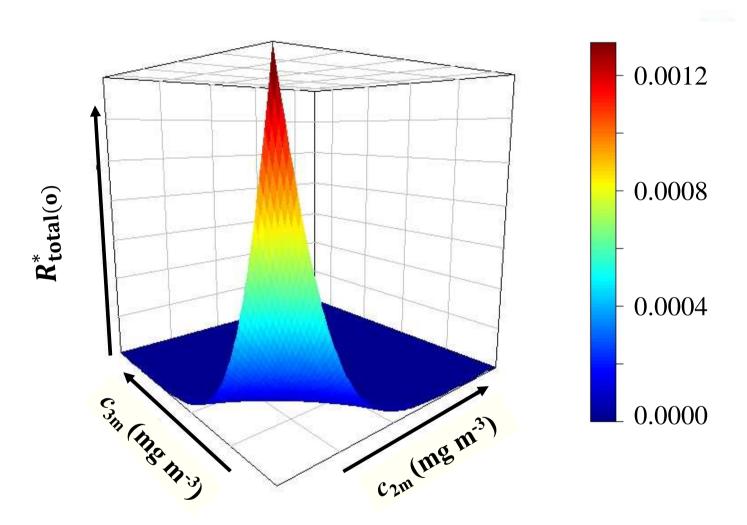


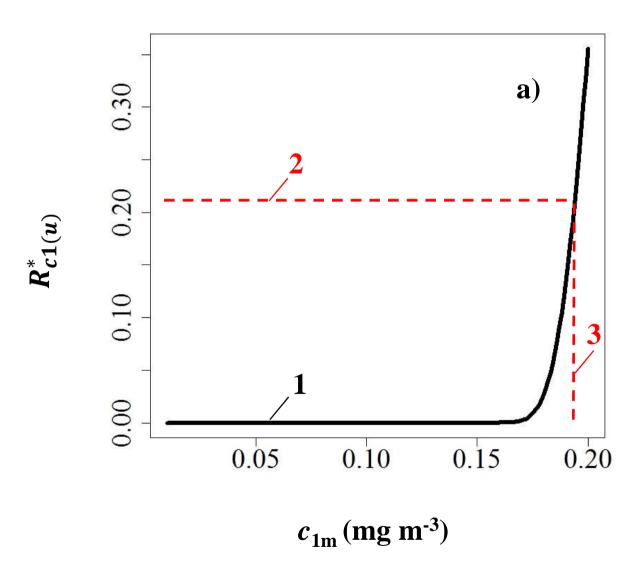












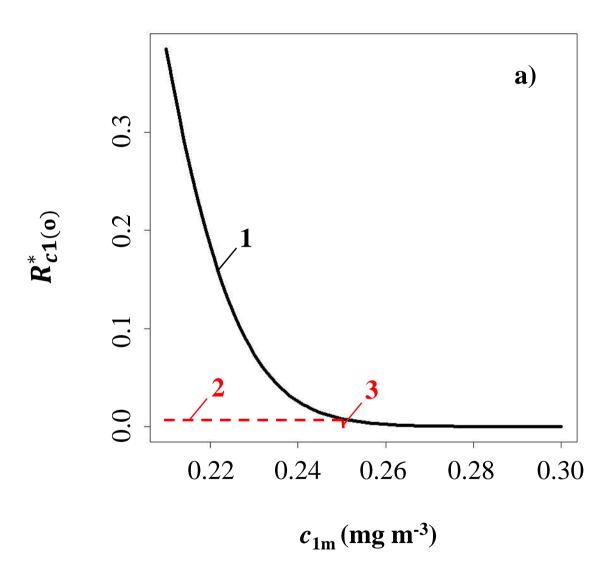


Figure captions

Fig. 1. Classification of the risks in conformity assessment of an environmental compartment due to measurement uncertainty. Specific risk refers to a specified compartment in a certain location at a certain time, whereas global risk – to the population of such compartments. Particular risk (specific R_{ci}^* or global R_{ci}) refers to i-th pollutant of the environmental compartment, i = 1, 2, ..., n, according to the JCGM Guide 106 (2012); and total risk (specific R_{total}^* or global R_{total}) – to the compartment as a whole. The total risk evaluation is the task of the IUPAC Project (2016), highlighted in the figure by an ellipse. These kinds of risks are relevant as for an underestimation of the pollutant concentration c_i , as for its overestimation, i.e. to the consumer' and producer's risks, respectively.

Fig. 2. Dependence of the total specific risks of underestimation $R^*_{\text{total}(\mathbf{u})}$ of TSPM concentration in air on the measurement results c_{im} . Fig. 2a is for a case when only the first quarry is active and the total risk $R^*_{\text{total}(\mathbf{u})}$ is equal to the particular risk $R^*_{c1(u)}$, shown by solid line 1. Dotted lines 3 and 2 point, as an example, a measured TSPM concentration $c_{1m} = 0.194$ mg m⁻³ and corresponding risk value $R^*_{c1(u)} = 0.211$. Fig. 2b is for a case when only the second and the third quarries are active. $R^*_{\text{total}(\mathbf{u})}$, presented as a color surface, is depending on both c_{2m} and c_{3m} in the range [0.010, 0.200] mg m⁻³. The meaning of the color is the total risk value according to the color scale of the bar on the right side of the plot. Fig. 2c illustrates a case when all the three quarries are active and $R^*_{\text{total}(\mathbf{u})}$ - the color surface - is depending on c_{2m} and c_{3m} in the range [0.010, 0.200] mg m⁻³ as in Fig. 2b, but $c_{1m} = 0.194$ mg m⁻³ (indicated in Fig 2a by dotted lines).

Fig. 3. Dependence of the total specific risks of overestimation $R_{\text{total}(o)}^*$ of the TSPM concentration in air on the measurement results c_{im} . Fig. 3a is for a case when only the first quarry is active and the total risk $R_{\text{total}(o)}^*$ is equal to the particular risk $R_{c1(o)}^*$, shown by solid line 1, while dotted lines 3 and 2 point, as an example, a measured TSPM concentration $c_{1m} = 0.250 \text{ mg m}^{-3}$ and corresponding risk value $R_{c1(o)}^* = 0.008$. Fig. 3b is for a case when only the second and the third quarries are active, as in Fig 2b, and the total risk $R_{\text{total}(o)}^*$ value is depending on both c_{2m} and c_{3m} in the range [0.210, 0.300] mg m⁻³. Fig. 3c illustrates a case when all the three quarries are active simultaneously as in Fig. 2c, but $c_{1m} = 0.250 \text{ mg m}^{-3}$ (indicated in Fig 3a by dotted lines).

HIGHLIGHTS

- Evaluation of total risks of false decisions on conformity of an environmental compartment is developed.
- The total risks due to measurement uncertainty of concentrations of two or more pollutants are considered.
- As a case study, the total risks are evaluated at control of total suspended particulate matter (TSPM) concentration in air.
- The study concerns three independent stone quarries as pollutant sources.
- The total probabilities of under- and overestimation of TSPM concentration in air are calculated.